Multi-Sensor Inversion via Rigorous Analysis of Underlying Waves

Qing H. Liu
Duke University
qhliu@ee.duke.edu

MURI Program Review
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Outline

• Summary of Progress
• Joint EM/Seismic Multi-Modality Inversion
• Through-Wall Imaging Results
• Inversion of data collected by Institut Fresnel in June 2004
• Inversion of data collected by Georgia Tech (W. Scott) in Dec. 2004
Summary of Research Progress

- Objective: To use the wave physics with fast algorithms to enhance multi-modality inversion and imaging.

- Developed
  - PSTD and spectral discontinuous Galerkin method for time domain simulation
  - Forward and inverse scattering methods for 3D objects in layered earth
  - Joint electromagnetic and acoustic inversion method with mutual information method
  - Novel diagonal tensor approximation (DTA)
  - Nonuniform fast Fourier transform (NUFFT) for imaging

- Inversion of measured 2D and 3D data sets
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Seismic/Electromagnetic Joint Inversion with Mutual Information Method

Joint Image $\alpha A + B$.  Seismic Image $A$.  EM Image $B$. 
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Through-Wall Imaging Results

Wall Imaging: Austria Profile in a 5-layer model
Configuration, source/receiver interval = 0.20 m

Air
\[ \varepsilon_w = 4.0 \quad \sigma_w = 0.01 \ S/m \]

\[ 0.2 \ m \]

Walls

32 sources and receivers
Single-Frequency Imaging

Single-frequency Wall Imaging: Austria Profile

The aperture size $L = 3$ m, a 2-side linear array

100 MHz

200 MHz

300 MHz

400 MHz

Poor due to Inadequate # of sensors
Multi-Frequency Adaptivity

Multifrequency Wall Imaging: Austria Profile

The aperture size $L = 3$ m, a 2-side linear array

100 MHz

100 MHz $\rightarrow$ 200 MHz

200 MHz $\rightarrow$ 300 MHz

300 MHz $\rightarrow$ 400 MHz
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Target-I : Geometry and Parameters

Air

Circular array
8 x 241

20 mm

60 mm

20 mm

D

200 mm

200 mm

$\epsilon_r = 1.45$

8 x 241

30 mm

$\epsilon_r = 3.0$
Multi-Frequency Inversion of Target I

Left: $\varepsilon_r$

Right: $\sigma$

$f = 2\text{GHz}$

$f = 2\text{GHz} \rightarrow 3\text{GHz}$
$f = 3\, \text{GHz} \rightarrow 4\, \text{GHz}$

$\ f = 4\, \text{GHz} \rightarrow 5\, \text{GHz}$
\[ f = 5 \text{GHz} \rightarrow 6 \text{GHz} \]

\[ f = 6 \text{GHz} \rightarrow 7 \text{GHz} \]
\[ f = 7 \text{GHz} \rightarrow 8 \text{GHz} \]

\[ f = 8 \text{GHz} \rightarrow 9 \text{GHz} \]
Target-III: Geometry and Parameters

Circular array
18 x 241

$D$

$\varepsilon_r = 3.0$

$\varepsilon_r = 3.0$

$\varepsilon_r = 1.45$

20 mm

60 mm

30 mm

200 mm

Air
Inversion Results: Target III

Left: \( E_r \)

Right: \( \sigma \)

\[ f = 2\text{GHz} \]

\[ f = 2\text{GHz} \rightarrow 3\text{GHz} \]
\[ f = 3\text{GHz} \rightarrow 4\text{GHz} \]

\[ f = 4\text{GHz} \rightarrow 5\text{Hz} \]
$f = 5\text{GHz} \rightarrow 6\text{GHz}$

$f = 6\text{GHz} \rightarrow 7\text{Hz}$
\[ f = 7\text{GHz} \rightarrow 8\text{GHz} \]
\[ f = 8\text{GHz} \rightarrow 9\text{Hz} \]
\[ f = 9 \text{GHz} \rightarrow 10 \text{Hz}. \]
\[ f = 9\,GHz \rightarrow 10\,GHz \]
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NUFFT for Subsurface Imaging

Nonuniform Fast Fourier Transform (NUFFT)
- To enable the discrete Fourier analysis of non-uniformly sampled data
- To take advantage of the Fast Fourier Transform (FFT) algorithm
- To retain the accuracy of DFT

Application in GPR Migration Signal Processing
- To enable Fourier reconstruction directly without linear interpolation
- To retain the computation accuracy with down-sampled data
NUFFT Method

DFT

\[ f_j = \sum_{k=0}^{N} \alpha_k e^{i \omega_k \cdot 2\pi t_j} \]

\[ \omega_k \in [-N/2, N/2], t_j = j/N \]

