7. Baseband Digital Transmission

Baseband transmission is the simplest form for the communication of information. Discrete information is communicated with specific symbols selected from a finite set of symbols. In baseband transmission, symbols are simply communicated as a pulse with a discrete voltage level and, for binary transmission, only two voltages are used. A series of pulses forms a pulse train that carries the full message. Prior to transmission, especially in radio systems, these pulses are shaped to limit their high frequency content so as to minimize crosstalk with adjacent communication channels. During transmission through a bandlimited channel, pulses are dispersed (spread) in time and can overlap with each other giving rise to intersymbol interference (ISI). When pulses reach the receiver, dispersion and other distortions can be partially compensated with an equalizer.

7.1 Impulse Response in a Bandlimited Channel

We first consider a series of narrow symbol pulses. Restricted channel bandwidth, disperses (or spreads) the pulse in time and necessitates an increased interval between symbols. The maximum rate at which symbols can be sent is proportional to channel bandwidth. Pulse dispersion relates directly to channel impulse response which can be determined through the Fourier transform of $H(f)$, the channel frequency response.

We now consider a single rectangular pulse with amplitude $h$ and interval $\tau$. The amplitude spectrum of the pulse is determined using the Fourier transform. As illustrated below, the spectrum has magnitude $h\tau$ at zero frequency and a $\sin(\pi \tau f / \pi \tau f)$ variation with frequency. In addition to amplitude spectrum, there is a phase spectrum (not shown) that has values 0 and $\pi$ when the pulse is centered on the $t = 0$ axis. Note the spectral nulls at $f = 1/\tau, 2/\tau, 3/\tau, \ldots$.

![Figure 7-1 Fourier transform of a single pulse waveform.](image)

Note that if the weight (i.e. area) of the pulse, $h\tau$, is held constant while the width is decreased, the spectral peak remains constant at $h\tau$ and the spectrum spreads out in frequency. Taken to the limit, the pulse approaches an impulse and the spectrum becomes "flat".

Next we consider a narrow rectangular pulse passing through a bandlimited transmission system modeled by an ideal lowpass filter (LPF). If the input pulse spectrum is approximately
“flat”, and the transmission system has a rectangular “brickwall” frequency response, the receiver will see a rectangular spectrum centered about 0 Hz. Applying the inverse Fourier transform to this rectangular spectrum, we determine that the received voltage response has peak amplitude $2f_c h \tau$ and $\sin \frac{\pi f c t}{\pi f c t}$ variation with time.

![Figure 7-2 Impulse response of a bandlimited channel](image)

Note that the impulse response tails oscillate at the LPF cutoff frequency. This can be related to a practical low pass filter where transfer function poles nearest the cutoff frequency have the lowest damping ratio (i.e. high Q) and, following an impulse, the filter continues to oscillate at the cutoff frequency because of the high Q poles.

**Example 7.1** - An impulse of amplitude 5 kV and duration 5 ns is input to a 50 ohm cable which includes an inline LPF with unity gain and 5 MHz cutoff frequency. The cable is terminated with a 50 ohm load. Determine the amplitude and zero crossing interval of the impulse response at the load. Show that the weight (or area) of the output response is independent of the LPF cutoff frequency.

**Solution:**

The impulse weight, $h \tau = (5 \text{kV})(5 \text{ ns}) = 25 \mu \text{V-s}$

Peak voltage of impulse response = $2f_c h \tau = 2(5 \text{MHz})(25 \mu \text{V-s}) = 250 \text{ V}$

Zero crossing interval (in the tails) = $1/2f_c = 1/10 \text{ MHz} = 100 \text{ ns}$

Area (or weight) $= \int_{-\infty}^{\infty} 2f_c h \tau \frac{\sin \pi f_c t}{\pi f_c t} dt$

$= \frac{2f_c h \tau}{\pi f_c} \left[ \frac{\sin \pi f_c t}{\pi f_c t} \right]_0^{\infty}$

$= \frac{2f_c h \tau}{\pi f_c} \int_0^{\pi/2} \frac{\sin x}{x} dx$

$= \frac{h \tau}{2} \left( \frac{\pi}{2} \right) = h \tau$ (which is independent of $f_c$)

Note that there is conservation of charge $\Rightarrow$ Impulse ampere-seconds = output response ampere-seconds

The integrals $\int_0^{\pi/2} \frac{\sin x}{x} dx = \frac{\pi}{2}$ and $\int_0^{\infty} \left( \frac{\sin x}{x} \right)^2 dx = \frac{\pi}{2}$ are useful for analysis of the above pulse spectra and impulse response.
7.2 **Maximum Signaling Rate**

Message symbols occur in a sequence at rate $R_s$, called the baud rate of the transmission; one baud equals one symbol per second. With limited channel bandwidth (as in a lowpass filter), symbol pulses are spread out in time (dispersed) and, if interference between successive pulses is to be avoided, there must be a minimum interval between pulses. The maximum symbol rate, $R_{\text{max}}$, is therefore limited by the channel bandwidth. We shall see that by setting the symbol rate equal to twice the lowpass filter bandwidth and by sampling the received signal at appropriate times, we have the maximum symbol rate that can be attained without intersymbol interference.

