A Novel Scattering Approximation for Rapid Multimodality Inversion

Qing H. Liu
Duke University
qhliu@ee.duke.edu

MURI Review, July 1, 2004
Outline

- Summary of Accomplishments
- A New 3D Scattering Approximation - The Diagonal Tensor Approximation
- The Rapid Forward Solver Based on the DTA
- The Rapid Inverse Solver Based on the DTA
- Summary and Future Work
Summary of Accomplishments

Objective of research: Multimodality inversion for characterization of subsurface targets
System 1

Raw data from system 2

Ground bounce

2D Migrated Result
Accomplishment 1

- Developed 3D fast EM forward solver based on biconjugate-gradient FFT method for inhomogeneous objects in multilayered media
  - Optimal computational efficiency: $O(N)$ memory and $O(N \log N)$ CPU time
  - First such model for layered media
Forward Results

Layer 1: (1, 0)
Layer 2: (1.21, 0)
Layer 3: (1.44, 0)
Object: (2, 0.2)
$z_1=3.25$, $z_2=2.75$ m
$d=0.35$ m
$a=b=1.6$ m, $c=1.15$ m
• RAM and CPU requirements for test example

**Figure 5: RAM and CPU requirements.**
Accomplishment 2

- Developed 3D spectral/high-order time-domain EM forward solvers based on the combination of
  - Pseudospectral time-domain method
  - Discontinuous Galerkin’s method
  - Embedded interfaces and enlarged cell methods
Illustration: A Cavity of 3 Rectangles

- Mixed Orders and Non-matching nodes

![Diagram of a cavity with mixed orders and non-matching nodes]
Error versus the changing order P2

\[ P1=8,\ P2=5,\ p3=6 \]

\[ H_z \]

Relative Error

\[ \text{Time (~ period)} \]

CFL = 1/4

\[ \text{Log}_{10}(\text{Error}) \]

\[ P_2 \]
Geometry of a buried UXO

Without rock

With rock

Air, \( K=1 \)

Ground, \( K=2 \)

UXO

Air, \( K=1 \)

Ground, \( K=2 \)

\( \varepsilon / \varepsilon \)

Rock, \( K=4 \)

K: dielectric constant

\( \bullet \): source \((0.12, 0, 0.06)\)

21 receivers

\((0.18:0.03:0.78, 0, 0.06)\)

Max Freq: 2.5G (Hz)

Problem Size:

30*15*10 (min wavelengths)
A 3D Spectral Element Solver

14249 tetrahedron 5th order elements
(~4,300,000 degrees of freedom)
Movie of Wave Propagation

Without rock

With rock
Accomplishment 3

- Developed 3D nonlinear inverse scattering solvers for inhomogeneous objects in multilayered media
  - Born iterative method (BIM)
  - Distorted Born iterative method (DBIM)
  - Contrast source inversion (CSI) method
SNR is now reduced to 20 dB

BIM Results

DBIM Results
Effect of the Aperture Size

**36 Tx and 36 Rx**

Spatial sampling increment, $\lambda$

**64 Tx and 64 Rx**

Spatial sampling increment, $\lambda$

Figure 6: Relative error of inverted complex permittivity as a function of aperture size.
Accomplishment 4

- Developed 2D joint nonlinear inversion of EM/seismic measurements in multilayered media
  - Based on mutual information method
  - Frequency diversity is utilized
  - Significant improvement of images versus single-modality inversion
Example: Joint EM/Seismic inversion of an underground facility

Joint Image $\alpha A + B$. Seismic Image $A$. EM Image $B$. 
Accomplishment 5

- Applied a migration method for the processing of field ground penetrating radar data (collected by NIITEK, Inc.)
  - Rapid imaging results can be obtained
  - However, no information of the target physical parameters is directly shown in the images
Migration Imaging for Metallic Landmine A

Cross-track processing
Left: Raw GPR data  Right: Migrated profile
Migration Imaging for Plastic Landmine A

Cross-track processing

Left: Raw GPR data

Right: Migrated Profile
Accomplishment 6 (To be Detailed in This Talk)

- Developed a migration method for the processing of field ground penetrating radar data (collected by NIITEK, Inc.)
  - Rapid imaging results can be obtained
  - However, no information of the target physical parameters is directly shown in the images
Outline

- Summary of Accomplishments
- A New 3D Scattering Approximation - The Diagonal Tensor Approximation
- The Rapid Forward Solver Based on the DTA
- The Rapid Inverse Solver Based on the DTA
- Summary and Future Work
The Diagonal Tensor Approximation (DTA)

- **Motivation**
  - Exact scattering solution is relatively time consuming, even with our new fast algorithms.
  - Typical 3D nonlinear inversion requires hours of CPU time, thus not practical for real-time processing.
  - On the other hand, the migration method is fast, but not accurate enough and has low resolution.