Non-uniform \( \omega_k \)

\[ \min \left| W_0^{m \omega_k j} - s^{-1} \sum_{l=[m \omega_k] - q/2}^{[m \omega_k] + q/2} \rho_{k-l} W_0^{lj} \right| \]

Uniform \( q+1 \)

\[ W_0 = e^{i 2\pi / mN} \]
NUFFT Method

Approximate each $W_0^{m\omega_kj}$ in terms of a $q+1$-term Fourier series

$$s_j W_0^{m\omega_kj} = \sum_{l=[m\omega_k]-q/2}^{[m\omega_k]+q/2} \rho_{k-l} W_0^{lj}$$

Approximate the value of a Fourier series at each in terms of values at the nearest $q$ uniform nodes

New Fourier coefficient:

$$\tau_l = \sum \alpha_k \rho_{k-[m\omega_k]}$$

Regular FFT:

$$\{\tau_l\} \rightarrow \{T_j\}$$
NUFFT Numerical Results

Graph showing the real part of the Fourier Transform for different methods: DFT, NUFFT, and Linear Interp., plotted against frequency.
NUFFT Numerical Results

![Graph showing numerical results for NUFFT and Linear Interp. The x-axis represents frequency, and the y-axis represents real part error. The graph compares the error for NUFFT and Linear Interp across different frequency ranges, with error values ranging from $10^{-8}$ to $10^0$.](image-url)
Formulation

\[ U(k_x, 0, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, 0, t)e^{-i(k_x x - \omega t)} \, dx \, dt \]

Wave-number Space (Non-uniform*)

\[ \omega^2 = v_0 (k_x^2 + k_z^2) \]
Formulation (Cont)

- **Inverse 2D Fourier Transform**

\[
u(x, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(k_x, z, \omega) \exp[i(k_x x - \omega t)] dk_x \, d\omega
\]

**Phase Shift**

\[
U(k_x, z, \omega) = U(k_x, 0, \omega) \exp[i \int k_z \, dz] = U(k_x, 0, \omega) \exp[i k_z z]
\]

**Migration**

\[
u(x, z, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(k_x, 0, \omega) \exp[i(k_x x + k_z z)] dk_x \, d\omega
\]

**NUFFT**

\[
u(x, z, 0) = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} U(k_x, 0, \omega) \exp[i k_z z] \, d\omega \right) \exp[i k_x x] \, dk_x
\]

\[
= \text{FFT} \left\{ \text{NUFFT} \left[ U(k_x, 0, \omega) \right] \right\}
\]
2D Results (Niitek)

Raw data

Migrated
Fast data acquisition by NUFFT

*NUFFT migration has good performance with down-sampled data, due to the non-uniformity of the wave-number space
3D NUFFT Imaging Results

- Niitek data: metallic and plastic landmines
- Georgia Tech data: GT plywood and chamber
- Cross section (horizontal plane) interested
- 3D animation

GT plywood

Chamber in sand
Acquisition Configuration (Georgia Tech)

Free space acquisition

![Diagram showing antenna configuration with phase center, array coordinate reference point, unit in cm, target, and styrofoam pedestal.]
3D Results (GT plywood)

Raw data  

Migrated image

Migration
3D Results (GT plywood)

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>width</td>
<td>38.5cm</td>
<td>38cm</td>
</tr>
<tr>
<td>height</td>
<td>46.5cm</td>
<td>44cm</td>
</tr>
<tr>
<td>thick</td>
<td>1.8cm</td>
<td>1.76cm</td>
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</table>

* Note: 1 pixel resolution = 2 cm (horizontal), 0.29 cm (vertical)
Acquisition Configuration (Georgia Tech)

Buried object acquisition

Chamber
3D Results (Buried chamber)

Raw data | Migrated image

At an estimated depth of 12cm from interface
3D Results (Buried chamber)

Raw data

Migrated image

At an estimated depth of 17 cm from interface
# 3D Results (Buried Chamber)

## Numerical verification

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Estimated</th>
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</thead>
<tbody>
<tr>
<td><strong>Width</strong></td>
<td>40.64 cm</td>
<td>42 cm</td>
</tr>
<tr>
<td><strong>length</strong></td>
<td>30.48 cm</td>
<td>34 cm</td>
</tr>
<tr>
<td><strong>height</strong></td>
<td>20.32 cm</td>
<td>16.1 cm</td>
</tr>
<tr>
<td><strong>depth</strong></td>
<td>9.5 cm</td>
<td>10.5 cm</td>
</tr>
</tbody>
</table>

*Note: 1 pixel resolution = 2 cm (horizontal), 0.16 cm (vertical)*
Summary

• 2D, 3D and multimodality inversion methods have been developed
• Inversion methods have been successfully applied to image some difficult targets from Institut Fresnel. These are 2D measured results for TM polarization.
• NUFFT has been applied to obtain high-fidelity 3D images from Georgia Tech data
• Through-wall imaging is investigated and is highly promising.