

Coordinate transformation and model reduction for efficient simulation of coupled circuit-field problems

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Abstract – We present a new algorithm to transform any complex system of linear first-order ODEs or second-order undamped ODEs, resulting from the discretization of field PDEs, into 1-D equivalent circuit system. This transformation algorithm offers an extremely simple and convenient way to incorporate field problems in circuit simulators to solve coupled circuit-field problems. We also present a model reduction strategy based on the proposed coordinate transformation algorithm. Numerical example involving a Voltage Regulator Module with MOSFET devices shows a significant reduction of simulation time compared to the solution without using the proposed transformation.

1. Introduction

Many applications in engineering require the solution of coupled circuit-field problems. With the recent advances in high-temperature semiconductor technology, there is growing need for power converters in high-temperature, high-power-density applications. Due to the increasing current density and higher power requirements of advanced power semiconductor devices such as IGBT (Insulated Gate Bipolar Transistor) and MOSFET (Metal-Oxide-Semiconductor Field-Effect-Transistor), the devices dissipate a considerable amount of heat. Regardless of the electrical quality of power semiconductor devices, devices will fail if heat cannot be removed effectively. In addition, the electrical performance of the system can be limited by device heating and thermal coupling between devices. Therefore, the coupling between circuit networks and the thermal field problem to examine potential self heating problems in power semiconductor devices requires an accurate electrothermal simulation of complete power electronic systems ([3], [7]). Most nonlinear semiconductor device models are implemented in circuit simulators that are widely used by engineers. Many of these simulators have highly efficient solvers. It is thus advantageous if coupled circuit-field problems using circuit simulators. To this end, methods to develop circuit networks equivalent to discretized field equations have been proposed ([3], [2]). However, the construction of this equivalent circuit network could be time consuming and take up considerable space in the circuit simu-

lator ([2], [3]). To remedy such pitfalls, we propose in the present work a new algorithm to transform the coordinates of the discrete equations of the field problem into extremely simple 1-D circuit networks that can be easily input into circuit simulators. The proposed coordinate transformation algorithm, which can be thought of as the inverse WYD algorithm, transform a general finite-element system, represented by possibly full system matrices, into a system with diagonal capacitance matrix and tridiagonal conductance system matrix. The resulting equivalent 1-D circuit network is simple, since it contains only resistors, capacitors, and current sources. We can solve either the resulting transformed ordinary differential equations, or the associated eigenvalue problem very fast using a general purpose electrical circuit simulator. Model reduction method based on the new coordinate transformation algorithm is presented to further reduce the equivalent 1-D electrical circuit system.

It should be noted that the proposed coordinate transformation algorithms are especially efficient when used in conjunction with a lumped (diagonal) capacitance matrix, since it is not costly to invert a diagonal matrix.

2. Coordinate transformation

To present our idea, we will use the electrothermal coupled problem as an example ([3]). We will focus on the heat problem to describe the construction of thermal component models for the thermal components (e.g., heat sink) of the overall electro-thermal system. These thermal models are obtained from a discretization of the heat diffusion equation, and can be expressed in several different coordinate systems: The original finite-element coordinate system, and other transformation coordinate systems that will make the subsequent analysis extremely efficient. We first state the heat diffusion problem, and summarize its finite-element formulation. We then review the traditional WYD algorithm, which can be thought of as coordinate transformation method itself. Finally, we introduce new coordinate transformation methods that transform the finite-element coordinate system in such a way to obtain a diagonal capacitance matrix and a tridiagonal conductance matrix. We also show that this new

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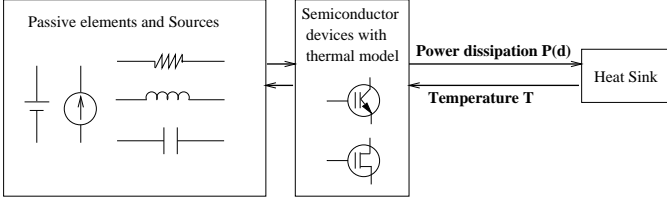


Figure 2.1. Coupled electrothermal system.

coordinate-transformation method can be thought of as the inverse WYD algorithm.