7.2.1 **Impulse signaling with an ideal filter**

Assume a first impulse (not a rectangular pulse) transmitted through an ideal lowpass channel with unity gain from 0 Hz to the cutoff frequency $f_c$. The received signal (the channel impulse response) will be of the form $\sin(\pi f_c t) / \pi f_c t$ with a main lobe having peak voltage at time $t_1$ and with zero crossings at time intervals $1/2f_c$ about time $t_1$. If the first impulse is followed by a second impulse, transmitted after delay $1/2f_c$, the second impulse response will have a main lobe with voltage peak at time $t_2$ occurring at a zero crossing of the first impulse response. The second symbol may then be received without interference provided that the received signal is sampled precisely at the zero crossing of the first impulse response. Further zero crossings of both impulse responses will be coincident at intervals of $1/2f_c$ and, if additional impulse symbols are transmitted at time intervals of $1/2f_c$, these too can be received without ISI.

Symbols may be communicated without inter-symbol interference at a rate of $f_{sy} = 2f_c$.

This symbol rate, $f_{sy}$, is known as the Nyquist signaling rate.

In Figure 7-3, five impulses with weights +3 and +1 are shown after transmission through a lowpass channel with cutoff frequency at 500 kHz. The received voltage is the superposition of all impulse responses from the transmitted symbol sequence. A sample taken at time $t = 8$ us, for example, contains no output from pulses $p_1$, $p_2$, $p_3$ and $p_5$ and is therefore responsive only to the amplitude of $p_3$. This condition of zero ISI is independent of the other pulse amplitudes.

For zero ISI we must exactly match the channel bandwidth to the reciprocal of the signaling rate. In this case, the channel includes the transmit filter, the transmission link and the receive filter. In practice, we use an interconnecting transmission link with somewhat larger bandwidth and then control the channel bandwidth with accurate transmitter and receiver filters with cutoff at $f_c$. Precise cut-off frequencies can be implemented with clocked digital filters.
7.2.2 Impulse signaling with practical filters

The sharp cut-off, rectangular “brickwall” filters illustrated in the previous section cannot be implemented in practice; the transition from passband to stopband must occur over some frequency range. Filters are implemented with a roll-off that is symmetric about $f_c$ extending up to $(1+r) f_c$ where $r$ is the channel roll-off factor. The transition region characteristic usually approximates the first $180^\circ$ of a raised cosine leading to the moniker “raised cosine filter”. With roll-off factor $r \approx 0.3$, the transmitted spectrum is $30\%$ in excess of what would be transmitted with a “brickwall” filter (i.e. $30\%$ “excess bandwidth”).

The gradual filter transition results shortening the ripple “tails” in the channel impulse response. This reduction of tail amplitude and duration significantly reduces ISI and relaxes the need for precise matching of signaling rate to the zero crossing rate. For small timing offsets, the previous impulse has a zero crossing near the desired sampling time. On the other hand, interfering impulses from the distant past may have displaced zero crossings however their amplitude has now become insignificant.

Figure 7-4 illustrates excess bandwidth and reduced impulse response duration. The filter characteristic follows a raised cosine function in the transition region and has a gain of $0.5$ (-6dB) at the frequency $f_c$. The total transmission bandwidth required is $f_b = (1+r) f_c$.

It is not possible to create a filter with perfectly sharp cut off in the frequency domain - all practical filters must have some excess bandwidth. The raised cosine pulse is defined in frequency by

$$H_2(f) = \begin{cases} 1 & 0 \leq |f| \leq \frac{1-r}{2T} \\ \cos^2 \left[ \frac{\pi T}{2r} \left( f - \frac{1-r}{2T} \right) \right] & \frac{1-r}{2T} < |f| < \frac{1+r}{2T} \\ 0 & \frac{1+r}{2T} \leq |f| \leq \infty \end{cases}$$

Additionally, an ideal filter must have an impulse response that extends to infinite time before and after the pulse peak and, if the filter is to be causal (output response occurs after the input is applied), the peak output would occur an infinite time after the input. This would
certainly not make for a useful communication system since one would need to wait more than a lifetime for the message to reach the receiver.

To this point we have considered filter structures (transversal filters, for example) with constant delay and therefore phase shift that increases linearly with frequency. Recursive digital filters and most analog filters do not have constant delay throughout the passband so the output impulse response is not symmetrical about the peak value. In rough terms, the ripples preceding the main lobe are eliminated and the ripples following the main lobe are approximately doubled in an analog filter impulse response.

Example 7.2 - A wireline transmission channel is modeled with a 1.0 MHz LPF with rolloff factor $r = 0.5$. What is the maximum symbol rate that can be transmitted without intersymbol interference? Above what frequency is there no received power?

**Solution:**

Maximum symbol rate $= 2f_c = 2,000,000$ symbols/sec.

Received power extends up to $(1+r)f_c = 1.5$ MHz.

