Electromagnetic/Seismic Joint Inversion in Multilayered Media

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I. Introduction

- Motivation
  - Electromagnetic and seismic measurements are complementary
  - Especially beneficial to combine these measurements for underground structures
  - Joint inversion can significantly improve inversion resolution

- Problem Considered
  - 2D and 3D EM/seismic scattering in multilayered media
  - Arbitrary number of layers are allowed
  - Such a model is necessary for the realistic situation where the soil and rock are heterogeneous
Problem Geometry

Layer 1 \( \varepsilon_1 \mu_1 \sigma_1 \)

\[ \vdots \]

Layer i \( \varepsilon_i \mu_i \sigma_i \)

\[ \vdots \]

Layer M \( \varepsilon_M \mu_M \sigma_M \)

Sensor Array A

\[ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \]

Sensor Array B

\[ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ \]

Target \( \varepsilon_r \sigma \)

Figure 1: EM/Seismic measurements in a multilayer environment.

- Sources and receivers are located above, on, or under the surface.
- Parameters for seismic properties are not shown but are similar.
- The objective is to obtain a high-resolution image of the target.
II. Theory

- Electromagnetic Waves: The electric field satisfies

\[-\nabla \times \mu_r^{-1} \nabla \times \mathbf{E} + k_0^2 \epsilon_r \mathbf{E} = -j\omega\mu_0 \mathbf{J}\]  

(1)

- Seismic Waves: The displacement vector satisfies

\[\frac{1}{2} \nabla \cdot \left[ \mathbf{C} \cdot \{\nabla \mathbf{u} + (\nabla \mathbf{u})^T\} \right] + \omega^2 \rho \mathbf{u} = -\mathbf{f}\]  

(2)

- Seismic Waves Become Scalar Acoustic Waves if Shear Waves are Neglected: The pressure field satisfies

\[\rho \nabla \cdot (\rho^{-1} \nabla p) + k^2 p = -j\omega c^{-2} f_s\]  

(3)
The Fast Forward Method: BCGS-FFT

- A stabilized biconjugate-gradient fast Fourier transform (BCGS-FFT) method has been developed for fast and accurate forward simulations of EM and acoustic waves in 3D and 2D multilayered media:

\[
\mathcal{L} [ \mathbf{D}(\mathbf{r}) ] = \mathbf{E}_{\text{inc}}(\mathbf{r}), \quad \mathbf{r} \in V
\]

\[
\mathcal{L}[ ] = \left[ \frac{1}{\varepsilon_i} \right] - (k_i^2 + \nabla \nabla \cdot) \frac{1}{\varepsilon_i} \int_V \mathbf{G}^{ii}(\mathbf{r}, \mathbf{r}') \cdot \chi[ ] d\mathbf{r}'.
\]

The BCGS-FFT method requires only \( O(N \log N) \) CPU time and \( O(N) \) memory.

**Refs:**

The Nonlinear Inverse Scattering Methods

- Three methods have been developed:
  - The Born Iterative Method (BIM)
  - The Distorted Born Iterative Method (DBIM)
  - The Contrast Source Inversion (CSI) Method

- Notations
  - $\chi = (\tilde{\varepsilon}_r/\tilde{\varepsilon}_{rb} - 1)$ defines the contrast of the target
  - $\delta\chi_{n+1} = \chi_{n+1} - \chi_n$ denotes the $(n + 1)$-th iteration’s update
  - $D$ denotes an enclosing box around the target domain; $S$ denotes the sensor surface.
- $\mathcal{L}$ denotes BIM’s nonlinear mapping from the contrast to the scattered field

$$E^{scat} = \mathcal{L}_D \chi, \quad r \in D$$

$$E^{scat} = \mathcal{L}_S \chi, \quad r \in S$$

- $\mathcal{G}$ denotes DBIM’s nonlinear mapping from the contrast to the scattered field

$$\delta E^{scat} = \mathcal{G}_D \delta \chi, \quad r \in D$$

$$\delta E^{scat} = \mathcal{G}_S \delta \chi, \quad r \in S$$
• Born Iterative Method

\[ F = \| f_{n+1} - \mathcal{L}_n \chi_{n+1} \|^2 + \gamma \| \chi_{n+1} \|^2 \]