2.1. Electrothermal coupled systems. Many electronic circuits contain semiconductor devices (see Fig. 2.1) can malfunction when heat generated during their operations goes beyond certain limit. It is therefore important to predict, so to avoid, such detrimental self heating effects. The nonlinear electrothermal coupled system can be generally expressed as:

$$\dot{\mathbf{d}} = \mathbf{f}(\mathbf{d}, \mathbf{e}, \mathbf{T}), \quad \mathbf{P} = \mathbf{P}(\mathbf{d}), \quad (2.1)$$

where \mathbf{d} is the matrix containing the nodal voltages or currents, \mathbf{e} the electrical input, \mathbf{T} the temperature, and \mathbf{P} the electrical power loss. The nonlinear electrical system is governed by the semiconductor equations (PDE's) and circuit equations (ODE's). The electrical power loss is originated from the heat generated inside the semiconductor devices. Since the semiconductor pn junction region, where most of the heat is generated, is small compared to the whole device region, and is close to the top surface of the *Si* chip, the electrical power loss can be assumed to be imported from the top boundary of the *Si* chip, i.e., the electrical power loss is treated as a boundary condition in the thermal problem.

The heat diffusion problem on a domain Ω with boundary $\partial\Omega = \overline{\Gamma_1} \cup \overline{\Gamma_2}$ is governed by the partial differential equation

$$\text{div}(\kappa \text{grad} T) = \rho c_p \frac{\partial T}{\partial t} \quad \text{in } \Omega \quad (2.2)$$

and the boundary conditions

$$\kappa \text{grad} T \cdot \mathbf{n} = \kappa \frac{\partial T}{\partial n} = \frac{\mathbf{P}}{A} \quad \text{on } \Gamma_1 \quad (2.3)$$

$$\kappa \text{grad} T \cdot \mathbf{n} = \kappa \frac{\partial T}{\partial n} = h(T_a - T) \quad \text{on } \Gamma_2 \quad (2.4)$$

and initial condition

$$T(x, 0) = T_a = 27^\circ C \quad (2.5)$$

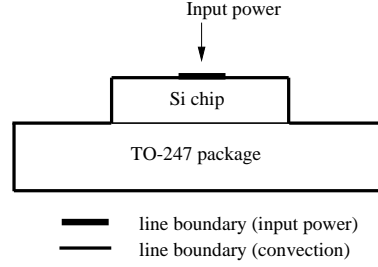


Figure 2.2. 2-D thermal model problem.

where $T : \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ is the temperature (a function of space and time), $\kappa : \Omega \rightarrow \mathbb{R}$ the thermal conductance, $\rho : \Omega \rightarrow \mathbb{R}$ the mass density, $c_p : \Omega \rightarrow \mathbb{R}$ the specific heat, $\mathbf{P} : \Gamma_1 \times \mathbb{R}_+ \rightarrow \mathbb{R}$ the input power from boundary Γ_1 , A the input power cross section area, h the convection coefficient, \mathbf{n} the outward normal vector to the boundary, and T_a the ambient temperature. In the above, the set of real numbers is denoted by \mathbb{R} , with $\mathbb{R}_+ := t \in \mathbb{R} \mid t \geq 0$, denoting the set of non-negative numbers. The domain of the heat sink is denoted by $\Omega \subset \mathbb{R}^s$, with $s = 1, 2, 3$ being the space dimension. The boundary $\partial\Omega$ of the heat sink is decomposed into two parts: Γ_1 for the input power boundary condition (2.3), and Γ_2 for the convective boundary condition (2.4).

2.2. Finite element formulation for thermal system. The semidiscrete equation of the thermal system, derived from a Galerkin finite element projection ([5], [3]), takes the form

$$\mathbf{C} \dot{\mathbf{d}} + \mathbf{K} \mathbf{d} = \mathbf{f} = \mathbf{I}_p + \mathbf{I}_c, \quad \text{and } \mathbf{d}(0) = \mathbf{d}_0, \quad (2.6)$$

where $\mathbf{C} \in \mathbb{R}^{n \times n}$ is the capacitance matrix, $\mathbf{K} \in \mathbb{R}^{n \times n}$ the conductance matrix, $\mathbf{f} \in \mathbb{R}^{n \times 1}$ the heat supply matrix, \mathbf{I}_p the input power supply from the electrical component, \mathbf{I}_c and the heat supply coming from the convective boundary condition, and $\mathbf{d} \in \mathbb{R}^{n \times 1}$ the column matrix of nodal temperatures d_i at the FE node i . Let N_i be the shape function associated with node i , then the temperature field over the domain of the heat problem is approximated by $T(x, t) \cong \sum_{i=1}^n d_i N_i$. The capacitance