### 7.2.3 Signaling with rectangular pulses

To this point we have considered the transmission of impulses (extremely short duration and extremely high amplitude). Many digital transmission systems transmit finite amplitude pulses of duration equaling the symbol interval or half the symbol interval. The pulse response presented to the receiver can be estimated by three methods:

a) convolve the impulse response with the transmitted pulse,

b) divide the transmitted pulse into narrow pulses determine the approximate impulse response of each then add (superposition) to get the received pulse response, and

c) differentiate the transmitted pulse to obtain leading and trailing impulses, obtain their impulse responses the integrate to get the pulse response.

![Figure 7-5 Calculation of pulse response](image-url)
Another method of analyzing rectangular transmitted pulses is to consider the process of flat-top sampling and to recall the inverse $\text{sin} \pi f_T / \pi f_T$ filter used to reverse the effects of flat-top reconstruction. With a similar inverse filter in the pulse transmission path, the received pulse response can be made to have the same waveshape (and zero crossings) as the channel impulse response.

Although wireline transmission systems such as Ethernet and digital telephone circuits transmit rectangular pulses directly, more elegant techniques are required for advanced communication systems. Pulses are shaped prior to transmission and this shaping becomes part of the channel frequency response.

### 7.2.4 Signaling with root-raised-cosine pulses

By allowing a little extra bandwidth in the channel’s equivalent lowpass filter, many difficulties associated with the “sinc” channel impulse response have been mitigated. In a communication system, there is need to associate some of this channel equivalent lowpass filter with the transmitter and some with the receiver. Transmit filtering is need to limit high frequency components in the symbol pulses and avoid interference with other communication channels. At the receiver, filtering is needed to remove incoming out-of-band noise prior to the detection process. In several cases, especially in wireless transmission, the physical channel is essentially “flat” and all of the channel filtering occurs in the transmitter and receiver. Since the transfer function of the overall channel is the product of the cascaded transmit filter and receive filter, each filter is implemented with a “root-raised-cosine” characteristic.

To this point, we assume that the transmitter generates an impulse then filters it prior to transmission. The transmitter is substantially simplified by directly generating the root-raised-cosine transmitted pulse. A reasonable truncated approximation of the desired pulse is synthesized from a series of stored samples covering the range 4-6 cycles on either side of the main lobe. A time domain expression [Chennakeshu/Anderson p26] for the root-raised-cosine pulse is

\[
v(t) = \begin{cases} 
\frac{1}{\sqrt{T}} \sin \left[ \pi \left(1 - r\right) t / T + (4 r t / T) \cos \left[ \pi \left(1 + r\right) t / T \right] \right] / \cos \left[ \pi / \left(1 - (4 r t / T)^2\right) \right] & t \neq 0, \quad t \neq \pm \frac{T}{4 r} \\
\frac{1}{\sqrt{T}} \left(1 - r + \frac{4 r}{\pi}\right) & t = 0 \\
\frac{r}{\sqrt{2} T} \left[ \left(1 + \frac{2}{\pi}\right) \sin \frac{\pi}{4 r} + \left(1 - \frac{2}{\pi}\right) \cos \frac{\pi}{4 r} \right] & t = \pm \frac{T}{4 r}
\end{cases}
\]
Example 7.3  
A 19.2 kb/s voiceband data modem has carrier frequency, \( f_0 \), of 1800 Hz and operates in a telephone channel with bandwidth 300 - 3300 Hz. The equivalent channel has unity gain at \( f_0 \) and has zero gain at frequencies outside \( f_0 \pm \Delta f \) where \( \Delta f = 1500 \) Hz. This can be considered as an equivalent baseband LPF centered at zero Hz that eliminates all power above 1500 Hz. What is the cutoff frequency, \( f_c \), for the equivalent raised cosine, \( r = 0.25 \) transmit/receive filter? What is the maximum symbol rate, \( f_s \), that can be transmitted without ISI and how many bits per symbol are required to give a data rate of 19.2 kb/s?

Solution: \( (1+r)f_c = 1500 \) Hz \( \Rightarrow f_c = 1500/1.25 = 1200 \) Hz. 
\( f_s = 2f_c = 2400 \) symbols/sec. \( \Rightarrow \) bits/symbol = 19200/2400 = 8 bits/sy

7.3  **Pulse Spectrum and Eye Patterns**

In this section we extend our study of pulse spectral density to a series of random pulses as would be used for message transmission. This spectrum is truncated by channel filters that are used to smooth the transmitted waveform and to eliminate much of the noise that would enter the receiver. The effect of channel filters on the received signal waveform is studied with the aid of a display known as an eye pattern. We begin by a calculation of the spectrum of a rectangular binary pulse sequence.

