Multi-Sensor Inversion via Rigorous Analysis of Underlying Waves

Qing H. Liu
Duke University
qhliu@ee.duke.edu

MURI Program Review
January 13, 2005

Outline

- Summary of Progress
- Joint EM/Seismic Multi-Modality Inversion
- Through-Wall Imaging Results
- Inversion of data collected by Institut Fresnel in June 2004
- Inversion of data collected by Georgia Tech (W. Scott) in Dec. 2004

Summary of Research Progress

- Objective: To use the wave physics with fast algorithms to enhance multi-modality inversion and imaging.
- Developed
  - PSTD and spectral discontinuous Galerkin method for time domain simulation
  - Forward and inverse scattering methods for 3D objects in layered earth
  - Joint electromagnetic and acoustic inversion method with mutual information method
  - Novel diagonal tensor approximation (DTA)
  - Nonuniform fast Fourier transform (NUFFT) for imaging
- Inversion of measured 2D and 3D data sets

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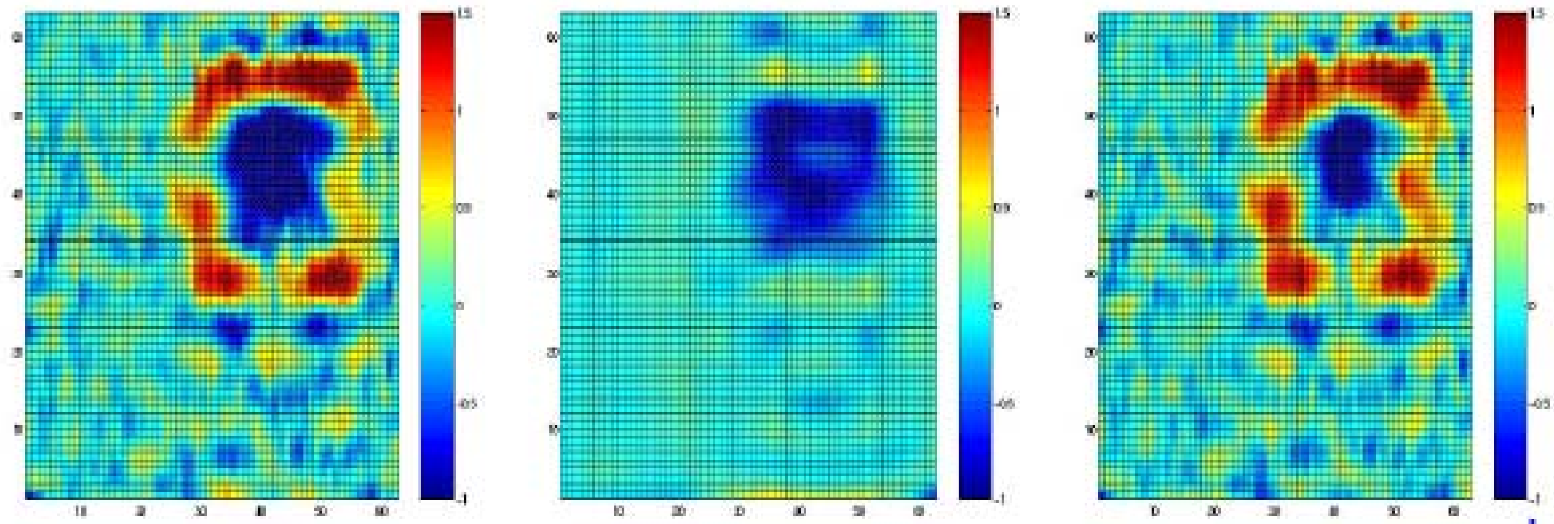
Seismic/Electromagnetic Joint Inversion with Mutual Information Method



Joint Image $\alpha A + B$. Seismic Image A. EM Image B.

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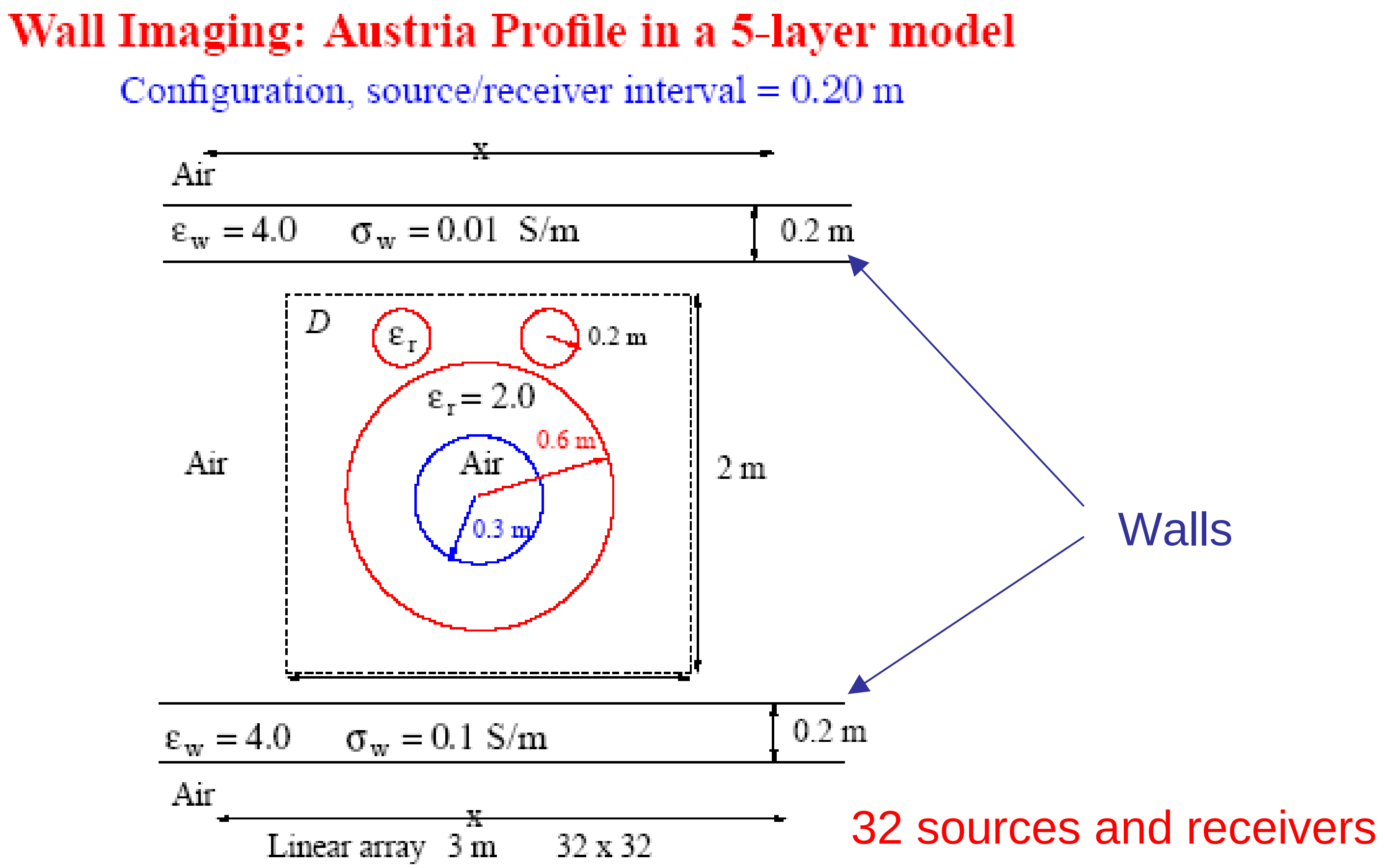
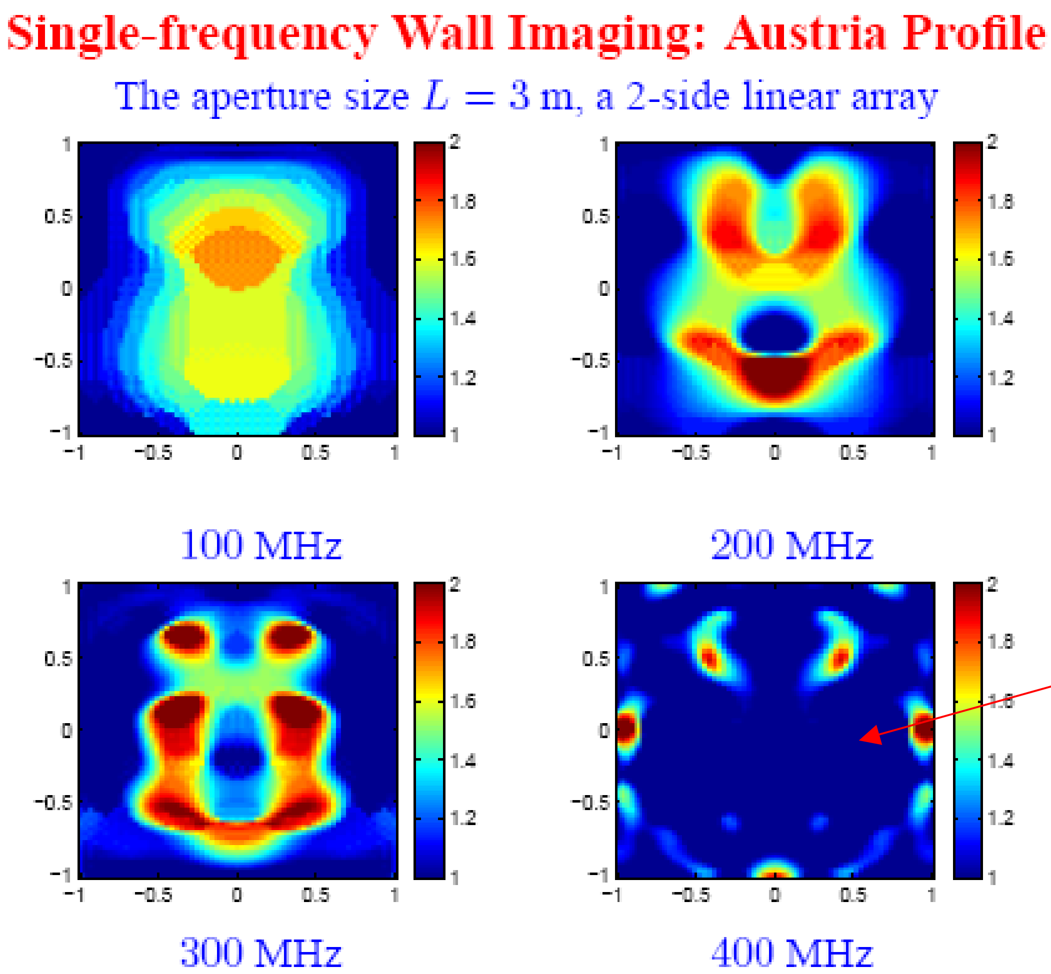
Through-Wall Imaging Results