• Distorted Born Iterative Method

\[ F = \| \delta f_{n+1} - \mathcal{G}_n \delta \chi_{n+1} \|^2 + \gamma \| \delta \chi_{n+1} \|^2 \]

• Contrast Source Inversion: First order approximation

\[ F = \frac{\sum_i \| f_i - \mathbf{G}_S \mathbf{w}_i \|^2_S}{\sum_i \| f_i \|^2_S} + \frac{\sum_i \| \chi \mathbf{E}^{inc}_i - \mathbf{w}_i + \chi \mathbf{G}_D \mathbf{w}_i \|^2_D}{\sum_i \| \chi \mathbf{E}^{inc}_i \|^2_D} \]

• The conjugate-gradient method is used to minimize these functionals. FFT is used to accelerate the iterations.

• DBIM is faster than BIM and CSI when the sources and receivers are colocated.
III. 3D EM Inversion

Figure 2: Typical configuration of an inhomogeneous object in a planarly layered medium. Right: Example of two targets ($\varepsilon_r = 10$, $\sigma = 0.3$ S/m).

- $64 \times 64$ sources/receivers in air. Single frequency at $f = 1$ GHz.
- The 3-layer medium models the presence of top soil.
Noise Performance of 3D Inversion

Figure 3: BIM (left) and DBIM (right) reconstruction for SNR = 40 dB. Top: dielectric constant $\varepsilon_r$. Bottom: conductivity $\sigma$. 
SNR is now reduced to 20 dB

Figure 4: BIM (left) and DBIM (right) reconstruction for SNR = 20 dB. Top: dielectric constant $\varepsilon_r$. Bottom: conductivity $\sigma$. 
Convergence of Data and Model Errors

Figure 5: Left: Data error convergence versus iterations for DBIM (solid) and BIM (dashed) at SNR = 40 dB.
Right: Relative error of inverted complex permittivity as a function of SNR.
Effect of the Aperture Size

Figure 6: Relative error of inverted complex permittivity as a function of aperture size.
Left: source/receiver numbers are 36/36.
Right: source/receiver numbers are 64/64.
IV. 2D Multi-Frequency EM and Seismic Imaging

Figure 7: Configuration for seismic imaging of underground structures.

- 16 × 16 sources/receivers in the soil.
- The 3-layer medium models the presence of top soil.
- Shear Waves are neglected for such deep imaging cases for the seismic case.
DBIM for Multi-Frequency Inversion

For single-frequency inversion, we solve following equation:

$$\delta f_{n+1} = \mathcal{L}_n \delta \chi_{n+1}$$

For multi-frequency inversion, we solve following equations:

$$\delta f_{n+1,\omega_1} = \mathcal{L}_{n,\omega_1} \delta \chi_{n+1}$$
$$\delta f_{n+1,\omega_2} = \mathcal{L}_{n,\omega_2} \delta \chi_{n+1}$$
$$\quad \hdots$$
$$\delta f_{n+1,\omega_K} = \mathcal{L}_{n,\omega_K} \delta \chi_{n+1}$$

The above equations can be further expressed as:

$$\delta F_{n+1} = \mathcal{L}_n \delta \chi_{n+1}$$

where $F_{n+1} = [f_{n+1,\omega_1}, f_{n+1,\omega_2}, \hdots, f_{n+1,\omega_K}]^T$;
$\mathcal{L}_n = [\mathcal{L}_{n,\omega_1}, \mathcal{L}_{n,\omega_2}, \hdots, \mathcal{L}_{n,\omega_K}]^T$
Single-Frequency Seismic Imaging of a Void

Figure 8: Left: 100 Hz. Right: 400 Hz

- The inversion at 100 Hz is reasonably good.
- At 400 Hz there are ghost images because of the inadequate sensors.
Multi-Frequency Seismic Imaging of a Void

Figure 9: Imaging with 5 frequencies between 100–1000 Hz.