7.3.1 **Spectrum of a continuous pulse sequence**

Assume a single rectangular pulse, \( w(t) \), of amplitude \( A \) and period \( T \) centered about \( t = 0 \). The amplitude spectral density \( W(f) \) is calculated using the Fourier Transform and the normalized energy spectral density \( E(f) \) is simply the square of the Fourier Transform.

\[
W(f) = AT \left( \frac{\sin \pi Tf}{\pi Tf} \right) \quad E_w(f) = A^2 T^2 \left( \frac{\sin \pi Tf}{\pi Tf} \right)^2
\]

Since power is energy per unit time, the single pulse power spectral density, \( S(f) \), is

\[
S_w(f) = \frac{E(f)}{T} = A^2 T \left( \frac{\sin \pi Tf}{\pi Tf} \right)^2
\]

In a sequence of pulses with random amplitude, spectral components of the message will add on a power basis. Power spectral density \( S_m(f) \) is the time average of the PSDs for each pulse.

\[
PSD = \langle S_w(f) \rangle = T \left( \frac{\sin \pi Tf}{\pi Tf} \right)^2 \langle A_n^2 \rangle \quad n = 1, 2, 3, \ldots
\]

\[
S_m(f) = A^2 T \left( \frac{\sin \pi Tf}{\pi Tf} \right)^2 \quad \text{if } A_n = A, -A
\]

When the pulse sequence is periodic, the PSD will be a line spectrum rather than a continuous spectrum. A square alternating sequence is a simple example.
7.3.2 Eye pattern measurement

The effect of bandwidth limitation, dispersion, distortion, intersymbol interference and timing impairments can be studied with the aid of an eye pattern display. This display is produced on an oscilloscope by superimposing many traces of the received signal and, when binary data is used, the resulting pattern resembles a human eye. At the sampling instant, the received signal should be well above or below the threshold voltage to ensure reliable detection of a binary one or zero and thus the eye opening indicates margin against possible errors caused by noise. It is evident that the best position for the receiver voltage threshold and sampling instant is in the center of the eye opening.

A test arrangement for generating eye patterns is shown below. The display shows one central symbol plus part of the preceding symbol and part of the following symbol. Since the oscilloscope is triggered by the data generator clock, the superposition of all possible data sequences can be shown on the screen. The eye diagram thus displays the 8 possible trajectories for 3 bits. It is preferable to operate the oscilloscope in storage mode.

7.3.3 Elimination of higher spectral frequencies

Channel lowpass filters eliminate the higher frequency components of the transmitted pulse sequence and smooth the sharp symbol transitions initially assumed. As the filter cut-off frequency approaches one half the symbol rate, the rise times become longer, the pulse broadens and the impulse response extends into the time allocated for the next symbol. This gives rise to the well-known eye diagram, illustrated below in binary form.
7.3.4 Loss of low frequencies

Many transmission systems introduce a component, such as a transformer, that eliminates a small portion of the spectral density extending upwards from zero hertz. This small loss of spectral power seems at first to be innocuous, however, it has a major effect on the eye pattern of an uncoded binary transmission. The beginning point of each trajectory in the eye diagram is influenced by the preamble of bits leading up to the 3 bit times shown in the display. In systems where there is loss of transmission at low frequencies, a positive preamble will result in a negative displacement of the trajectory while a negative preamble will have the opposite effect. This variation in the trajectories reduces the eye opening as illustrated below in Figure 7-8.

Transmission systems are tested with pseudo-random sequences. (These are also known as pseudo-noise (PN) data sequences). Commercial test units provide a variety of PN sequence lengths and one choice might be $2^{11}-1 = 2047$. Most instruments can provide much longer sequences. Although the $2^{11}-1$ sequence repeats every 2047 bits, the longest string of sequential ones (or zeros) is 11. The PN code provides the complete variety of 11-bit preambles.
7.4 **BASEBAND LINE CODING**

Binary digital signals may be formatted such that they are more suitable for transmission on a particular medium such as a transmission line. This formatting is referred to as line coding and several examples are illustrated in Figure 7-11. Line coding may simply define the pulse shape for an individual symbol (for example NRZ or RZ) or it may also define the pulse sequence format for successive binary symbols (for example BPRZ). Factors influencing the choice of line code include high frequency and low frequency cut-off of the medium, signal-to-noise ratio and the phase linearity of the channel. Popular 2 level line codes are non-return to zero (NRZ), biphase (Bi∅) and delay modulation. A common 3 level line code is bipolar return to zero (BPRZ), a code that incorporates alternate-mark-inversion and features a spectral null at zero hertz. This make is suitable for ac coupled media. We are specifically interested in dc balanced baseband codes that minimize the baseline wander illustrated in the previous section.

![Figure 7-9 Baseband Line Coder / Decoder](image)

A selection of line codes can be generated by the following simple logic circuits. Conventional silicon logic circuits generate a unipolar output (for example, 0v and 5v) that must be level translated to give a polar line signal (+2.5v and -2.5v).