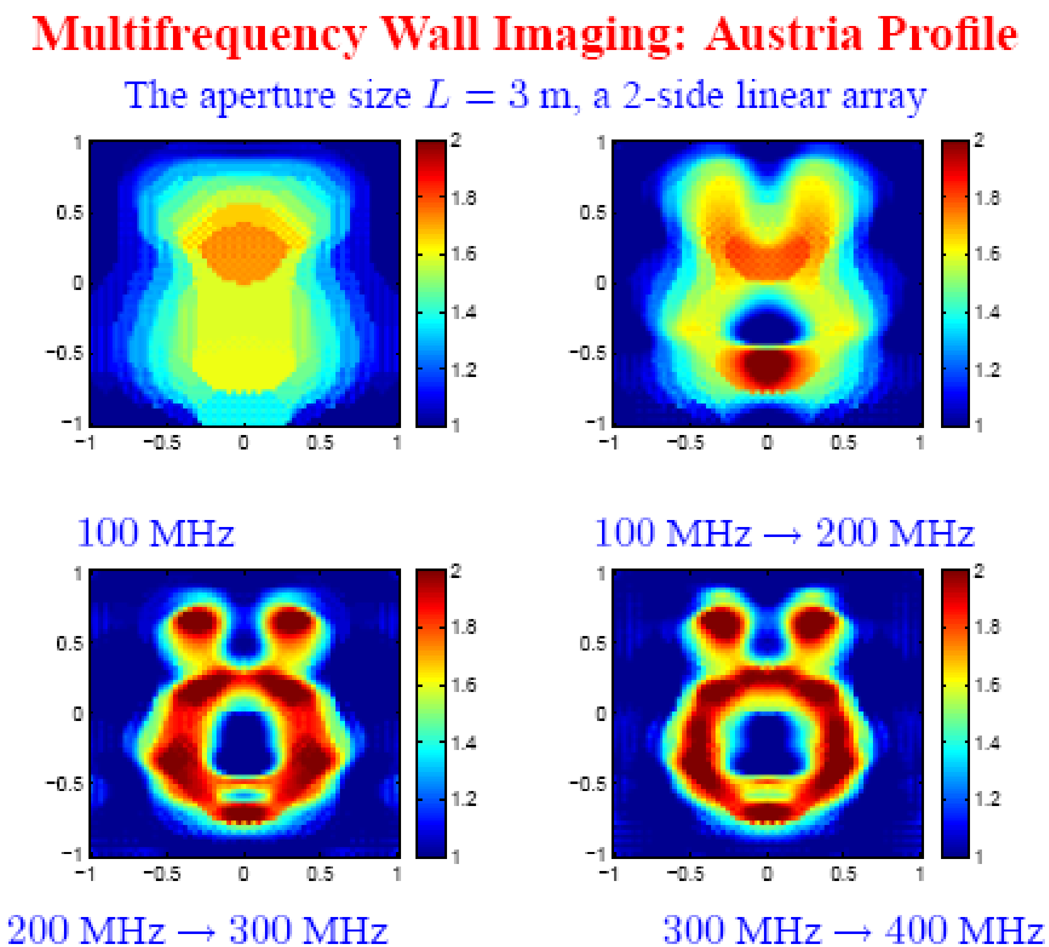


Single-Frequency Imaging



Poor due to Inadequate # of sensors

Multi-Frequency Adaptivity



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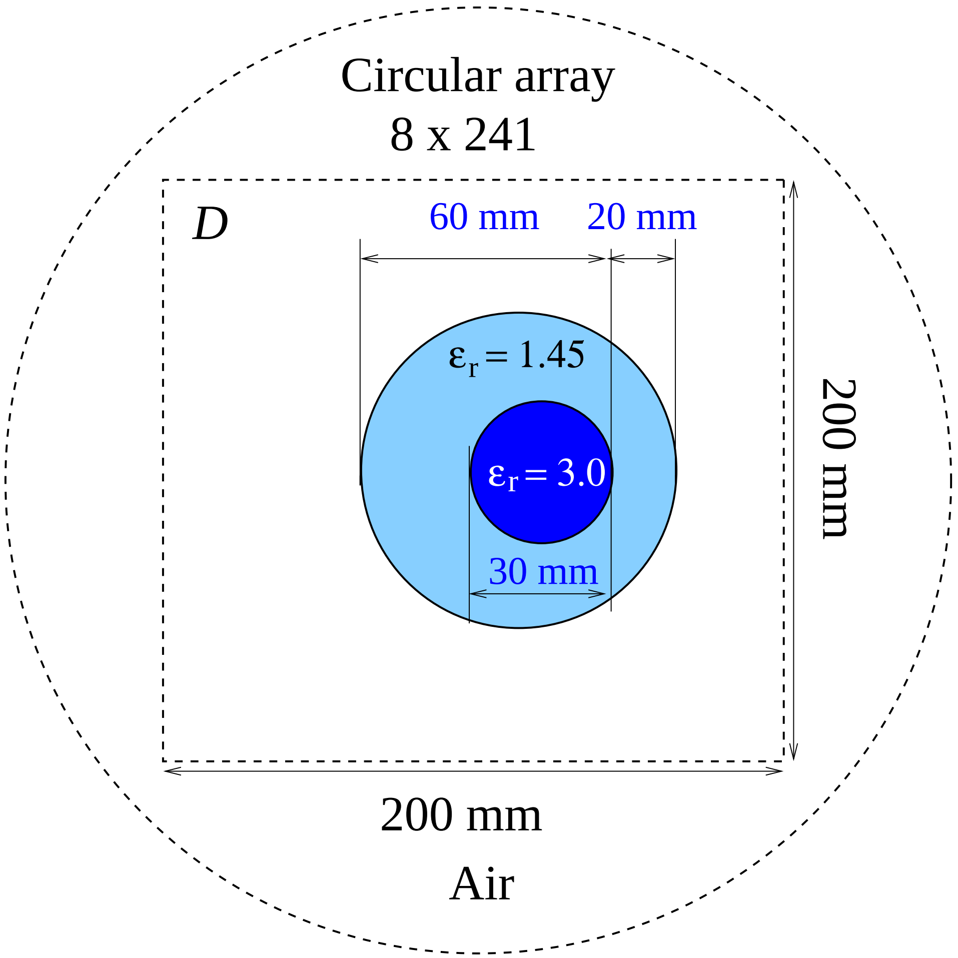
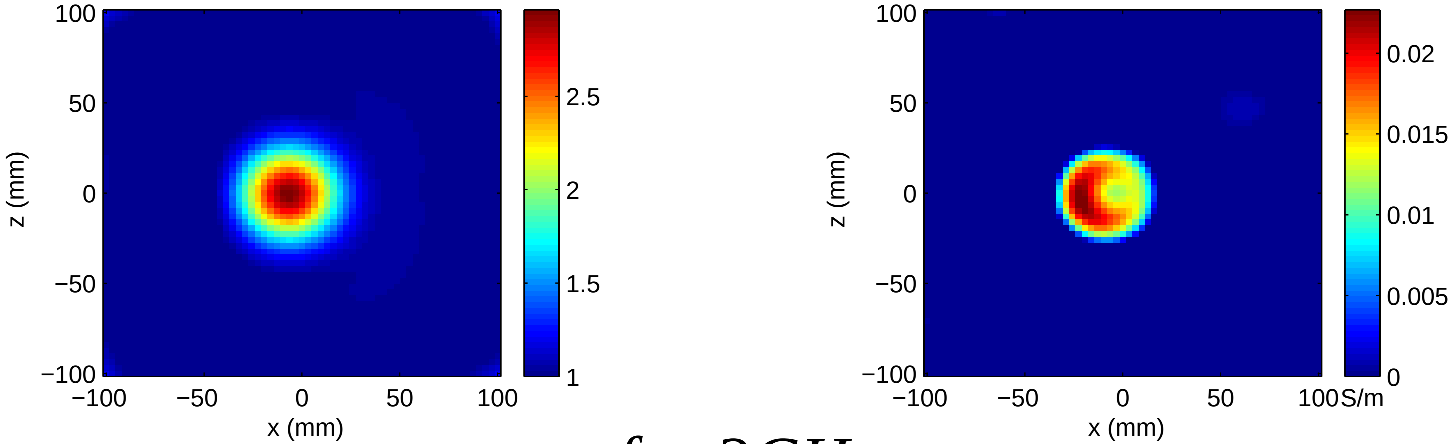
Target-I : Geometry and Parameters

