A Fast Inverse Solver and Tunnel Imaging

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Outline

- Introduction
- Diagonal tensor approximation (DTA)
- A fast rigorous inversion method with DTA-CSI
- Inversion of experimental data
- Imaging of tunnels
- Summary
Introduction: The Problem

\[ z_{N-1} \frac{N}{N-1} \]

\[ z_i \]

3D inhomogeneous object

\[ z_{i-1} \]

\[ z_2 \]

\[ z_1 \]

\[ \begin{array}{c}
1 \\
2 \\
i \\
i
\end{array} \]

\[ D \]

\[ \begin{array}{c}
x \\
y \\
z
\end{array} \]
Layered Media in Sensing and Imaging Problems

• Applications are ubiquitous:
  – Landmine detection
  – Underground tunnel imaging
  – Biomedical imaging

• Key issues:
  – Optimal sensing configuration
  – How to obtain best resolution for a given data set?
Conflicting Factors in EM Imaging: Resolution and Probing Volume

- Resolution is limited by:
  - Frequency (wavelength)
  - Aperture size
- High resolution requires high frequency:
  - Shallow penetration depth because of larger attenuation

- Probing volume is limited by:
  - Attenuation
  - Signal-to-noise ratio (SNR)
- Large probing volume requires low frequency because attenuation increases with frequency
Maxwell’s Equations for Layered Media

\[ \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \]

\[ \nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \]

- We will reconstruct permittivity and conductivity of the 3D target:

\[ \tilde{\epsilon}(r) = \epsilon(r) - \frac{j \sigma(r)}{\omega} \]
Electrical Field Integral Equation

• Forward problem: Inside the target domain D

\[ E(r) = E^{\text{inc}}(r) + \int_D G^i_E(r, r') \cdot \chi(r')E(r') \, dr' \]

where \( \chi = (\tilde{\varepsilon} - \tilde{\varepsilon}_i) / \tilde{\varepsilon}_i \) and \( G^i_E(r, r') \) is the dyadic Green’s function of the layered medium.

• Numerical solution of E requires inversion of a large matrix. This is time consuming.

• Previously, we used BCGS-FFT method to speed up this solution. To further improve the speed, we propose a new scattering approximation.
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Scattering Approximations to Avoid Matrix Inversion

• Born Approximation, $E \approx E_{inc}$
  – Valid for the case of weak scattering
• The Extended Born Approximation (EBA) by Habashy et al (1993)
• The Quasi-Linear (QL) approximation Zhdanov and Fang (1996)
• The Quasi-Analytical (QA) approximation by Zhdanov et al (2000)
The Diagonal Tensor Approximation (DTA)

Introducing a scattering tensor

\[ E(r) - E^{\text{inc}}(r) = E^{\text{sct}}(r) \equiv \Gamma(r) \cdot E^{\text{inc}}(r) \]

Then the integral equation becomes

\[ \Gamma(r) \cdot E^{\text{inc}}(r) = \int_D G_{E}^{ii}(r, r') \cdot \chi(r') [I + \Gamma(r')] E^{\text{inc}}(r') dr' \]
Approximate the scattering tensor as a diagonal tensor

\[ \Gamma \approx \begin{bmatrix} \gamma_x & 0 & 0 \\ 0 & \gamma_y & 0 \\ 0 & 0 & \gamma_z \end{bmatrix} \]

\[ E^{scat}(\mathbf{r}) \approx \Gamma(\mathbf{r}) \cdot E^{inc}(\mathbf{r}) \]
Now noting the Green’s function is highly peaked at the source point, inside the integrand we can approximate $\Gamma(r') \approx \Gamma(r')$

$$\Gamma(r) \cdot E^{inc}(r) \approx \int_D G_{E}^{ii}(r, r') \cdot \chi(r')[I + \Gamma(r)]E^{inc}(r')dr'$$

• **Note:** The first term of the RHS is the scattered field in Born approximation $E^{sct}_{B}$

• The equation can be solved in closed form
• Closed form solution

\[
\begin{bmatrix}
\gamma_x \\
\gamma_y \\
\gamma_z
\end{bmatrix} = \begin{bmatrix}
E_x^{inc} - g_{xx} & -g_{xy} & -g_{xz} \\
-g_{yx} & E_y^{inc} - g_{yy} & -g_{yz} \\
-g_{zx} & -g_{zy} & E_z^{inc} - g_{zz}
\end{bmatrix}^{-1} \begin{bmatrix}
E_{B,x}^{sca} \\
E_{B,y}^{sca} \\
E_{B,z}^{sca}
\end{bmatrix}
\]

where

\[
g_{pq} = \int_D G_{E,pq}(r, r') \chi(r') E_q^{inc}(r', r_s) d^3 r'
\]