![Figure 7-10 Baseband line encoder circuits](image)

### 7.4.1 Frequency Spectrum of Line Codes

Waveforms for common line codes are shown below along with an expression for the power spectral density, $S(f)$, for positive and negative frequencies. The pulse symbol rate is $1/T$ and 50% pulse width ($T/2$) is assumed for RZ and BPRZ. Note that $A$ is not always equal to the peak pulse voltage.
\[ S(f) = A^2 \left[ \left( \frac{\sin \pi f T}{\pi f T} \right)^2 + \frac{A^2}{4} \delta(f - n T) \right] \]

**Figure 7-11 Line Code Waveforms**

**Figure 7-12 Line Code Power Spectral Density (PSD)**

- **Random Data**
  - A = 1v
  - R = 1kb

- **BPRZ**

- **BiØ**

- **NRZ**

- **DM-M**

- **Power Spectral Density**

- **BPRZ**

- **BiØ**

- **NRZ**

- **DM-M**

- **Frequency (Hz)**
  - 0
  - 500
  - 1000
  - 1500
  - 2000

- **PSD (mW/Hz)**
  - 0.0
  - 0.5
  - 1.0
  - 1.5

- **Random Data**
  - A = 1v
  - R = 1kb
7.4.2 Application of the biphase (Bi∅) and BPRZ codes

The biphase (Bi∅) line code is also widely known as the Manchester code. It has widespread application in 10 Mb/s Ethernet® transmission both on coaxial cable and on twisted pair. The very low spectral content near zero hertz and the high bit rate allow transmission lines to be coupled to the electronic circuits through very small (and cheap) transformers. Another feature of this code is the regular voltage transitions that facilitate recovery of date rate clock in the receiver. The LocalTalk® transmission format, standard on Apple® computers for many years, uses the (Bi∅) line code together with differential encoding.

The bipolar-return-to-zero (BPRZ) line code has been widely applied for paired wire transmission of 1.544 Mb/s DS1 signals in the telephone system. These signals are transmitted on wire sections as long as 1.8 km (6000 feet) and there are potentially damaging foreign voltages introduced in the transmission link. Transformer coupling is used to protect transmitter and receiver electronics from lighting induced surges and ground potential differences arising from 60 Hz power distribution. Although both codes have a spectral null at zero hertz, BPRZ was chosen over Bi∅ because, for the same bit rate, BPRZ requires less of the high frequency spectrum which is greatly attenuated in long transmission lines. Another advantage of dc free codes is that there is less spectral energy in the low frequency voice band. When compared to NRZ coding, this results in less interference to voice signals due to crosstalk between adjacent wire pairs.

Example 7.4 - Random data bits are transmitted by 50% BPRZ (bipolar return to zero) coding. Assume 50 kb/s rate and ±10 volt amplitude. Sketch the power spectrum indicating the peak power spectral density and the frequency of any spectral nulls. Also estimate the transmitted power in the voiceband frequency range 300-3300 Hz.

Solution:

\[
Power_{300-3300\ Hz} = 2 \int_{300}^{3300} A^2 T \left( \frac{\sin \pi f T / 2}{\pi T} \right)^2 (\sin \pi f)^2 \, df
\]

recall that \( \sin \theta \cong \theta \) for small \( \theta \)

\[
Power \cong 2A^2 T \int_{300}^{3300} \pi^2 T^2 f^2 \, df
\]

\[
Power = 2(5)^2(\pi^2)(2 \times 10^{-5})^3 \left[ \frac{3^{3300}}{3^{300}} \right] = 47.2 \, mW \text{ normalized}
\]
7.4.3 BPRZ eye pattern with loss of low frequencies

The following eye patterns show the two level NRZ and the three level BPRZ cases. For the BPRZ code and there is need for two thresholds and this results in less voltage margin for errors introduced by noise.

The robustness of the BPRZ line code to loss of low frequency transmission is shown in the series of patterns resulting from progressive removal of low frequency content. For the 255 bit PN code sequence used in these tests, BPRZ can withstand at least 10 times more bandwidth loss at low frequencies. For longer PN data sequences, the NRZ code has even more problems with removal of low frequencies.

Figure 7-13 Loss of low frequencies in NRZ and BPRZ eye patterns
7.4.4 Differential Coding

In twisted pair baseband systems and in BPSK carrier systems it is convenient if the data can be correctly decoded even when there is a polarity reversal in the transmission medium. Bipolar return to zero (BPRZ) coding is inherently able to function with reversed polarity. Other coding formats can be differentially coded. A disadvantage of this method is that one transmission error will cause two decoded errors.

![Figure 7-14 Differential encoder / decoder (Mark)](image)

In differential “mark” encoding, a logic one in the source data results in a transition in the differential data. In "space" encoding, a logic zero in the source data results in a transition in the differential data.

**Example 7.5** In differential biphase mark encoding, a transition in the channel data corresponds to logic one in the source data. A one in the source data therefore causes a longer pulse in the channel signal. The channel signal can also be viewed as digital FM where long pulses (low frequency) corresponds to logic 1 (mark) and short pulses (high frequency) corresponds to logic 0 (space). This is also known as FM-0 encoding.