- Significant improvement is observed over the single-frequency imaging.
Multi-Frequency EM Imaging of an Underground Room

Figure 10: Imaging with 4 frequencies between 10–70 MHz. Left: Reconstructed Image from multi-frequency EM data. Right: The ground truth.

- The wall is well reconstructed because of its large EM contrast with background ($\chi = 1.5$).
Multi-Frequency Seismic Imaging of an Underground Room

Figure 11: Imaging with 4 frequencies between 100–1000 KHz. Left: Reconstructed Image from multi-frequency seismic data. Right: The ground truth.

- The air inside is well reconstructed because of its large acoustic contrast with background.
Convergence Curves in Multi-Frequency Imaging

Figure 12: Data fitting error versus iteration number. Left: EM inversion. Right: Seismic inversion.
V. 2D EM/Seismic Joint Inversion

- Joint Inversion using the Mutual Information (MI) Theory

MI of two random variables A and B can be obtained as:

\[ I(A, B) = H(A) + H(B) - H(A, B) \]

where \( H(A) \) and \( H(B) \) are the entropies of A and B, and \( H(A, B) \) is their joint entropy

\[
\begin{align*}
H(A) &= \sum -P_A(a) \log P_A(a) \\
H(B) &= \sum -P_B(b) \log P_B(b) \\
H(A, B) &= \sum -P_{A,B}(a, b) \log P_{A,B}(a, b)
\end{align*}
\]

The MI based criterion states that the images shall be registered when \( I(A, B) \) is maximal.
The Mutual Information Theory

The probability density functions

\[ P_{A,B}(a, b) = \frac{h(a, b)}{\sum h(a, b)} \]

\[ P_A(a) = \sum_b P_{A,B}(a, b) \]

\[ P_B(b) = \sum_a P_{A,B}(a, b) \]

\( P \) is the probability density function, \( h(a, b) \) is the number of the corresponding pairs having intensity value \( a \) in the first image and intensity value \( b \) in the second image.
Mutual Information Theory in Joint EM/Seismic Inversion

Two modalities:
A–Seismic
B–EM

There exists an operator $L_{BA}$ such that

$$L_{BA}B = A$$

The MI based criterion states that the images shall be registered when $I(A, L_{B,A}B)$ is maximal.
Mutual Information Theory in Joint EM/Acoustic Inversion

The output image is:

From the view of the “Seismic Contrast”, we have the combined image

\[
\frac{A + L_{BA}B}{2}
\]

From the view of the “EM Contrast”, we have the combined image

\[
\frac{B + L_{AB}A}{2}
\]

The consistency of \(L_{AB}\) and \(L_{BA}\) is measured by

\[
< dp >= \sum \| A - L_{BA}L_{AB}A \|
\]

which is zero if the models are completely consistent. For simplicity we assume that

\[
L_{BA} = LI
\]

in our preliminary study. This is an approximate model.
Joint EM/Seismic Inversion from the Seismic View

Figure 13: Joint inversion in the seismic view.
Left: The mutual information vs. L from the seismic view.
Right: The combined seismic image.
Joint EM/Seismic Inversion from the EM View

Figure 14: Joint inversion in the EM view.
Left: The mutual information vs. L from the EM view.
Right: The combined EM image.
The Joint EM/Seismic Image versus Single-Modality Images

Joint Image $\alpha A + B$.  Seismic Image $A$.  EM Image $B$.

- The joint image is significantly better than single-modality images.
Summary and Future Work

- We have developed several new nonlinear inversion capabilities in layered media
  - 3D single-frequency EM inversion
  - 2D multi-frequency EM and seismic inversion
  - 2D EM/seismic joint inversion
- Multi-frequency inversion can significantly improve the resolution
- Joint inversion of multi-modalities has been demonstrated with EM and seismic data.
- Preliminary results are very encouraging in these new inversion models
- More thorough investigation is needed, especially in the area of combining multiple modalities. Inclusion of *a priori* correlation information.