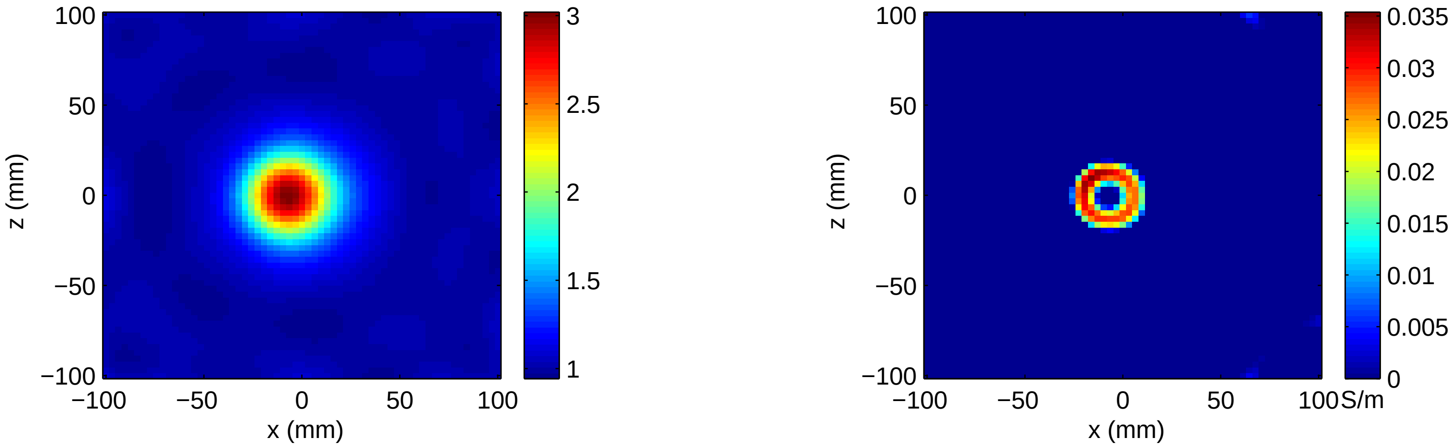


Multi-Frequency Inversion of Target I

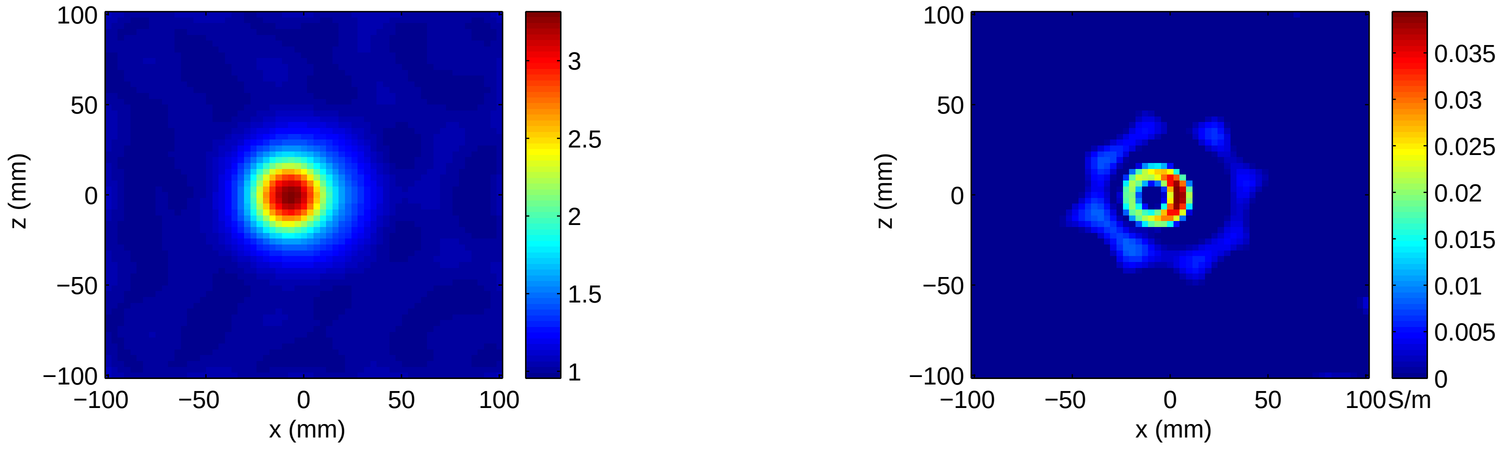
Left: ε_r Right: σ



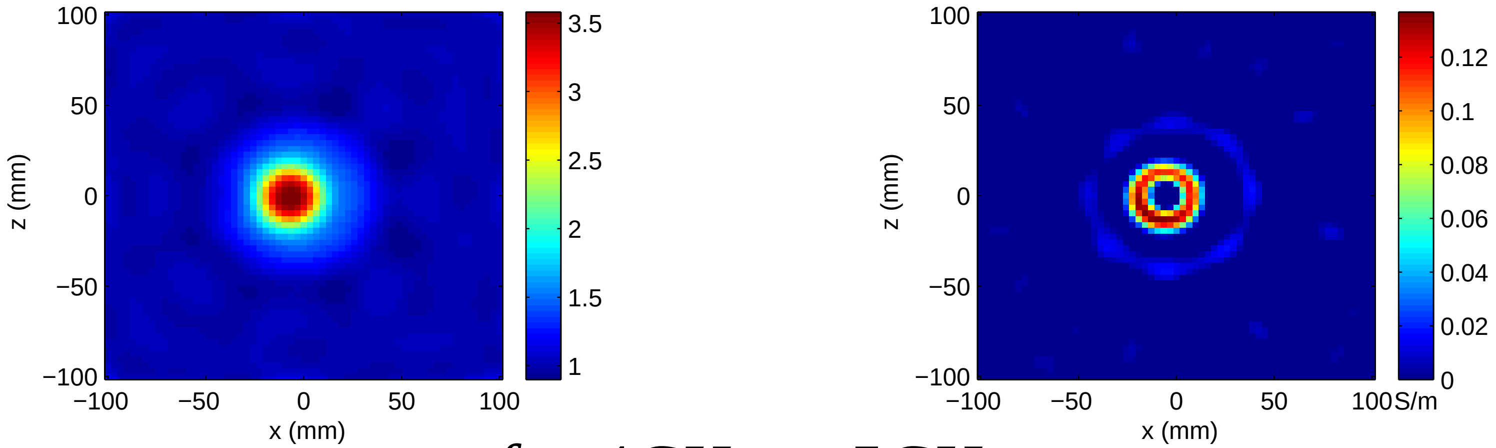
$$f = 2GHz$$



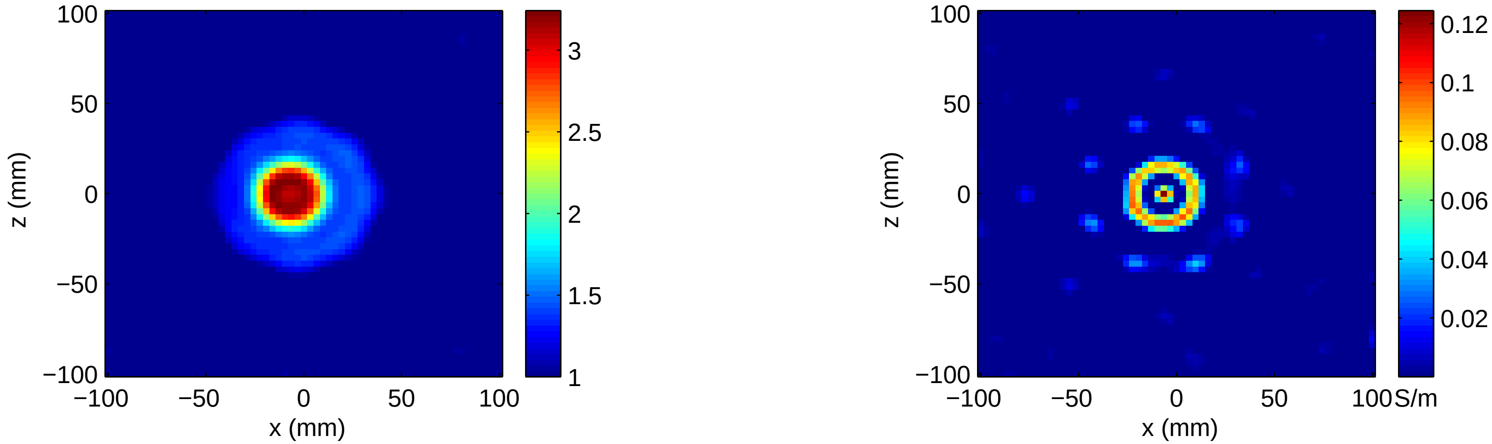
$$f = 2GHz \rightarrow 3GHz$$



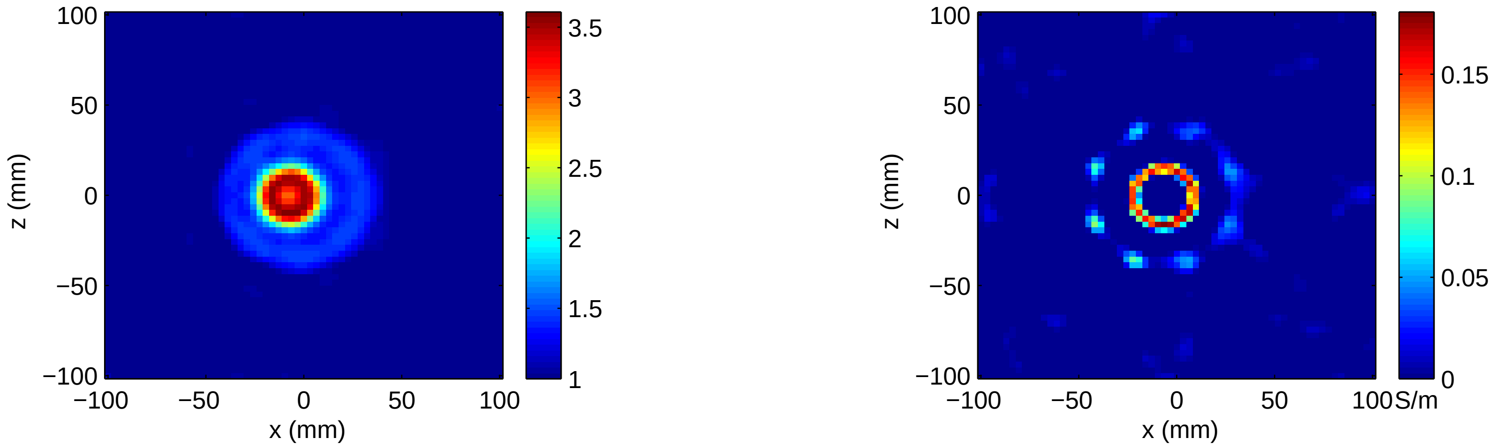
$$f = 3GHz \rightarrow 4GHz$$



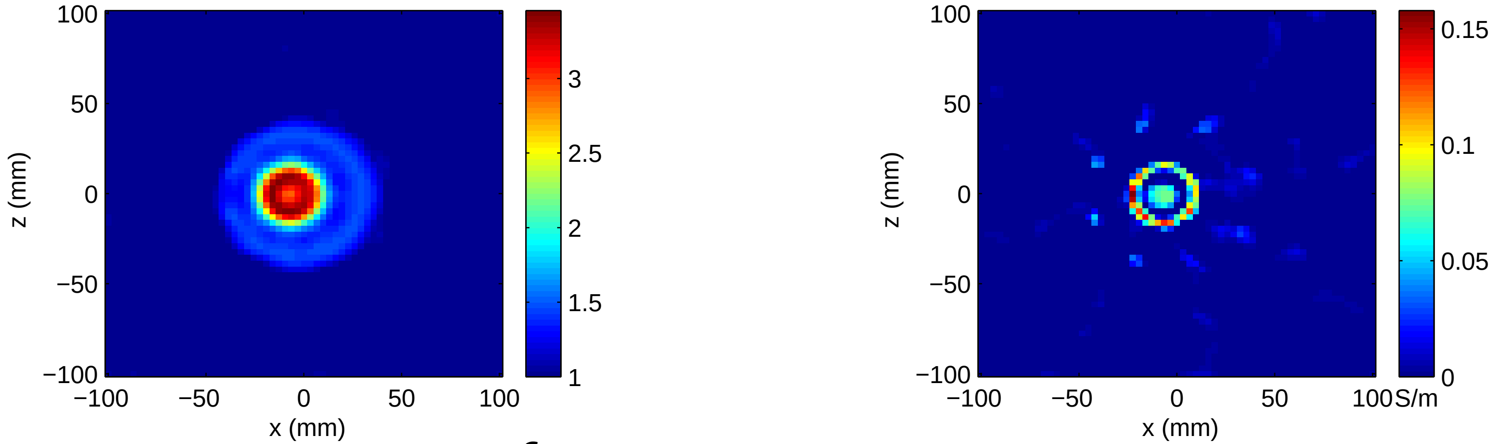
$$f = 4GHz \rightarrow 5GHz$$



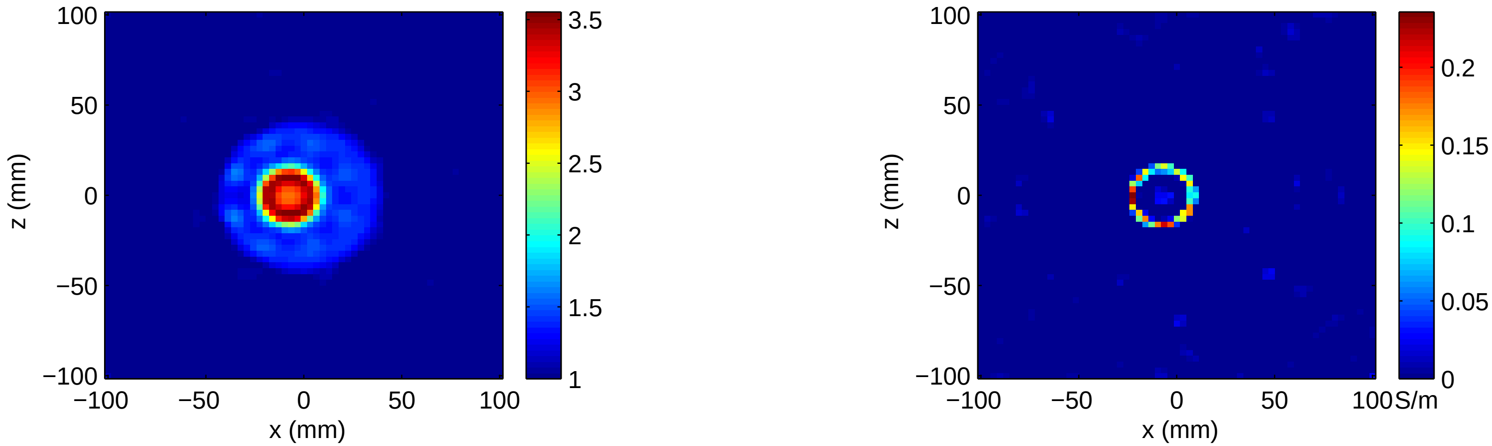
$$f = 5GHz \rightarrow 6GHz$$



$$f = 6GHz \rightarrow 7GHz$$

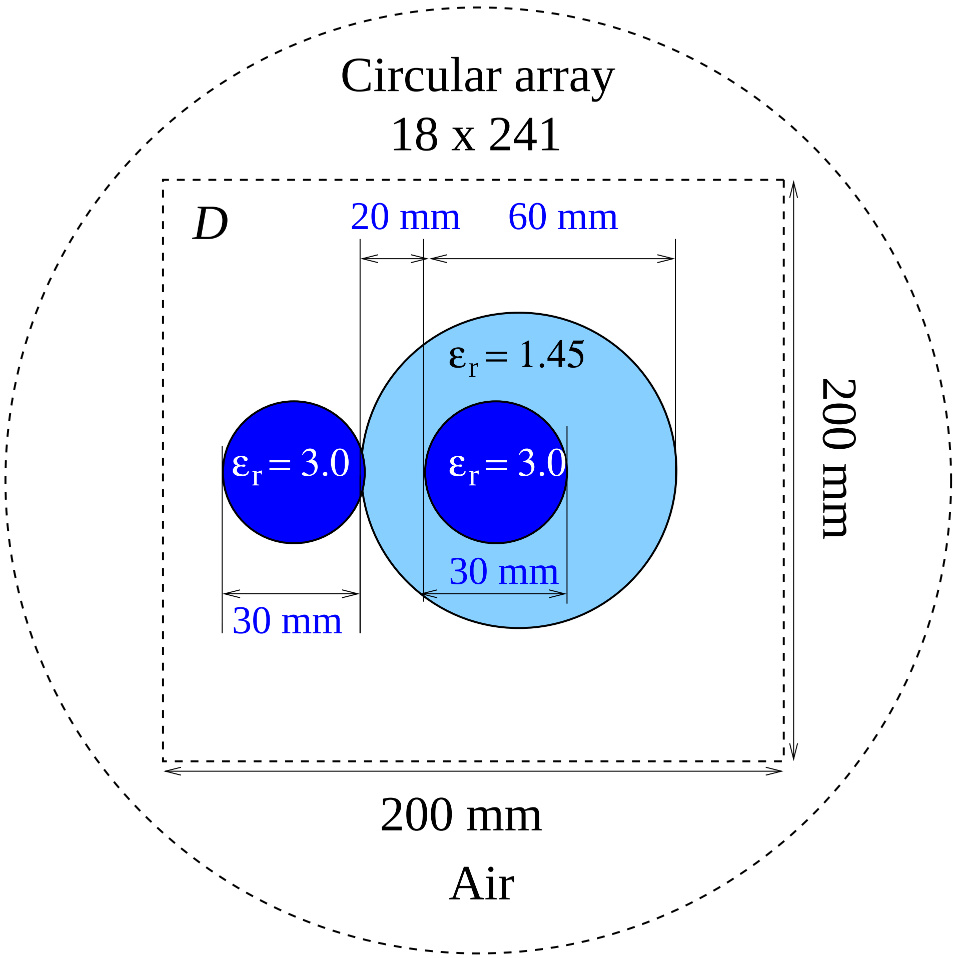