**Solution:**

<table>
<thead>
<tr>
<th>Source</th>
<th>Tx</th>
<th>Rx</th>
<th>Decoded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 1 1 1 0 0 0 1 1 0 1</td>
<td>1 1 1 1 0 1 1 1 1 0 1</td>
<td>1 0 1 1 1 0 0 0 1 1 0 1</td>
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</tr>
</tbody>
</table>

![Figure 7-15 Differential Biphase encoding/decoding](image)
7.5 PN SEQUENCES, SCRAMBLING AND ERROR MEASUREMENT

Pseudorandom sequences are periodic signals that are deterministic and not at all random. Nevertheless, these signals appear to have the statistical properties of a random signal (such as sampled white noise). Pseudorandom binary sequences (PRBS) are used in testing to “exercise” a communication system by providing a variety of data sequences with a “full” frequency spectrum. Because the sequences are deterministic and fully predictable by the receiver, the error rate of the system can be measured while the PRBS is being sent. Another use of these sequences is to randomize message information making it more suitable for transmission.

7.5.1 Pseudorandom Sequence Generation

A PBRS sequence generator uses an n bit shift register with a feedback structure containing modulo-2 adders (i.e. exclusive OR gates) and connected to appropriate taps on the shift register. The generator generates a maximal length binary sequence of length $2^n-1$. The maximal length (or "m" sequence) has nearly random properties and is classed as a pseudo noise (PN) sequence. Properties of "m" sequences are as follows:

1) The Balance Property - in each period of the sequence, the number of '1's and the number of '0's differ by at most one. (In a 31 bit sequence, there are 16 '1's and 15 '0's).

2) The Run Property - among the runs of '1's and of '0's in each period, one half the runs of each kind are of length one, one quarter are of length two, one eighth are of length three, etc. as long as these fractions give meaningful numbers of runs. (In a 31-bit sequence there are 16 runs).

3) Shift and add property - The modulo-2 sum of an "m" sequence and any cyclic shift of the same sequence results in a third cyclic shift of the same sequence.

4) The Correlation Property - if a full period of the sequence is compared, term-by-term, with any cyclic shift of itself, the number disagreements is one more than the number of agreements.

5) Spectral Properties - The sequence is periodic, and therefore the spectrum consists of a sequence of equally-spaced harmonics where the spacing is the reciprocal of the period. Prior to sinx/x shaping due to 100% pulse width, and with the exception of the dc harmonic, the magnitude of all the harmonics are equal. Aside from the spectral lines, the frequency spectrum of a maximal length sequence resembles that of a random sequence.

![Figure 7-16 Example spectrum of a length 31 "m" sequence](image)
Example 7.6 – Maximal Length sequence generator with length 31 – is formed using an X-OR gate and a 5-bit shift register. Feedback taps (2, 5) are taken after 2 delays and after 5 delays. This corresponds to the generating polynomial $1 + x^2 + x^5$. During the maximal length sequence, the content of the register passes through every possible 5-bit combination except for the all zero state.

![Figure 7-17 Five Stage PN Sequence Generator(1+x^2+x^5)](image)

**Solution:** The output sequence 11111001 10100100 00101011 1011000 has a period of 31 ($2^n - 1$). The sequence has 16 ‘1’s and 15 ‘0’s and has 16 runs: 4 runs of a single "0", 4 runs of a single "1", 2 runs of double "0", 2 runs of double "1", 1 run of triple "0", 1 run of triple "1", 1 run of quadruple "0" and 1 run of quintuple "1".

The shift register generator normally generates a maximal length sequence, however, it can also produce an alternate sequence of constant logic zero. With specific tap settings the maximal length ($2^n - 1$) sequence is generated. Tap settings other than those indicated in the following table result in a shorter length sequence and the alternate sequence is correspondingly longer. These non-maximal length sequences do not have pseudorandom properties.

The Table below lists required tap settings for a maximal length sequence. All possible “forward sequence” tap settings are listed for register lengths up to 8 and only selected coefficients are given for register lengths 9, 10 and 11. Reverse sequences can be generated by using "symmetric" taps. ie, $x^6 + x^5 + 1$ instead of $1 + x + x^6$.

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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Example 7.7 – Maximal Length sequence generator with length 63 – A PN sequence generator with feedback taps taken after 5 delays and 6 delays is shown below. A 100 kHz clock is supplied to the shift register. a) What is the generating polynomial? b) What is the maximal length PN sequence and is there any other possible output? c) Sketch the PN sequence frequency spectrum using a calibrated frequency scale. d) How many ‘runs’ of each type are there in the PN sequence? Classify the runs as to number of each pattern

Solution: a) \(1 + x^5 + x^6\)  
b) The maximal length sequence is \(2^{6} - 1 = 63\), the other sequence is constant “0”.  
c) Sketch the PN sequence frequency spectrum.  
d) How many ‘runs’ of each type

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7.5.2 Scrambling

Scrambling is a randomizing method that achieves dc balance and breaks up long sequences of zero’s or one’s that might originate from a message source such as a computer keyboard. Long strings of zeros or ones introduce baseline wander in the receiver eye pattern and present difficulty for timing recovery systems that require frequent transitions. In scramblers, a maximal length sequence is modulo-2 added to the data sequence and this is attractive because it requires no extra bandwidth. Since the PN sequence is modulo-2 added to the message sequence, scrambling is the equivalent of modulation by the set of harmonics and we expect the result to be approximately white. This “whitening” disperses any large low frequency signal component that would seriously degrade transmission in a system with ac coupling. A serious problem can occur if the input stream has period equal the PN sequence since the scrambled stream itself can then have a large d.c. component resulting in severe baseline wander at the receiver. If the PN sequence has large period, this pathological situation will arise with very low probability.