matrix \mathbf{C} can be realized by capacitor elements, the conductance matrix \mathbf{K} by resistor elements, and the heat supply vector \mathbf{f} by current sources, so that thermal effects for the semiconductor devices like IGBT, MOSFET can be dynamically incorporated into circuit simulators with thermal connection. The thermal network based on the FEM is an equivalent circuit network that yields the same semidiscrete equations (ODE's) resulting from a Galerkin projection using FE basis functions. Similar to FE global matrices, the global thermal network is assembled from

elemental networks equivalent to the elemental matrices \mathbf{c}_e , \mathbf{k}_e and \mathbf{f}_e . The coupled electrothermal system is shown in Fig. 2.1, from which it can be seen that the coupling between these two systems is through the power loss \mathbf{P} and the nodal temperature \mathbf{d} . Recall that the nodal temperatures in \mathbf{d} are system unknowns in a circuit simulator such as Saber, and are solved for simultaneously with the electrical unknowns.

As an example of the thermal problem, we consider the *Si* chip and package shown in Fig. 2.2 ([3]), where thermal networks equivalent to a discretization of the heat equation by the finite element method (FEM) were presented. The proposed methodology not only solve the field problem, but also provides a natural way to solve the coupled circuit-field problem as a single circuit problem. Depending on the order of interpolation employed, the elemental equivalent circuit networks could be complex, thus making the overall thermal circuit network extremely involved, and not simple to implement in a circuit simulator. If we can transform the original discrete field model into an equivalent simple model (diagonal capacitance matrix and tridiagonal conductance matrix), then the circuit network equivalent to this simple model can be easily constructed.

2.3. WYD method. The WYD method, first introduced by **Wilson, Yuan and Dickens** ([8]), has become popular in structural dynamics, is based on the direct superposition of a special class of Ritz vectors generated from the spatial distribution $\mathbf{f}(s) \in \mathbb{R}^{n \times 1}$ of the dynamic load that can be put under the form $\mathbf{f}(s)g(t)$, where $g(t)$ is function of time. This method eliminates the requirement for exact evaluation of the free vibration frequencies and mode shapes. The sequence of WYD vectors is generated by taking into account the spatial distribution of the external excitation, which is important information neglected by direct use of exact mode shapes. The first WYD vector is the steady-state vector obtained from a steady-state analysis using the spatial distribution \mathbf{f} of the dynamic excitation as input. The other vectors are generated from a recurrence relationship in which the capacitance matrix \mathbf{C} is multiplied by the last WYD vector; the resulting vector is then used as the excitation for the next steady-state solution.

WYD Algorithm

1. Given \mathbf{C} , $\mathbf{K} \in \mathbb{R}^{n \times n}$, and $\mathbf{f} \in \mathbb{R}^{n \times 1}$
2. Triangularize $\mathbf{K} = \mathbf{L} \mathbf{D} \mathbf{L}^T$
3. Solve for starting vector \mathbf{w}_1

$$\mathbf{K} \mathbf{w}_1^* = \mathbf{f} \Rightarrow \mathbf{w}_1^* = \mathbf{K}^{-1} \mathbf{f}$$

$$\mathbf{w}_1 = \frac{\mathbf{w}_1^*}{\sqrt{\mathbf{w}_1^{*T} \mathbf{C} \mathbf{w}_1^*}} \quad (\mathbf{C} - \text{normal.})$$

4. Solve for remaining vectors $i = 2, \dots, r$

$$\mathbf{K} \mathbf{w}_i^* = \mathbf{C} \mathbf{w}_{i-1} \Rightarrow \mathbf{w}_i^* = \mathbf{K}^{-1} \mathbf{C} \mathbf{w}_{i-1}$$

$$c_{i,j} = \mathbf{w}_j^T \mathbf{C} \mathbf{w}_i^*$$

$$\mathbf{w}_i^{**} = \mathbf{w}_i^* - \sum_{j=1}^{i-1} c_{i,j} \mathbf{w}_j \quad (\mathbf{C} - \text{orthog.})$$

$$\mathbf{w}_i = \frac{\mathbf{w}_i^{**}}{\sqrt{\mathbf{w}_i^{**T} \mathbf{C} \mathbf{w}_i^{**}}} \quad (\mathbf{C} - \text{normal.})$$

