NUFFT for Medical and Subsurface Image Reconstruction

Qing H. Liu
Department of Electrical and Computer Engineering
Duke University
Duke Frontiers 2006
May 16, 2006
Acknowledgment

• Jiayu Song – main contributor for MRI and subsurface imaging applications
• Nhu Nguyen – collaborator for forward NUFFT
• Susan X. Tang – collaborator for inverse NUFFT. Discussion w/ X. Sun
• A. Johnson’s group – MRI data
• W. Scott’s and L. Collins’ groups – subsurface imaging data

• Research supported by NIH and DARPA
Outline

• Introduction: History and Motivation
• NUFFT algorithm (Liu/Nguyen, 1998)
• MRI and CT applications
• Subsurface imaging applications
• Summary
What is NUFFT?

- **Nonuniform Fast Fourier Transform**
- Fast algorithm for discrete Fourier transform sum when sample points are nonuniform

\[ F_j = \sum_{k=0}^{N-1} f_k e^{i\omega_k t_j}, \quad j = -M/2, \ldots, M/2 - 1 \]
Brief History of NUFFT

• Many “slow” summation methods before 1993. Cost: $O(MN)$
• Dutt and Rokhlin (1995) first fast algorithm with cost of $O(N \log N)$
• Other fast algorithms: Beylkin (1995); Dutt & Rokhlin (1996); Anderson & Dahleh (1996)
• “NUFFT” coined in 1998 (Liu & Nguyen, 1998; Nguyen & Liu, 1999)
• Fessler/Sutton (2003); Greengard/Lee (2005)
• MRI application: Sha, Guo and Song (2003) and others
Motivation

Medical Imaging: MRI, CT
Raw data is acquired in nonuniform Fourier space (MRI) or can be related to Fourier space (CT)
MRI Magnetization Process

Transverse Magnetization Decay

\[ M_{xy}(t) = M_0 e^{-t/T_{2^*}} \]

Vertical Magnetization Recovery

\[ M_z(t) = M_0 (1 - e^{-t/T_1}) \]
Phase and Encoding

\[ B = B_0 + G_{xx}x + G_{yy}y + G_{zz}z \]

\[ \phi(x, t) = \int_{\tau=0}^{t} -\gamma B(x, \tau) d\tau \]

\[ = \omega_0 t - 2\pi x \cdot k \]

\[ k(t) = \frac{\gamma}{2\pi} \int_{\tau=0}^{t} G(\tau) d\tau \]

**K-space trajectory**

**Signal Model**

\[ S(k) = \iiint_{\text{FOV}} m_{xy}(x, 0) e^{i\omega_0 t} e^{-2\pi x \cdot k} dx \]

\[ = e^{i\omega_0 t} \iiint_{\text{FOV}} m_{xy}(x, 0) e^{-2\pi x \cdot k} dx \]
Non-Cartesian Trajectories

- Radial
- Rosette
- Spiral
- Lissajou
Motivation (2)

Ground penetrating radar (GPR) or seismic sensors

Raw data acquired can be related to nonuniform Fourier space data
A Phenomenological Model Example: Radar Waves

Radar freq. $f_c = 1 \text{ GHz}$

$\varepsilon_r = 4$

$1.5 \text{ m}$

$2 \text{ m}$

$2 \text{ m}$

$1.5 \text{ m}$
Movie of Radar Waves

0.96 ns interval

• First arrival: Direct head waves with speed in air
• Second arrival: Wave speed in soil
• Third arrival: Reflection from tunnel
Waveforms at Receivers

Air  Soil  Tunnel
GPR Sensor Scan

Sensor Scan \((x,y)\)

Wave travel time \((t)\)

Wavenumber Space \((k_x,k_y)\)

Nonlinear dispersion relation

\[
k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}
\]
Common problem: Nonuniform Samples in K Space

- 1D Uniform versus Non-Uniform Samples

- Fast Fourier Transform: $O(N \log N)$ CPU time

\[
\text{FFT: } F_j = \sum_{k=0}^{N-1} f_k e^{i2\pi kj/N}
\]

- NUFFT: Fast algorithm to achieve $O(N \log N)$ CPU time

\[
\text{NUFFT: } F_j = \sum_{k=0}^{N-1} f_k e^{i2\pi \omega_k t j/N}
\]
Outline

• Introduction: History and Motivation
• NUFFT algorithm (Liu/Nguyen, 1998)
• MRI and CT applications
• Subsurface imaging applications
• Summary
NUFFT Algorithm (Liu/Nguyen)

- Optimal
- Uses least-square interpolation of exponential function
- A new class of matrices – regular Fourier matrices – have been discovered
  - Independent of nonuniform points
- Closed-form solution for an excitation (data) vector
NUFFT Procedures

• Preprocessing: Calculates interpolation coefficients for exponential functions
• Interpolation: Calculates interpolated Fourier coefficients at an oversampled grid
• Regular FFT: Uses regular FFT to calculate the DFT on the oversampled grid
• Scaling: Evaluates the NUDFT from the DFT on the oversampled grid
Comparison of NUFFT with previous algorithm [Dutt/Rokhlin, 1993]: More than one order of magnitude improvement

Computation cost

\[ O(N \log N) \]
Outline

• Introduction: History and Motivation
• NUFFT algorithm
• MRI and CT applications
• Subsurface imaging applications
• Summary
MRI Applications

- Scanner
- Phantom

Source: Center for In Vivo Microscopy
Computer Simulation

256x256

|IDFT| INUFFT|
---|---|
100000X

|IDFT-INUFFT|
e = 0.034%

64x64x64

|IDFT|
---|
1000X

|IDFT-INUFFT|
e = 0.042%

64x64x64

|IDFT|
---|
1000X

64x64x64

|IDFT|
---|
1000X
Phantom Scan

2D

$\mathbf{I}_{\text{NUFFT}}$

3D

$\mathbf{I}_{\text{NUFFT}}$
In Vivo Scan

2D

3D

Source: cardiac data from Elizabeth Bucholz
Outline

• Introduction: History and Motivation
• NUFFT algorithm
• MRI and CT applications
• Subsurface imaging applications
• Summary
NUFFT Imaging of 3D Objects

3D Configuration (W. Scott, Georgia Tech)
3D Imaging Results (GT plywood)

Raw Data
A Field Example of IED Imaging
Summary

• NUFFT has widespread applications in medical imaging and subsurface sensing
• It allows arbitrary sampling patterns
• Improved accuracy enables faster data acquisition – important for functional imaging