$$f = 7GHz \rightarrow 8GHz$$



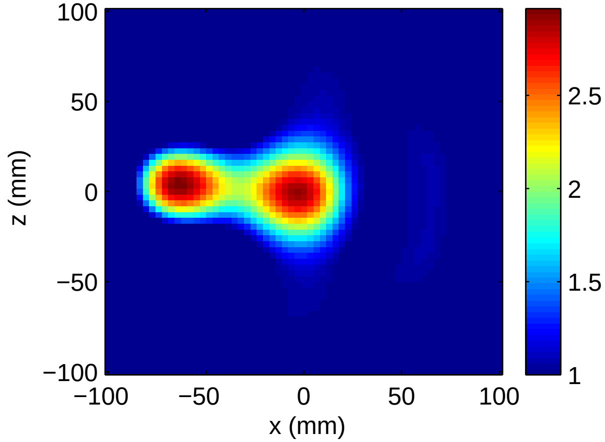
$$f = 8GHz \rightarrow 9GHz$$

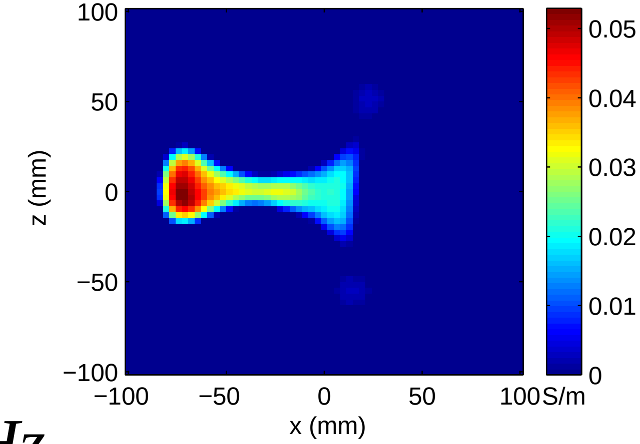
Target-III: Geometry and Parameters



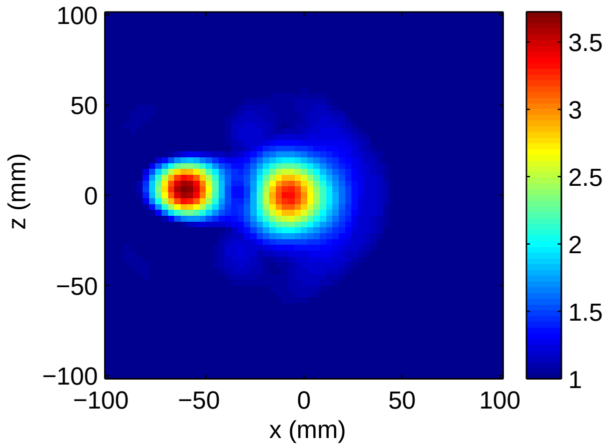
Inversion Results: Target III

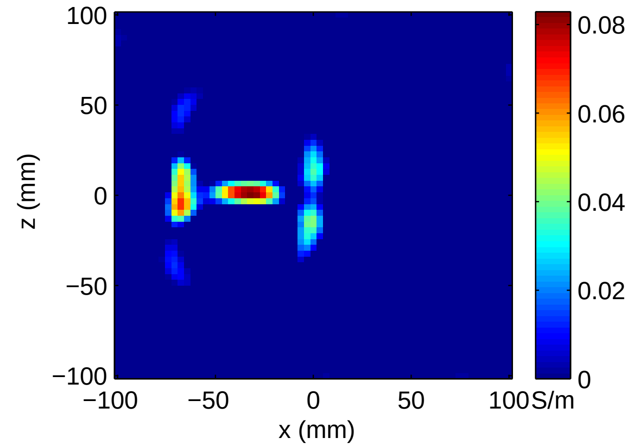
Left: ε_r **Right:** σ



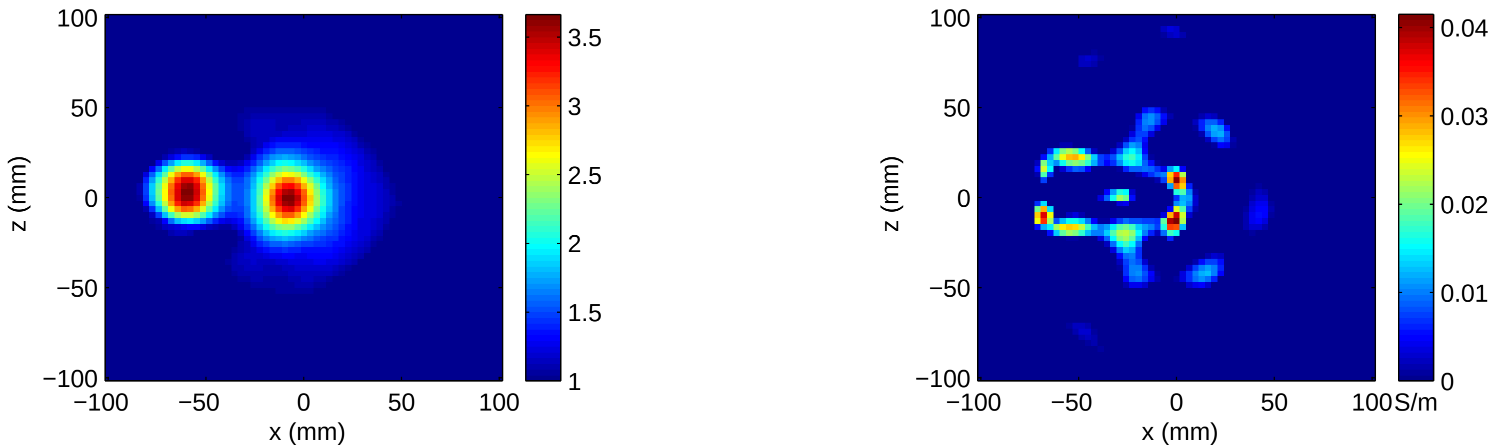


$$f = 2GHz$$

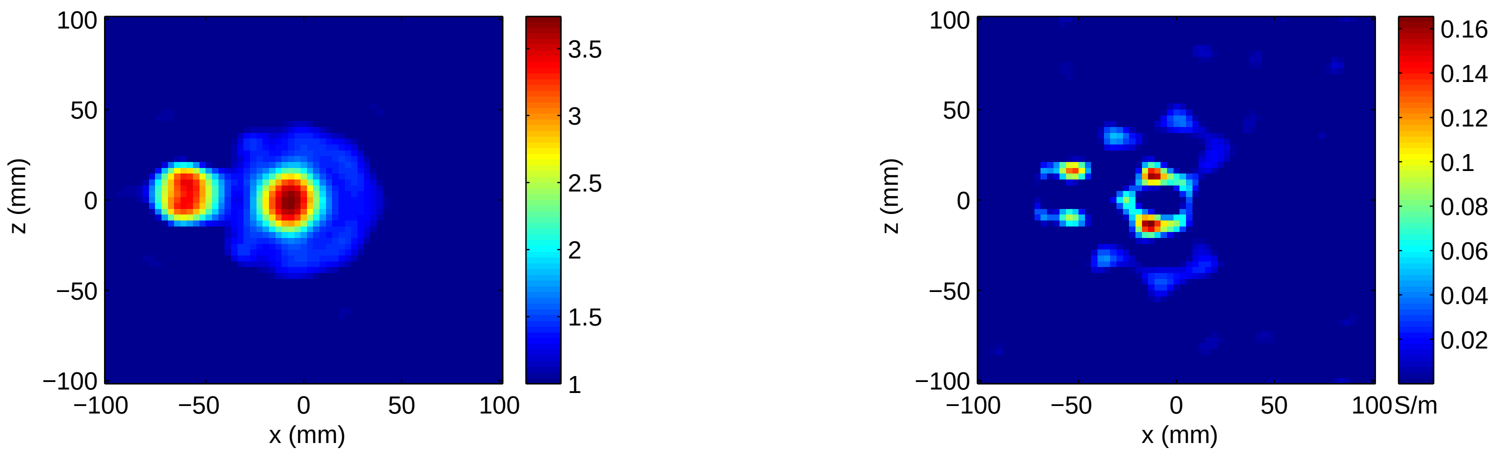




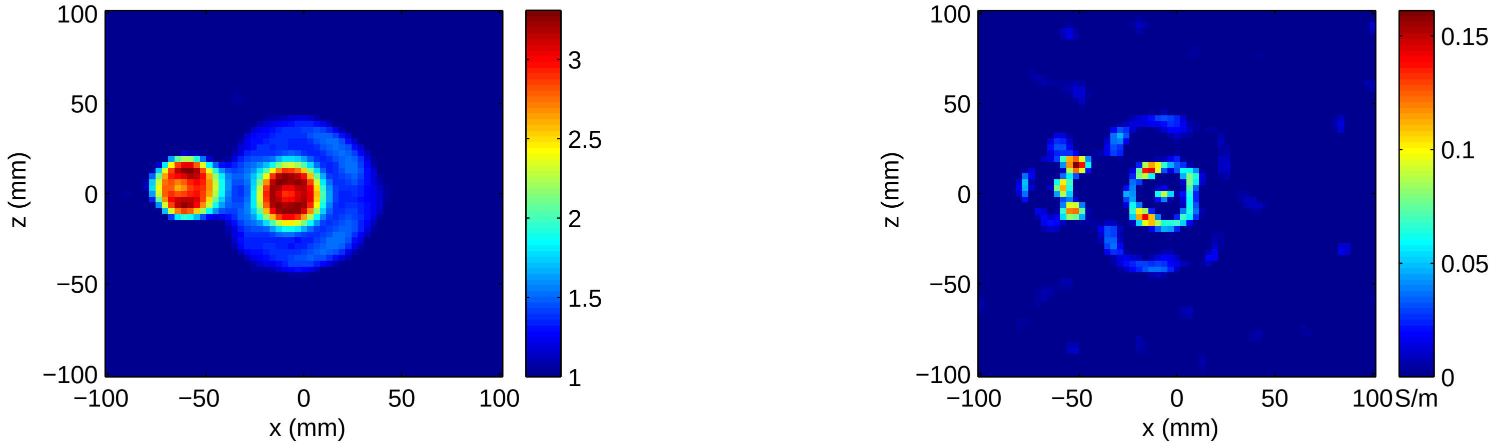
$$f = 2GHz \rightarrow 3GHz$$



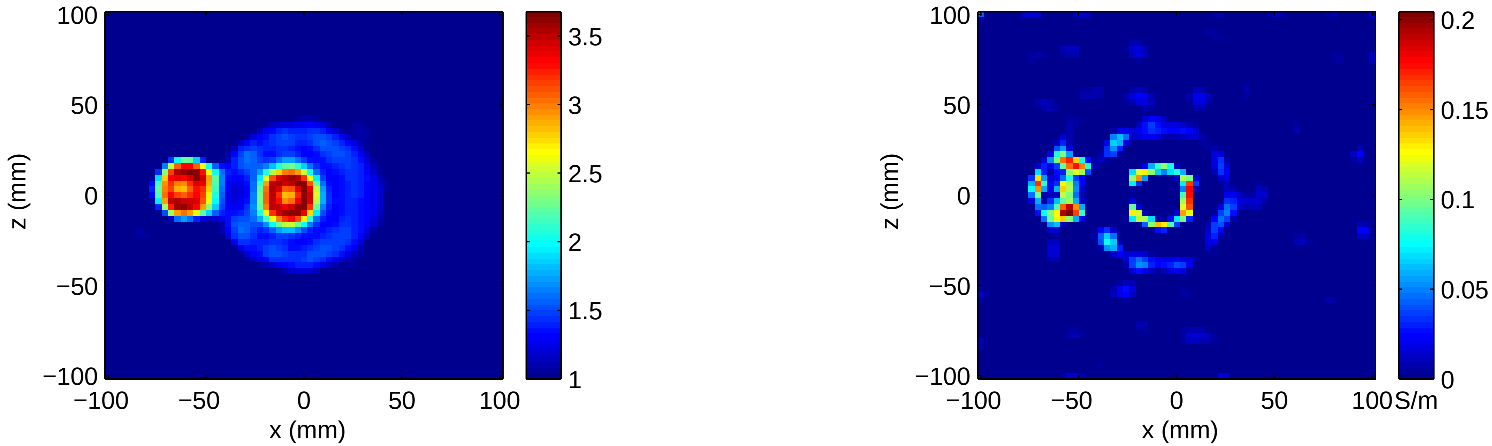
$$f = 3GHz \rightarrow 4GHz$$