2.4. Inverse WYD method. As mentioned in the introduction, when the capacitance (mass) is diagonal, tridiagonal, or even pentadiagonal, and the conductance (stiffness) matrix is full, it is more efficient to transform the system into a diagonal capacitance matrix and a tridiagonal conductance matrix prior to an implementation in a circuit simulator, since the transformed system can be represented by an extremely simple 1-D equivalent circuit network. Unlike the WYD method, which works with the matrix $\mathbf{K}^{-1} \mathbf{C}$, our method works with the matrix $\mathbf{C}^{-1} \mathbf{K}$ to generate the Krylov sequence. The trial vectors are generated from the Krylov subspace

$$\mathcal{K}^{(k)}(\mathbf{K}^{-1} \mathbf{C}, \mathbf{z}_1) = \text{span}[\mathbf{z}_1, (\mathbf{K}^{-1} \mathbf{C}) \mathbf{z}_1, \dots]. \quad (2.7)$$

here we choose to generate the Ritz vectors from the Krylov subspace

$$\mathcal{K}^{(k)}(\mathbf{C}^{-1} \mathbf{K}, \mathbf{z}_1) = \text{span}[\mathbf{z}_1, (\mathbf{C}^{-1} \mathbf{K}) \mathbf{z}_1, \dots]. \quad (2.8)$$

To generate a linearly independent vector basis, Gram-Schmidt orthogonalization is used in both the WYD method and the inverse WYD method. Our proposed method can be put in the form of a conceptual algorithm as follows:

Inverse WYD Algorithm

- 1 **Given** : \mathbf{C} , $\mathbf{K} \in \mathbb{R}^{n \times n}$, and $\mathbf{f} \in \mathbb{R}^{n \times 1}$.
- 2 $\mathbf{K} \mathbf{w}_1^* = \mathbf{f} \Rightarrow \mathbf{w}_1^* = \mathbf{K}^{-1} \mathbf{f}$.
- 3 \mathbf{C} -normal. $\mathbf{w}_1^* \Rightarrow \mathbf{w}_1 = \frac{\mathbf{w}_1^*}{\sqrt{\mathbf{w}_1^{*T} \mathbf{C} \mathbf{w}_1^*}}$.
- 4 Decompose $\mathbf{C} = \mathbf{L} \mathbf{L}^T$.
- 5 Generate the remaining vectors \mathbf{w}_i .
- 6 **do** $i = 2, \dots, n$
- 7 $\mathbf{C} \mathbf{w}_i^* = \mathbf{K} \mathbf{w}_{i-1} \Rightarrow \mathbf{w}_i^* = \mathbf{C}^{-1} \mathbf{K} \mathbf{w}_{i-1}$.
- 8 Compute $c_j = \mathbf{w}_j^T \mathbf{C} \mathbf{w}_i^*$, for $j = 1, \dots, i-1$.
- 9 \mathbf{C} -orthog. $\mathbf{w}_i^* \Rightarrow \mathbf{w}_i^{**} = \mathbf{w}_i^* - \sum_{j=1}^{i-1} c_j \mathbf{w}_j$.
- 10 \mathbf{C} -normal. $\mathbf{w}_i^{**} \Rightarrow \mathbf{w}_i = \frac{\mathbf{w}_i^{**}}{\sqrt{\mathbf{w}_i^{**T} \mathbf{C} \mathbf{w}_i^{**}}}$.
- 11 **end**

It is important to note here that while the WYD method uses the load-dependent trial vectors to form a basis to obtain a reduced eigenvalue problem, whose solution leads to the Ritz vectors (or approximated eigenvectors), our methodology bypasses the solution of the reduced eigenvalue problem and the use of the Ritz vectors, and uses directly the trial vectors. It should also be noted that the traditional WYD method leads to a diagonal capacitance matrix \mathbf{C} , but a *full* conductivity matrix \mathbf{K} , thus making the construction of an equivalent circuit complicated. The inverse WYD method, On the other hand, leads to an identity capacitance matrix $\mathbf{C} = \mathbf{I}$, and a tridiagonal conductance matrix \mathbf{K} .