• After \( \Gamma = \text{diag}[\gamma_x, \gamma_y, \gamma_z] \) is found, fields everywhere can be obtained by Green’s function.
Observations

\[ E^{sc}(r) \approx \int_{D} G_{E}^{ii}(r, r') \cdot \chi(r') [I + \Gamma(r_s, r')] E^{inc}(r') \, dr' \]

- Cross polarization is included
- The DTA is a source-dependent approximation
- Born approximation and quasi-analytic approximation are special cases of DTA
- No need to invert a large system matrix to obtain the field solution.
- The cost is almost as inexpensive as the Born approximation.
### 100 MHz

![Graph](image)

### 400 MHz

![Graph](image)

<table>
<thead>
<tr>
<th></th>
<th>100 MHz</th>
<th>400 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTA</td>
<td>3.52%</td>
<td>4.01%</td>
</tr>
<tr>
<td>QA</td>
<td>9.22%</td>
<td>9.15%</td>
</tr>
<tr>
<td>EBA</td>
<td>23.77%</td>
<td>24.16%</td>
</tr>
</tbody>
</table>
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Inverse Solver Based on DTA

• Use the Born iterative method
• During each iteration, Green’s operation is accelerated by FFT (a nice property of the Green’s function even in layered media)
• For high contrasts, DTA inversion result can serve as the initial solution of full nonlinear inversion methods (CSI and DBIM) to speed up the solution
Example of Hybrid DTA/BCGS-FFT Method for Inversion

800 MHz; 45 sources and 45 receivers on five faces

- Dielectric constant and conductivity have different contrast patterns
DTA/BCGS
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Imaging from Measured Data

(Data collected by K. Belkebir and M. Saillard, Institut Fresnel)
Imaging Results for Dielectric Constant

2 GHz

2 GHz to 3 GHz

\[ \frac{\lambda}{4} = 2.5 \text{ cm at } 3 \text{ GHz} \]
3 GHz to 4 GHz

4 GHz to 5 GHz

5 GHz to 6 GHz

6 GHz to 7 GHz
7 GHz to 8 GHz  

8 GHz to 9 GHz
• **Observation:** Super-resolution is achieved. *(At 3 GHz, the well-resolved wall thickness of the outer cylinder is less than 1/4 wavelength.)*
NUFFT Imaging of 3D Objects

3D Configuration (W. Scott, Georgia Tech)
3D Imaging Results (GT plywood)
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GEOMETRY OF TUNNEL

1.5m

earth

$\varepsilon_r = 1$

4m x 4m

$\varepsilon_{r1} = 4, \sigma_1 = 0.01$

$\varepsilon_{r2} = 20, \sigma_2 = 0.02$
Single-Frequency Imaging

1. True Profile
2. 20 MHz
3. 50 MHz
4. 200 MHz (Inadequate Sampling)
Multi-Frequency Imaging
(Low-Frequency Results as Initial Solution of High-Frequency Inversion)

1. True profile

2. 50 MHz

3. 50 MHz with 100 MHz

4. 100 MHz with 150 MHz
Multi-Frequency Imaging (Cont.)

- 150 MHz with 200 MHz
- 200 MHz with 250 MHz
- 250 MHz with 300 MHz
- 300 MHz with 350 MHz
THEORETICAL DATA ERROR

![Graph showing data error against iteration number for different frequencies: 50 MHz, 100 MHz, 150 MHz, and 200 MHz. The y-axis represents data error on a logarithmic scale, and the x-axis represents iteration number.]
Acoustic Imaging of a Tunnel

**Air:**
- Density: \( \rho = 1.168 \text{ kg/m}^3 \)
- Velocity: \( v = 340 \text{ m/s} \)

**Soil 1:**
- Density: \( \rho = 1400 \text{ kg/m}^3 \)
- Velocity: \( v = 2000 \text{ m/s} \)

**Soil 2:**
- Density: \( \rho = 2700 \text{ kg/m}^3 \)
- Velocity: \( v = 3000 \text{ m/s} \)
Wave Propagation for One Source
Movie of Wave Propagation

Air

Top soil

Bottom soil
Waveforms at Different Locations
Summary

• A novel diagonal tensor approximation has been developed to accelerate forward and inversion solutions.
• Experimental data inversion confirms high fidelity and super-resolution of the method.
• Electromagnetic and acoustic imaging of tunnels has been simulated.
• Optimal configuration for tunnel imaging is under investigation.