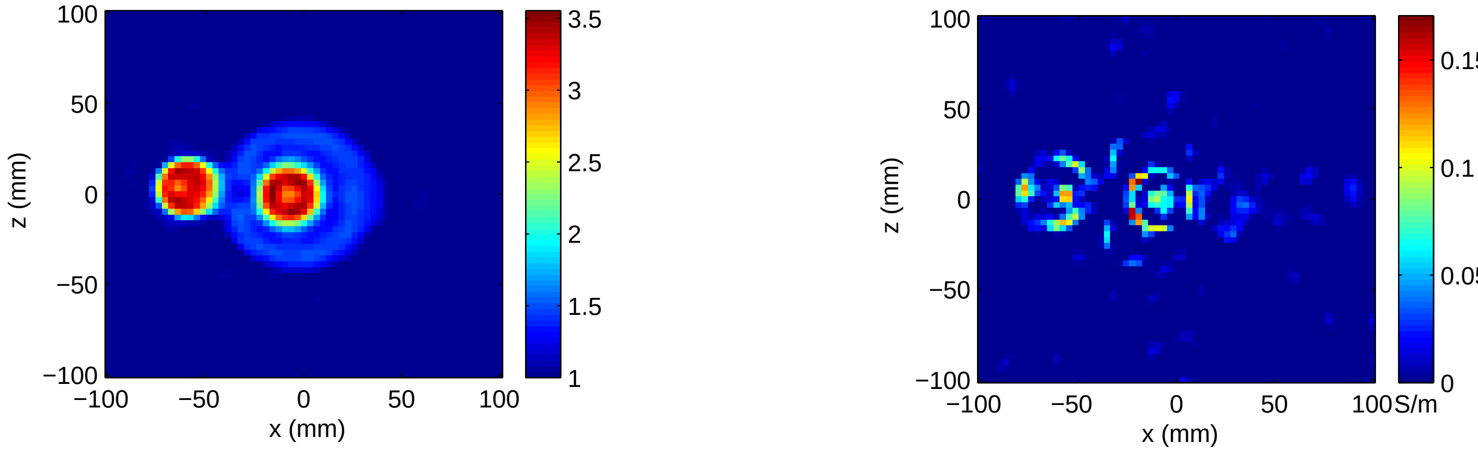
$$f = 4GHz \rightarrow 5Hz$$



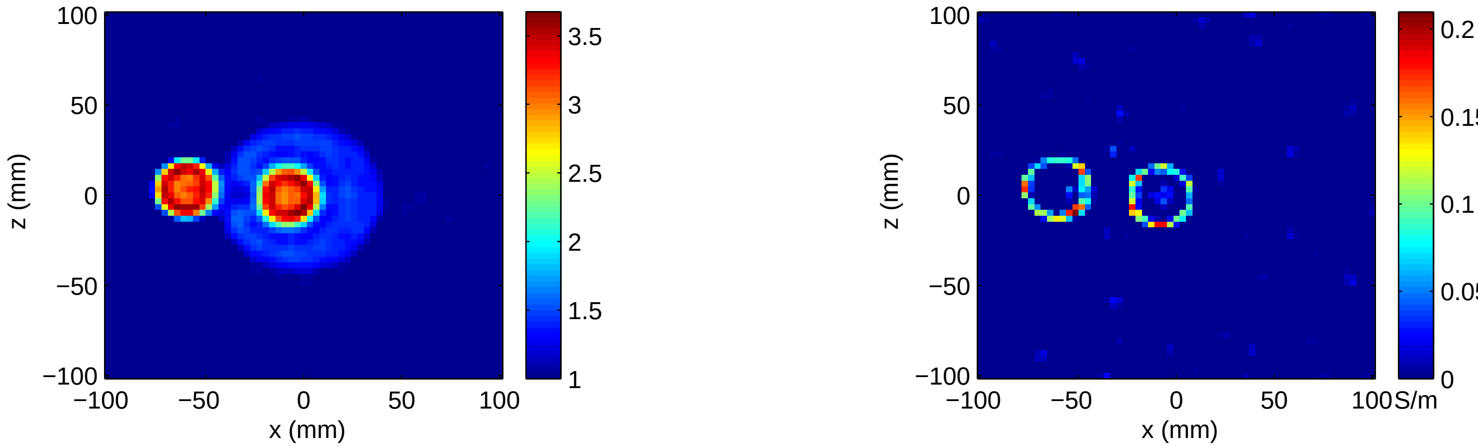
$$f = 5GHz \rightarrow 6GHz$$



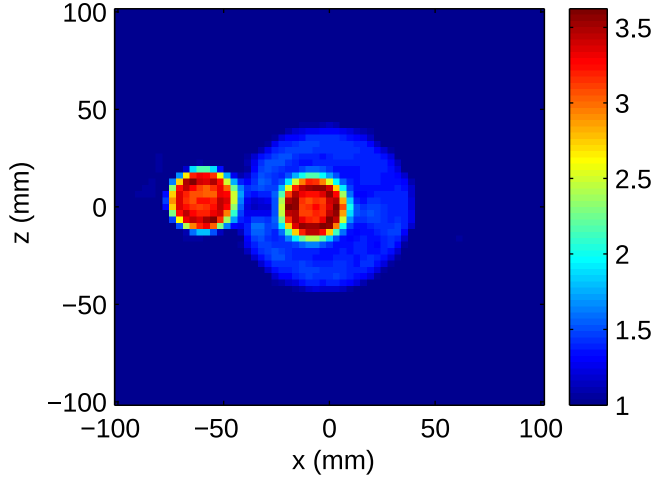
$$f = 6GHz \rightarrow 7Hz$$

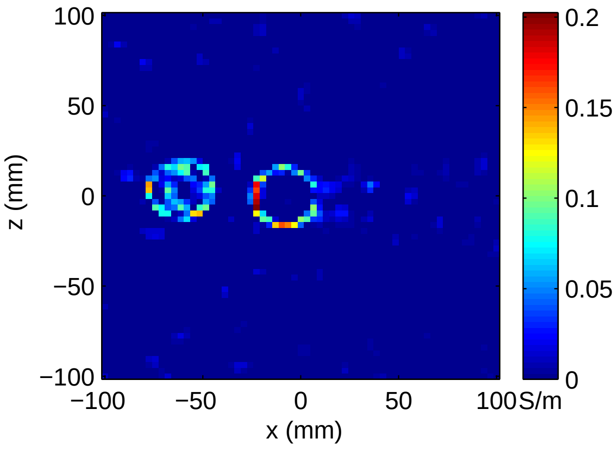


$$f = 7GHz \rightarrow 8GHz$$

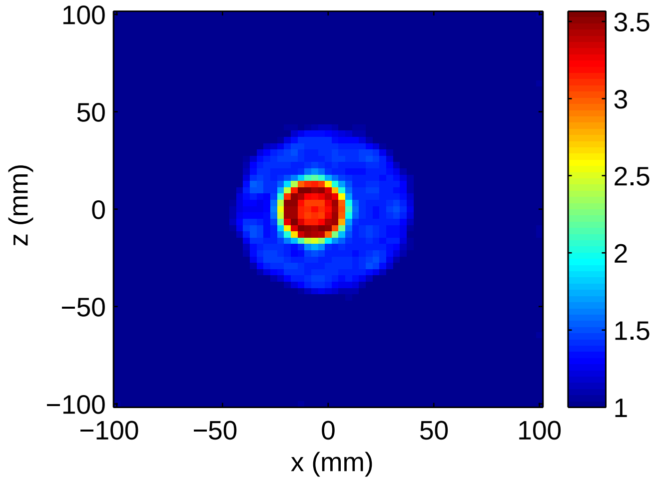


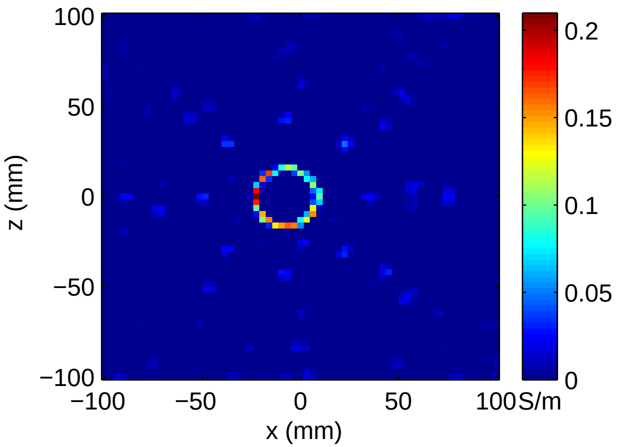
$$f = 8GHz \rightarrow 9Hz$$





$$f = 9GHz \rightarrow 10Hz$$





$$f = 9GHz \rightarrow 10GHz$$

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NUFFT for Subsurface Imaging