3. Electrothermal simulation

Since self-heating effects of the semiconductor devices play an important role in the electrical characteristics and power dissipation, effective simulation of power electronic circuits requires the availability of accurate models for the power semiconductor devices. **Saber** electrothermal models ([1], [7]) for the semiconductor devices couple the electrical network with the thermal network represented by thermal network components. The electrothermal semiconductor models use the instantaneous device temperature (temperature at the silicon chip surface T_j) to evaluate the temperature-dependent properties of silicon and the temperature-dependent model parameters. These temperature-dependent values are then used by the physics-based semiconductor device model to describe the instantaneous electrical characteristics and the instantaneous dissipated power. A thermal component model is based upon a finite element discretization of the heat diffusion equation. Once a thermal network component is developed, it can be used in any electrothermal coupled problems.

3.1. Equivalent circuit after inverse WYD transformation. Reference [3] presents a rational approach

to construct thermal circuit networks equivalent to a discretization of the heat equation by the finite element method. These thermal circuit networks are to be connected to the electrical networks of power electronic systems to provide complete electrothermal models that can be conveniently used in any circuit simulator. However, the element equivalent circuit may be complex, and thus the resulting circuit network could be extremely complex. If we transform the original discrete field model into an equivalent simple model, then the circuit network equivalent to this model can be constructed easily. For any complex thermal component model after a discretization of the heat equation by the finite element method, Fig. 2.3 shows the equivalent 1-D circuit after inverse WYD transformation. Since after inverse WYD transformation, we can obtain a system with capacitance matrix being diagonal and conductance matrix being tridiagonal. Therefore, in the 1-D equivalent circuit, there is no capacitor between nodes and there will be one capacitor and one resistor between nodes and ground.

3.2. Implementation of Saber thermal models.

The thermal network after inverse WYD transformation is written as element templates in **Saber** circuit simulator. Thermal package models can be built by these element templates. A **Matlab** code was written to automatically generate Saber netlists or templates. A 2-D thermal problem in Fig. 2.2 was used to test the simplified circuit networks. The modeled templates will read in trial vectors generated from Matlab code. In addition, CCCS's (Current Controlled Current Sources) and a VCVS (Voltage Controlled Voltage Source) are used in the equation section of the Saber template for the recovery of nodal temperature from the transformed coordinate to physical coordinate. The controlled sources are used to allow only one node of the thermal component models to connect with the thermal terminal of the Saber electrothermal models.

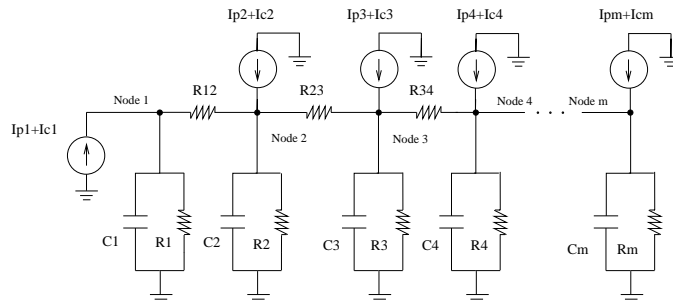


Figure 2.3. Equivalent circuit after inverse WYD transformation. I_p is power input for the node and I_c is convective B.C. for that node.

4. Model reduction

To minimize the computational effort while obtaining good accuracy, the WYD method uses only Ritz vectors having large participation factors in mode superposition analysis ([8]). The WYD process can be truncated based on certain measure to obtain reduced-order models. The inverse WYD process can also be truncated, and the generated trial vectors can be used to construct reduced-order models. There is a trade-off in the solution of the method to use to reduce the order of a large system in a cost-effective manner, while maintaining high accuracy. The selection of the trial vectors that form the basis for the transformation of coordinates of the original system to a reduced-order system can be based on the participation factors of these trial vectors to the dynamic response of the system. Once a reduced-order model is obtained, the dynamic response can be computed by direct numerical integration or by modal superposition method ([4]).