Figure 7-18 Six Stage PN Sequence Generator

Figure 7-19 Polarity reversal using X-OR gate
Chapter 7: Baseband Digital transmission

There are two forms of scrambling - self-synchronizing and frame-synchronized. Both types of scramblers use maximal-length shift-register sequences and in this discussion we will focus on the **self-synchronized scrambler**. The input data stream is modulo-2 added in the feedback path of a PN sequence generator as illustrated in Figure 7-20. At the receiver descrambler, the received stream is applied to the input of an identical shift-register. Since both shift-registers have identical inputs (in the absence of transmission errors), their outputs (C and C1) will be identical and, from the operation of the modulo-2 adders, it follows that the output stream will be identical to the input data stream.

![Figure 7-20 Self synchronized scrambler/descrambler](image)

Two problems exist with this simple scrambling structure. The first problem is that a transmission error causes an initial error in the output A1 and then, after traveling through the shift register, it causes two more errors in the output. In this case there is error multiplication by a factor of three. The second problem has to do with scrambler lockup. When the input A is logic zero, the scrambler generates a PN sequence and the shift register takes on all states except the all zero state. When the input A is logic one, the register takes on all states except the all one’s state. During this latter case, an input transition can occur while the register has all zero’s and after this input transition, the scrambler remains in the all zero state (locked up) until there is another input transition. The output descrambler works perfectly under these conditions, however, the transmitted data is no longer randomized and the receiver may make errors due to baseline wander and the loss of clocking transitions.

### 7.5.3 Error Measurement Test Set

An elegant application of the scrambler concept is used in the measurement of errors on a digital transmission link. The data input to the transmitter scrambler is held at ‘0’ thus the descrambler output should always be ‘0’ in the absence of channel errors. When the switch in Figure 7-21 is in position ‘a’ the circuit operates as a self-synchronized descrambler. After some period of operation without errors, the receiver shift register is known to have an error free sequence and can therefore re-circulate independently by moving the switch to position “b”. In this latter mode, the counter registers the true number of errors since the errored sequence no longer enters the shift register.
**Problems**

*7.1* An impulse of weight 5 mV-s enters a transmission system which is modeled by an ideal LPF with cutoff at 40 kHz. What is the peak amplitude of the impulse response and what is the interval between zero crossings in the tails?

*7.2* A single impulse has an energy spectral density that is constant at all frequencies. Review Figure 7-2 and Example 7.1 and then show that impulse response energy is directly proportional to the filter bandwidth. (We would also expect this as a result of Parseval’s theorem)

*7.3* A pulse transmission system has a “total” lowpass bandwidth of 3000 Hz and a -6 dB roll-off frequency at 2400 Hz. What is the maximum PAM signaling rate that can be sent without intersymbol interference?

*7.5* A voice signal with bandwidth 300 to 3300 Hz is sampled at 8 ksamples/s. What minimum transmission bandwidth is required to send the PAM samples without intersymbol interference?

*7.7* Random binary data is transmitted on a transmission channel with nominal -6dB bandwidth of 128 kHz. The channel frequency response has r=0.5 raised cosine roll off with excess transmission bandwidth up to 192 kHz.

a) Provide a sketch of the transmission channel frequency response.

b) Determine the maximum symbol rate (ks/s) which can be received without intersymbol interference (ISI).

c) Assuming NRZ coding, provide a rough sketch of the transmitted signal spectrum and the received signal spectrum.

*7.9* Random data bits are transmitted using NRZ coding, 200 kb/s rate and ±5 volt amplitude.

i) Calculate the normalized power in the data signal.

ii) Sketch the power spectrum, indicating the peak power spectral density and the frequency of any spectral nulls.

iii) Estimate the data signal power through integration of the spectral density from minus infinity to plus infinity. (Parseval’s theorem)
*7.11 Illustrate the waveform when the sequence 0100 1111 0100 is transmitted with the following code schemes. Assume ±10 volt amplitude and 100 kb/s rate.

NRZ (non return to zero)   BPRZ (bipolar return to zero)   BiØ-L (biphase level)

*7.13 Illustrate non-return to zero (NRZ), alternate mark inversion bipolar return to zero (BPRZ) and biphase level (BiØ-L) coding for the following binary sequence.

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*7.15 Consider a ±2 volt NRZ random data signal at rate 2400 b/s.

a) Sketch the two sided normalized PSD on calibrated axes.
b) Use Parseval's theorem to verify your PSD result. Your solution may be aided by a table of definite integrals found in Appendix A-5 of Couch-5.
c) For a symbol rate of 2400 b/s, what is the minimum channel bandwidth required in theory for zero intersymbol interference?
d) Revise the sketch a) to show the result of passing through a channel which meets the Nyquist conditions for zero intersymbol interference (ISI) and has a raised cosine rolloff factor of 0.3.