Nonuniform Fast Fourier Transform (NUFFT)

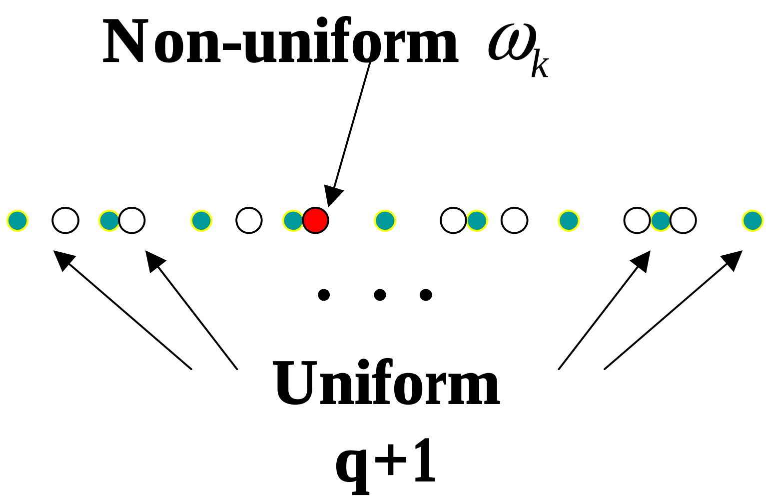
- **To enable the discrete Fourier analysis of non-uniformly sampled data**
- **To take advantage of the Fast Fourier Transform (FFT) algorithm**
- **To retain the accuracy of DFT**

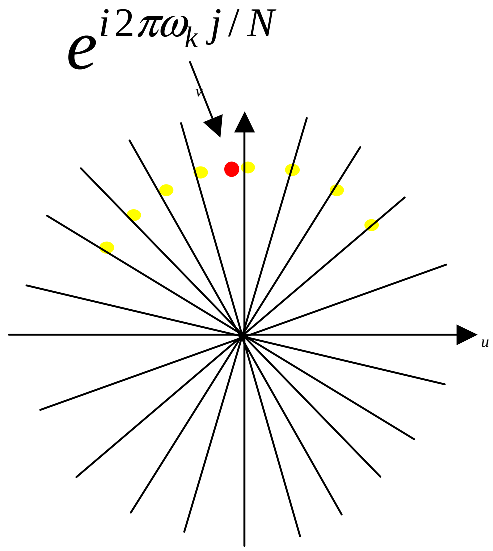
Application in GPR Migration Signal Processing

- **To enable Fourier reconstruction directly without linear interpolation**
- **To retain the computation accuracy with down-sampled data**

NUFFT Method

DFT $$f_j = \sum_{k=0}^{N} \alpha_k e^{i\omega_k \cdot 2\pi t_j}$$ $$\omega_k \in [-N/2, N/2], t_j = j/N$$





$$\min \left| W_0^{m\omega_k j} - s_j^{-1} \sum_{l=[m\omega_k]-q/2}^{[m\omega_k]+q/2} \rho_{k-[m\omega_k]} W_0^{lj} \right|$$ $$W_0 = e^{i2\pi/mN}$$

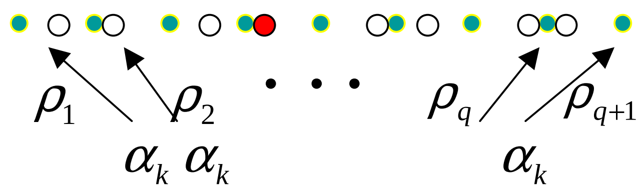
NUFFT Method

Approximate each $W_0^{m\omega_k j}$ in terms of a q+1-term Fourier series

$$s_j W_0^{m\omega_k j} = \sum_{l=[m\omega_k]-q/2}^{[m\omega_k]+q/2} \rho_{k-[m\omega_k]} W_0^{lj}$$

Approximate the value of a Fourier series at each in terms of values at the nearest q uniform nodes

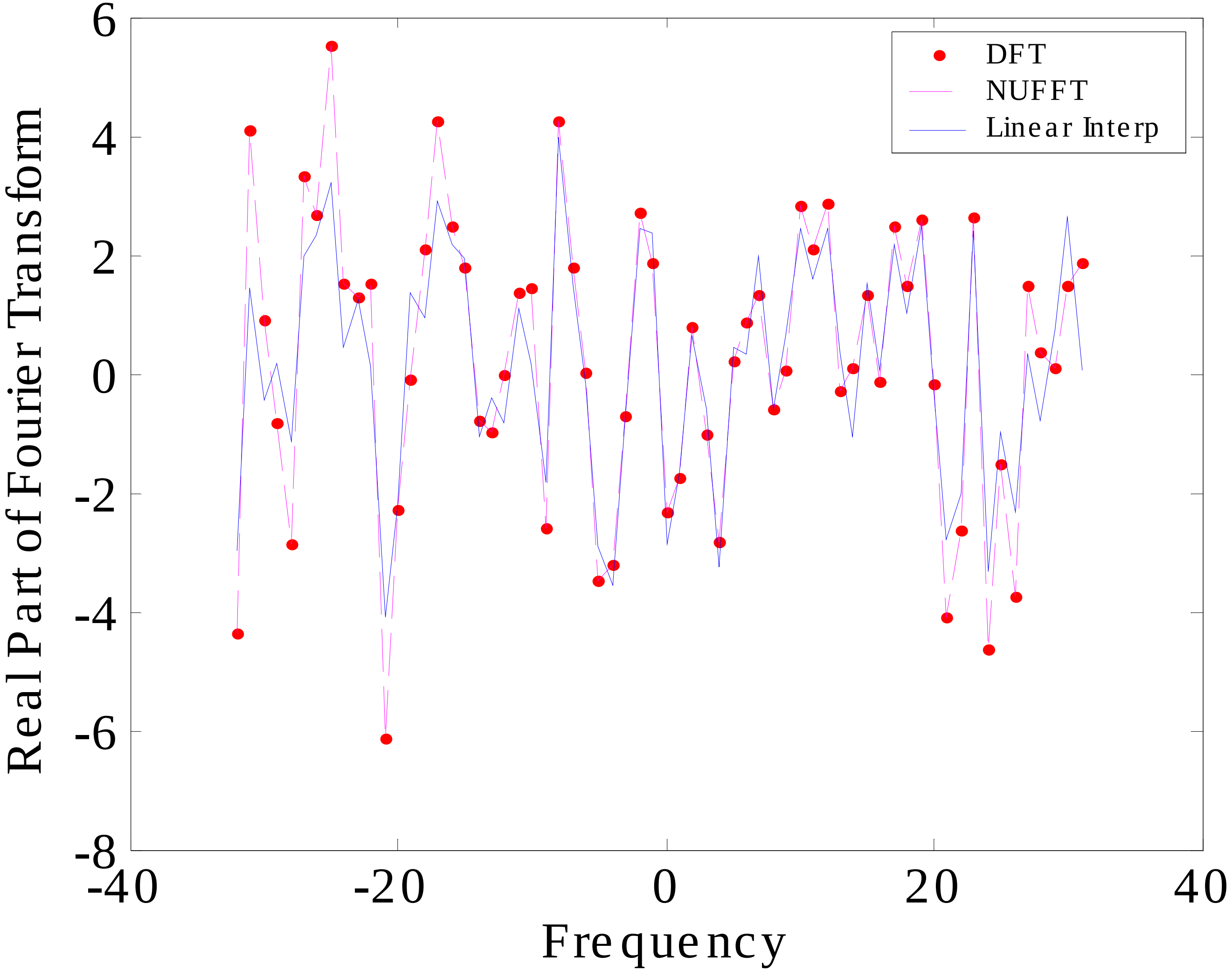
New Fourier coefficient:



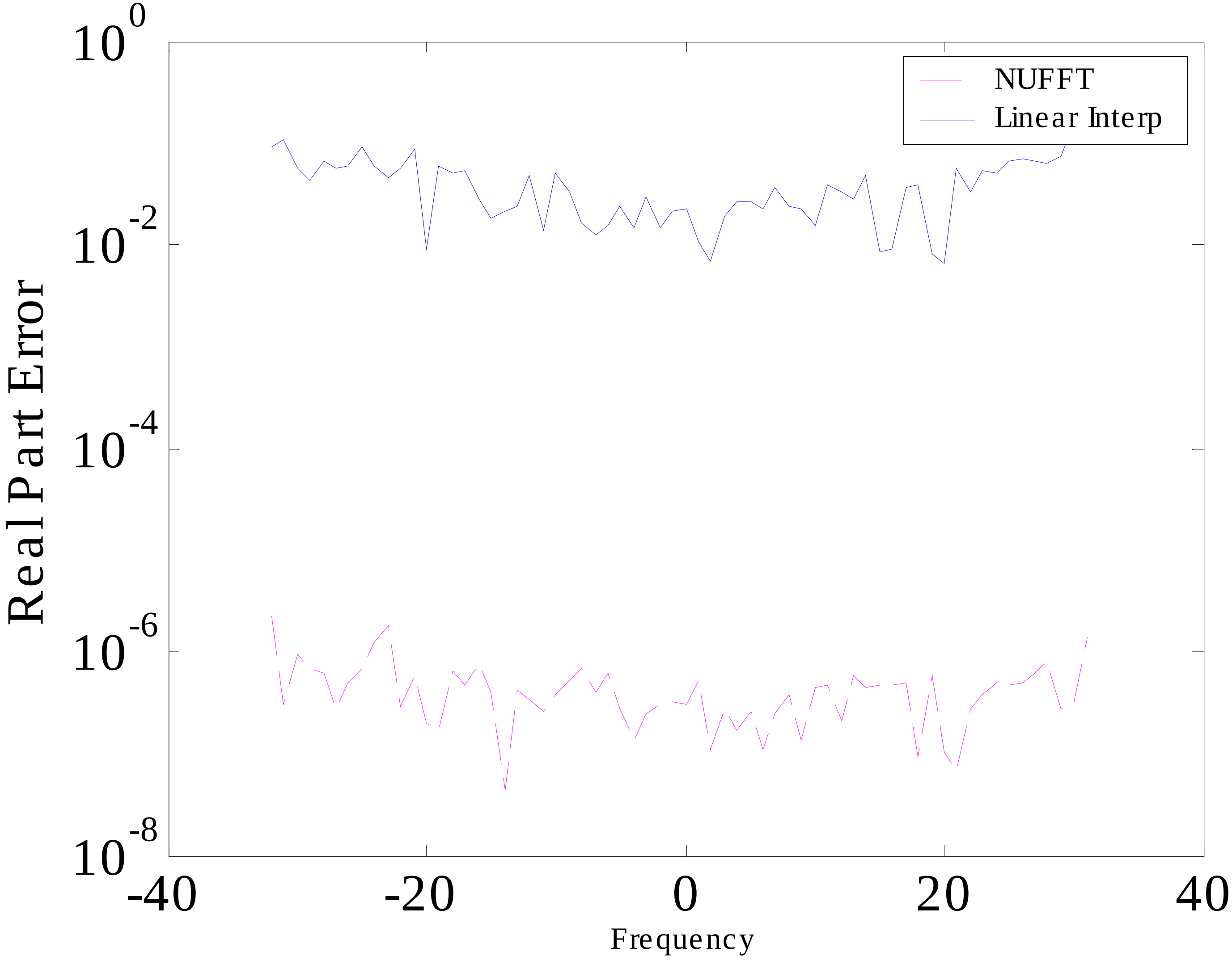
$$\tau_l = \sum \alpha_k \rho_{k-[m\omega_k]}$$