Based on the algorithm presented above, a model-reduction strategy using a combination of forward and inverse methods can be introduced. First, using the (forward) WYD method to obtain a reduced-order model by truncating the generation of the WYD trial vectors, followed by the use of the inverse WYD method to transform the resulting reduced-order model to a very simple form (diagonal capacitance, tridiagonal conductance). Actually, we can even obtain a further reduced-order model by truncating the generation of the inverse WYD trial vectors. Such truncation can be based on the use of the response participation of each trial vector. At the end of this two-stage model-reduction strategy, we obtain a reduced-order model with an identity capacitance matrix and a tridiagonal conductance matrix, which is represented by a simple 1-D equivalent circuit, as discussed in previous sections.

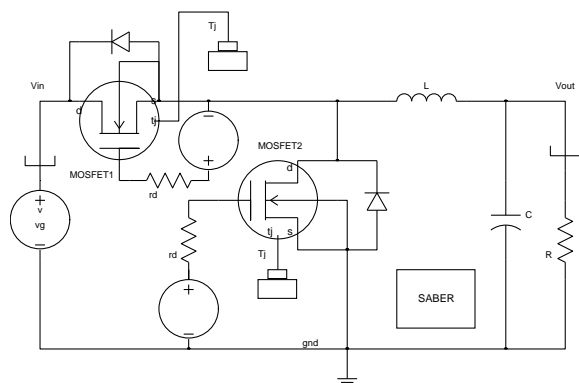


Figure 3.1. Buck converter with synchronous rectifier.

5. Numerical examples

The proposed new thermal models are applied in the electrothermal simulation of power electronic circuits and systems to increase efficiency. A buck converter with synchronous rectifier shown in Fig. 3.1, referred to [6], was simulated in **Saber** using our equivalent circuit model for the thermal component. Our tested 2-D thermal model is shown in Fig. 2.2. We use 2 different meshes in the finite element discretization. Mesh1 is finite element discretization with 16 triangular elements and 16 nodes; mesh2 is finite element discretization with 368 triangular elements and 215 nodes. After inverse WYD transformation, we obtain the transformation models from the full order models. Apply WYD method followed by inverse WYD methods on the full order models we obtain the reduced order models. In our example (see Table 5.2), we show a reduced-order model with 51 nodes obtained from mesh2 using the combination of forward WYD and inverse WYD methods. The operating frequency for the simulated VRM is 500kHz. The following major components of the VRM power stage were selected: switch SW - 2XIRF7811, SR - 4XIRF7811, and inductor $L = 500nH$. The capacitor $C = 100uF$, the load resistor $R = 6\Omega$. Fig. 3.2 shows the MOSFET bulk-drain total energy dissipation obtained from Saber transient simulation. It clearly shows the increase of the energy dissipation versus the transient simulation time. Table 5.1 gives the comparison of the transient simulation time between the original 2-D thermal model and the inverse WYD transformed models of the described VRM. Table 5.2 gives the comparison of the transient simulation time between the original 2-D thermal model and the reduced-order model by the combined WYD/inverse WYD method. The MOSFET bulk-drain total energy losses shown in Fig. 3.2 using our proposed thermal models, will be ignored by the

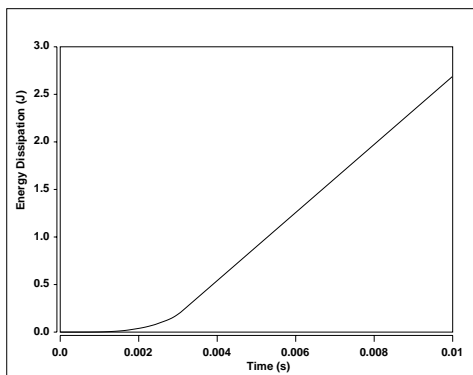


Figure 3.2. MOSFET bulk-drain total energy dissipation versus simulation time.

Table 5.1. *Saber transient simulation time for the original model and for the inverse WYD transformed model*

Models	Mesh1	Mesh2
original	1740s	10200s
transformed	648s	2680s

simulations without thermal effects.

6. Conclusion

A new coordinate transformation algorithm was presented to transform any complex system resulting from the discretization of field equations into 1-D equivalent circuit system. The proposed algorithm is used to build thermal circuit models for efficient electro-thermal simulations of power electronic circuits and devices. A model reduction strategy based on the proposed algorithm was also presented.

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Table 5.2. *Saber transient simulation time for full-order model and for the reduced-order model*

Models	Mesh2
original	10200s
transformed	2680s
reduced-order	1300s

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