*7.17 Random data bits are transmitted by biphase-level coding. Assume 50 kb/s rate and ±10 volt amplitude.

a) Sketch the power spectrum indicating peak power spectral density and the frequency of spectral nulls.
b) estimate the transmitted power in the voiceband frequency range 300-3300 Hz.

*7.19 A 4 level 2B1Q (two binary, one quaternary) data signal can be developed as the voltage sum of two independent NRZ data streams. If the data in the two NRZ signals are uncorrelated, the power spectral density (PSD) of the 2B1Q signal will simply be the sum of the NRZ power spectral densities.

a) Consider a ±2 volt NRZ random data signal at rate 2400 b/s. Sketch the two sided normalized PSD on calibrated axes.
b) Add to the signal in part a) a second NRZ random ±1 volt data signal and sketch the resulting 2 sided PSD on calibrated axes.
c) What are the voltage levels of the resulting 2B1Q signal?
d) Revise the sketch b) to show the result of passing through a channel which meets the Nyquist conditions for zero intersymbol interference (ISI) and has a raised cosine rolloff factor of 0.3.

*7.21 Cellular telephony radio channels have 30 kHz frequency spacing and an equivalent baseband transmission channel extending from 0 Hz to 15 kHz. Signals must not exceed this bandwidth, otherwise there will be crosstalk with adjacent frequency channels. Assume that a r = 0.25 raised cosine roll-off filter is used for binary digital transmission.

a) What is the maximum symbol rate which can be sent without ISI (intersymbol interference)?
b) If each symbol can have 4 states, what is the transmission bit rate?
c) What are the practical advantages of using roll-off factor r = 0.25 versus r = 0?

*7.23 A 300 to 3300 Hz voice signal is sampled at 10 ksamples/s. What is the minimum transmission bandwidth required to send the PAM samples without intersymbol interference? Sketch the frequency
response, $|H(f)|$, of a practical transmission channel which could carry the above PAM samples without intersymbol interference. Put numerical values at significant points on the horizontal and vertical axes.

*7.25 Illustrate the waveform when the sequence 1111 0010 0010 is transmitted using the following code schemes. Assume 100 kb/s rate and ±10 volt amplitude.

- NRZ (non return to zero)
- BiØ-L (biphase level)

*7.27 Show differential encoding and decoding for the sequence shown below. Assume that the initial reference digit is a binary "1".

0 0 1 1 0 0 0 1 0 0 0 1

*7.29 Illustrate biphase mark differential coding (BiØ-M or FM-0) for the following binary sequence. Assume logic low initial state.

| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |

*7.31 Commercial data test sets use PN sequence lengths of 127, 511 and 2047 bits (as well as much longer sequences). The PN sequences are generated with shift registers and X-OR gates. Estimate what shift register length is required to generate each of the 3 sequence lengths.

*7.33 A Biphase mark encoded 56 kb/s data signal at +3v level travels on one pair of a subscriber distribution cable. Determine the noise induced in baseband telephone circuits which share the same bundle in the cable. Assume a crosstalk loss of 80 dB throughout the voice band and assume flat noise weighting in the voice band.

*7.34 Further to Example 7.6, a reverse order sequence is generated when the order of the modulo-2 adder taps is reversed. In the 5 stage register, feedback taps are taken after 3 delays and 5 delays. This corresponds to the generating polynomial $1 + x^3 + x^5$. Assuming that the register starts in the all one’s state, determine the output sequence.

Fig. P.7-34

*7.35 A PN sequence generator is constructed with a single exclusive OR gate and feedback taps taken after 5 and 6 delays. A 1 MHz clock is supplied to the shift register which has output voltage levels of ±2 v.

a) Sketch a logic diagram of the PN generator.

b) What is the period of the PN sequence in ms?

c) Sketch the PN sequence frequency spectrum using a calibrated frequency and amplitude scales.

d) How many 'runs' are there in the PN sequence? Classify the runs as to number of each pattern.

*7.37 a) A partial eye diagram is shown below. i) complete the remaining traces in the diagram and ii) show a block diagram of a test setup to produce an eye diagram.

b) Illustrate how reduced (or excess) low frequency response reduces the eye opening (noise voltage margin) and introduces jitter in the received digital signal.
*7.39* One trace of a 1 Mb/s eye diagram is shown in Diagram A to represent the bit pattern 010 with a preamble of 10101.

a) Assuming the same 10101 preamble in each case, how many traces are required to complete the eye diagram? Sketch the result.

b) Assuming original upper bandlimit and a lower bandlimit of 10 kHz, sketch the resulting trace in diagram B when the preamble xxxxx is 11111 preceeded by alternating bits and also when the preamble is 00000 preceeded by alternating bits.

c) Why is a pseudorandom (PN) sequence used to generate an eye diagram? How does the PN shift register length affect the traces in diagram B?