Regular FFT: $\{\tau_l\} \rightarrow \{T_j\}$

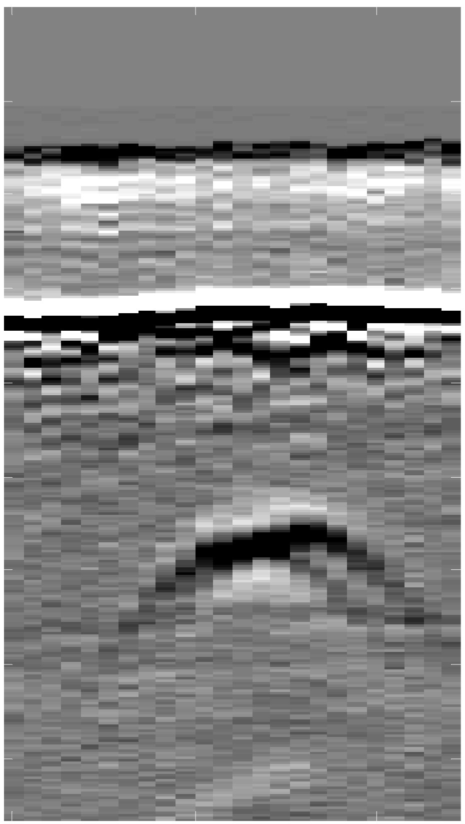
NUFFT Numerical Results



NUFFT Numerical Results

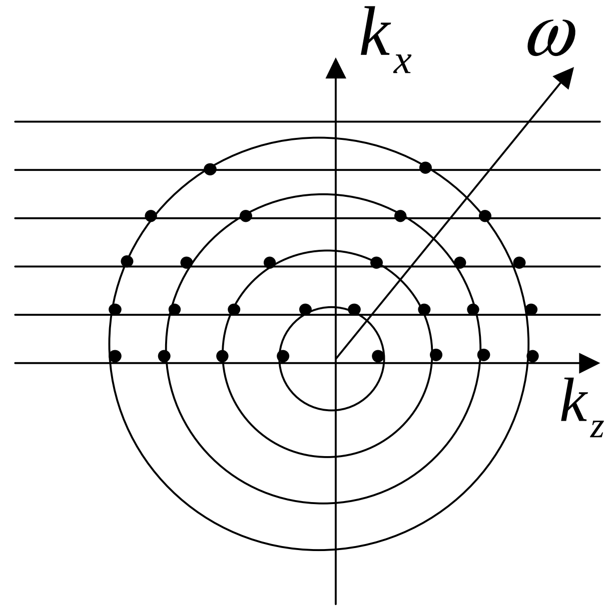


Formulation



$$U(k_x, 0, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, 0, t) e^{-i(k_x x - \omega t)} dx dt$$

Fourier Transform



Wave-number Space
(Non-uniform*)

$$\omega^2 = v_0 (k_x{}^2 + k_z{}^2)$$

Formulation (Cont)

- Inverse 2D Fourier Transform

$$u(x,z,t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(k_x,z,\omega)\exp[i(k_x x-\omega t)]dk_x d\omega$$

Phase Shift $U(k_x,z,\omega)=U(k_x,0,\omega)\exp[i\int k_z dz]=U(k_x,0,\omega)\exp[ik_z z]$

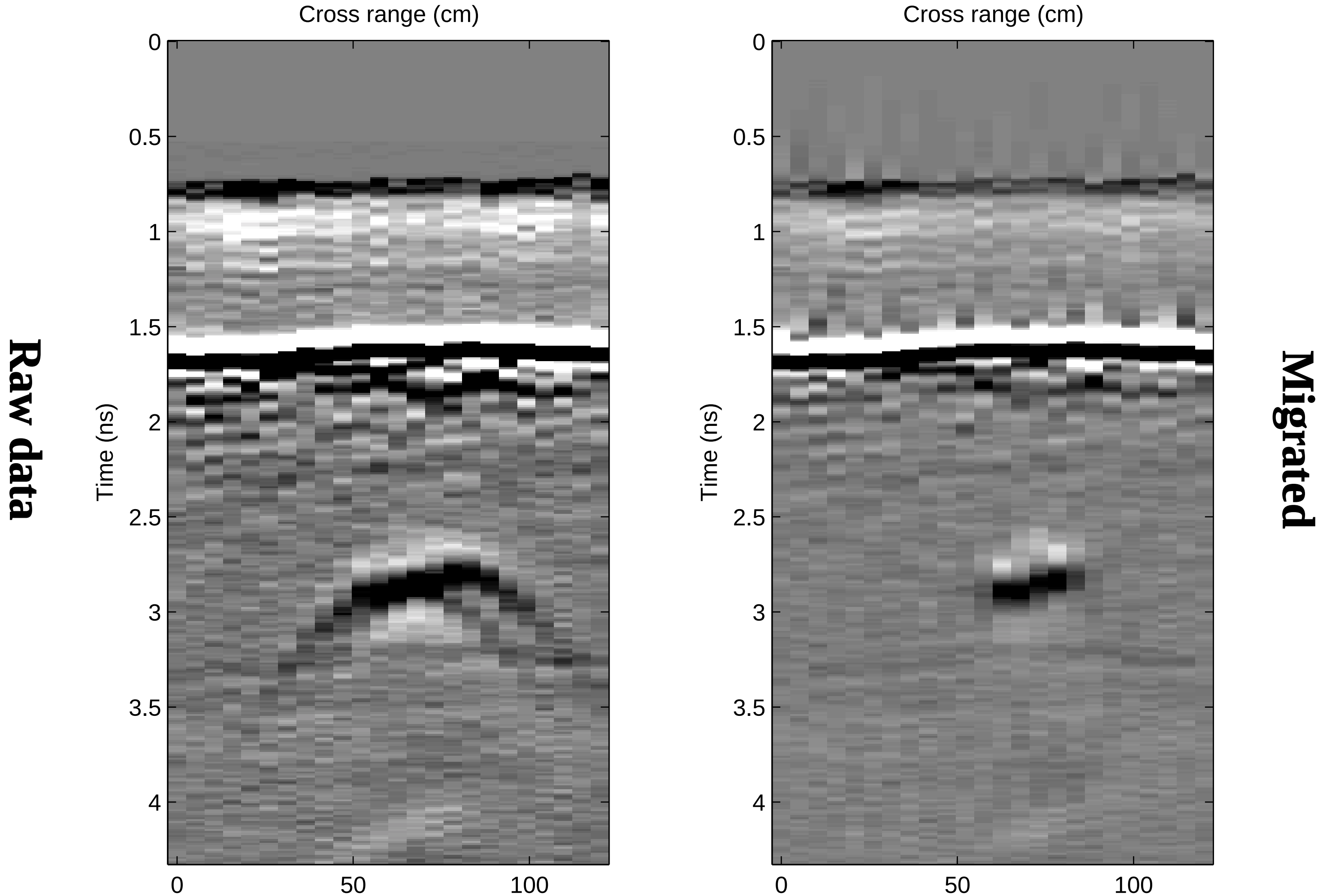
Migration $u(x,z,0)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(k_x,0,\omega)\exp[i(k_x x+k_z z)]dk_x d\omega$

NUFFT

$$u(x,z,0)=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}U(k_x,0,\omega)\exp[ik_z z]d\omega\right)\exp[ik_x x]dk_x$$

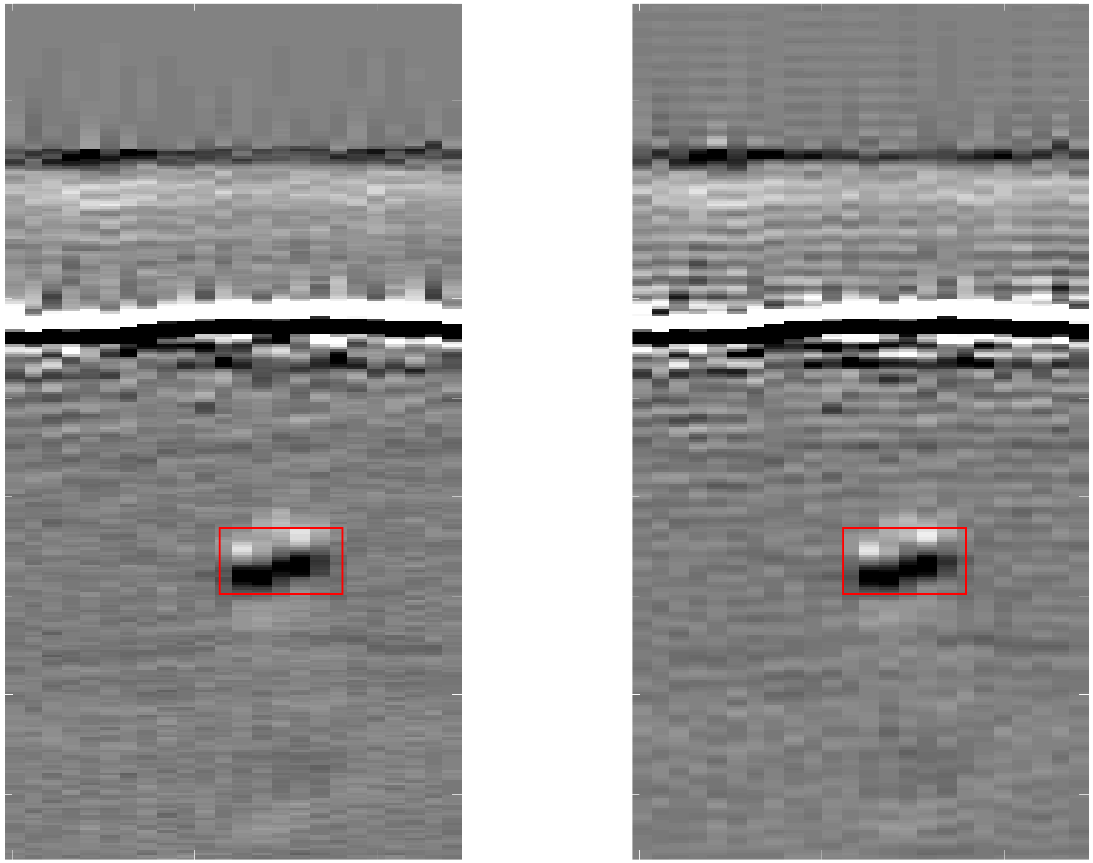
$$=FFT\left\{NUFFT\left[U(k_x,0,\omega)\right]\right\}$$

2D Results (Niitek)



Fast data acquisition by NUFFT

All data



¼ data

*NUFFT migration has good performance with down-sampled data, due to the non-uniformity of the wave-number space

3D NUFFT Imaging Results

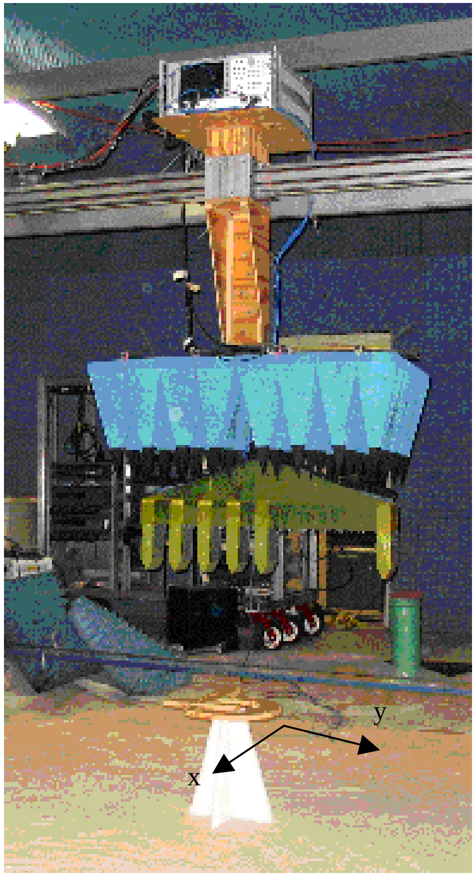
- Niitek data: metallic and plastic landmines
- Georgia Tech data: GT plywood and chamber
- Cross section (horizontal plane) interested
- 3D animation

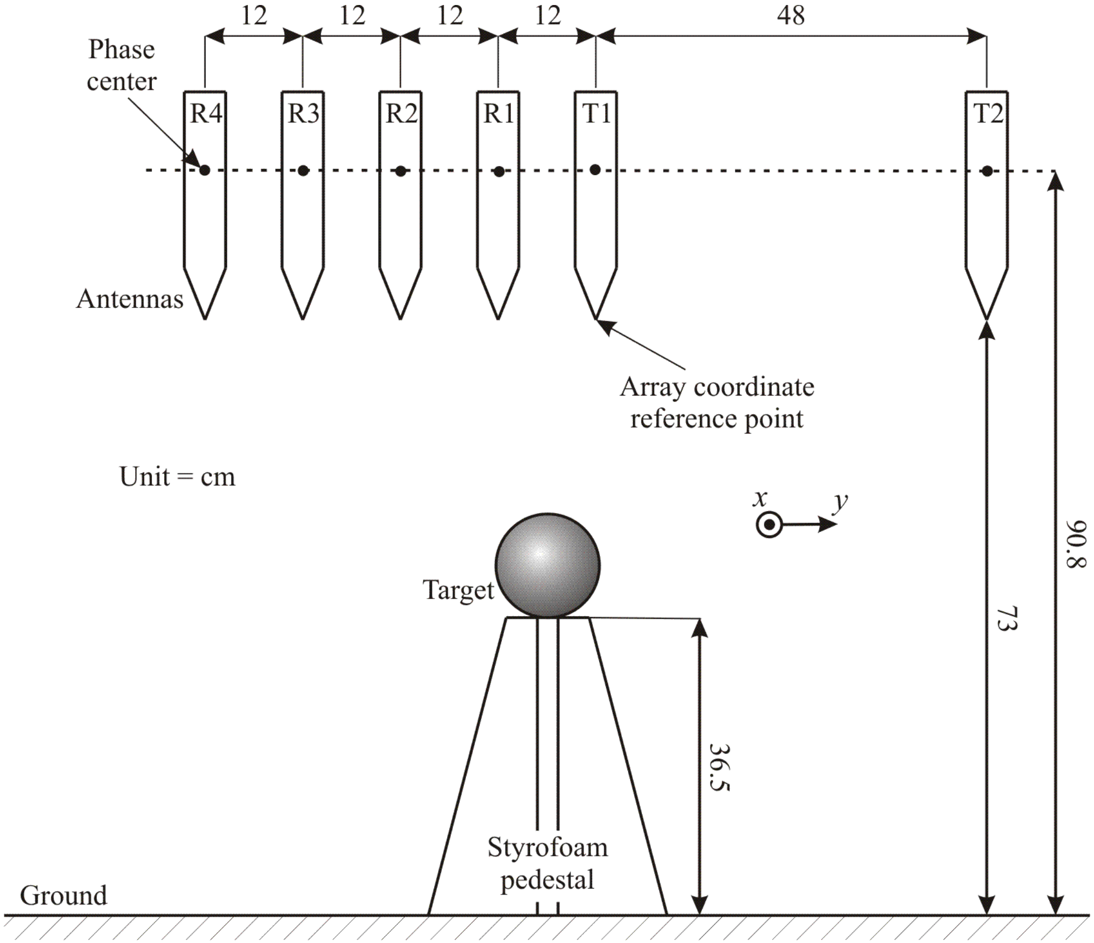
GT plywood

Chamber in sand

Acquisition Configuration (Georgia Tech)

Free space acquisition





3D Results (GT plywood)

Raw data

Migration

Migrated image

3D Results (GT plywood)

Numerical verification

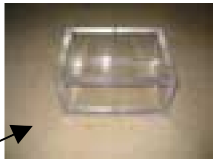
	Actual	**Estimated**
width	38.5cm	38cm
height	46.5cm	44cm
thick	1.8cm	1.76cm

3D animation

*** Note: 1 pixel resolution = 2 cm (horizontal), 0.29cm (vertical)**

Acquisition Configuration (Georgia Tech)

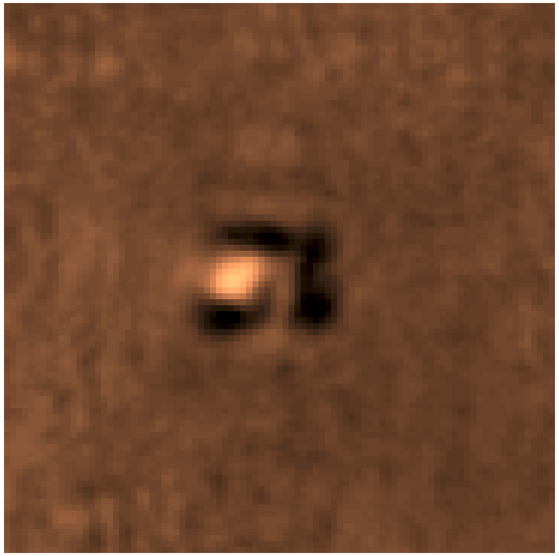
Buried object acquisition



Chamber

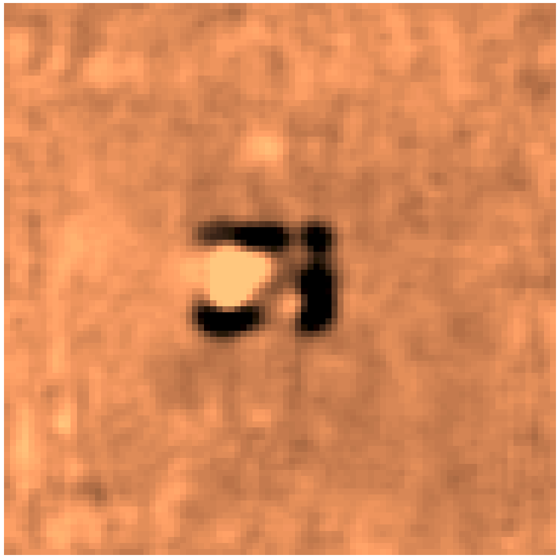
3D Results (Buried chamber)

Raw data



Migration

Migrated image

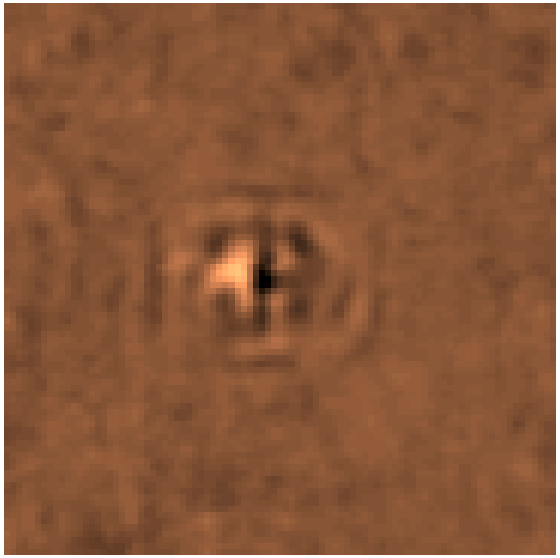


At an estimated depth of 12cm from interface

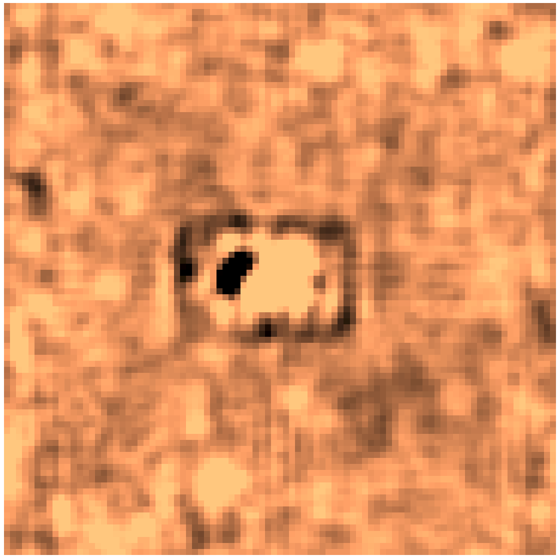
3D Results (Buried chamber)

Raw data

Migrated image



Migration



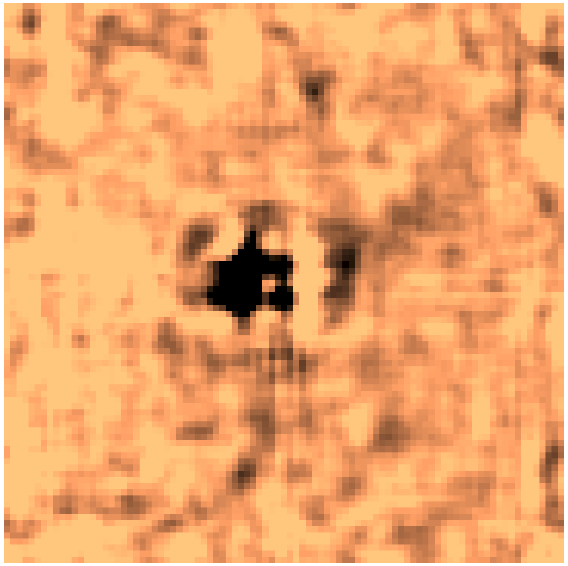
At an estimated depth of 17cm from interface

3D Results (Buried Chamber)

Numerical verification

	Actual	Estimated
Width	40.64cm	42cm
length	30.48cm	34cm
height	20.32cm	16.1cm
depth	9.5cm	10.5cm

3D animation



*** Note: 1 pixel resolution = 2 cm (horizontal), 0.16cm (vertical)**

Summary

- 2D, 3D and multimodality inversion methods have been developed
- Inversion methods have been successfully applied to image some difficult targets from Institut Fresnel. These are 2D measured results for TM polarization.
- NUFFT has been applied to obtain high-fidelity 3D images from Georgia Tech data
- Through-wall imaging is investigated and is highly promising.