- The objective of the DTA is to achieve high accuracy and efficiency with a novel approximation.
The Problem Geometry:
A Target in a Multilayered Medium

<table>
<thead>
<tr>
<th>Layer 1</th>
<th>$\varepsilon_1 \mu_1 \sigma_1$</th>
<th>Sensor Array A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$z_1$</td>
</tr>
<tr>
<td></td>
<td>$\vdots$</td>
<td>Sensor Array B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z_{i-1}$</td>
</tr>
<tr>
<td>Layer i</td>
<td>$\varepsilon_i \mu_i \sigma_i$</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z_i$</td>
</tr>
<tr>
<td></td>
<td>$\vdots$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z_{M-1}$</td>
</tr>
<tr>
<td>Layer M</td>
<td>$\varepsilon_M \mu_M \sigma_M$</td>
<td></td>
</tr>
</tbody>
</table>

Objective: To rapidly reconstruct dielectric constant and conductivity from the measured field
The Equations

- Forward problem

\[ E(r) = E^{inc}(r) + \int_D G_{E}^{qq}(r, r') \cdot \chi(r')E(r') \, dr', \quad r \in D \]

Once the \( E(r) \) is solved, the measured scattered field on some surfaces \( S \) in layer \( m \) can be found

\[ E_{m}^{sct}(r) = \int_D G_{E}^{mq}(r, r') \cdot \chi(r')E(r') \, dr', \quad r \in S \]

However, numerical solution of \( E \) requires inversion of a large matrix. This is time consuming.
Scattering Approximations to Avoid Matrix Inversion

- Born Approximation, $E \approx E^{inc}$
  - Valid for the case of weak scattering
- The Extended Born Approximation (EBA) by Habashy et al (1993)
- The Quasi-Linear (QL) approximation by Zhdanov and Fang (1996)
The Diagonal Tensor Approximation (DTA)

- The principle of superposition

\[ E_i(r, r_s) = E_i^{inc}(r, r_s) + E_i^{sca}(r, r_s), \quad i = x, y, z \]

- \( r_s \) is a source position vector and \( E_i^{inc}(r, r_s) \neq 0 \)

Introducing a scattering tensor

\[ E^{sca}(r) = \Gamma(r) \cdot E^{inc}(r), \quad r \in D \]

\[ E^{sca}(r) = \int_{D} G_{E}^{qq}(r, r') \chi(r') \cdot [I + \Gamma(r')] \cdot E^{inc}(r') dr' \]
Approximate the scattering tensor as a diagonal tensor

\[ \Gamma = \begin{bmatrix} \gamma_x & 0 & 0 \\ 0 & \gamma_y & 0 \\ 0 & 0 & \gamma_z \end{bmatrix}, \quad \gamma_i(r, r_s) = \frac{E_{i}^{\text{scat}}(r, r_s)}{E_{i}^{\text{inc}}(r, r_s)}. \]
Now noting the Green’s function is highly peaked at the source point, inside the integrand we have:

\[ \Gamma(r') \cdot E_{inc}(r') \approx \Gamma(r) \cdot E_{inc}(r') \]

Now denoting

\[ E_B = \int_D G_E^{qq}(r, r') \cdot \chi(r') E_{inc}(r', r_s) dr', \quad r \in D, \]

\[ g_{ij} = \int_D G_{E,ij}^{qq}(r, r') \chi(r') E_{j}^{inc}(r', r_s) dr', \quad r \in D. \]

the integral equation leads to ...
• The New approximation (DTA)

\[
\begin{bmatrix}
\gamma_x \\
\gamma_y \\
\gamma_z
\end{bmatrix} = \begin{bmatrix}
E_x^{inc} - g_{xx} & -g_{xy} & -g_{xz} \\
-g_{yx} & E_y^{inc} - g_{yy} & -g_{yz} \\
-g_{zx} & -g_{zy} & E_z^{inc} - g_{zz}
\end{bmatrix}^{-1} \begin{bmatrix}
E_{B,x} \\
E_{B,y} \\
E_{B,z}
\end{bmatrix}
\]

• The EBA (Habashy et al, 1993)

\[
\Gamma_{EBA}(r) = \left[ I - \int_D G_E^{qq}(r, r')\chi(r')dr' \right]^{-1}
\]

• The QA approximation (Zhdanov et al, 2000)

\[
\gamma_{QA} = (E_B \cdot \bar{E}^{inc})(E^{inc} \cdot \bar{E}^{inc} - E_B \cdot \bar{E}^{inc})^{-1}
\]
Why do we expect the DTA to be a better approximation?

- Compared to the EBA (extended Born approximation), the DTA is source dependent, and thus better captures the source signature in the scattered fields.

- Compared to the QA (quasi-analytical approximation), the DTA is a tensor rather than a scalar scattering factor. Thus it has better polarization effects.

- The computational cost is essentially the same for all these approximations (i.e., inexpensive).
Once the scattering tensor $\Gamma(r')$ is found, we have

$$E^{\text{Scal}}(r) = \int_D G_{E}^{qq}(r, r') \chi(r') \cdot [I + \Gamma(r')] \cdot E^{\text{inc}}(r') \, dr'$$

- That is, there is no need to invert a matrix to obtain the field solution.
- The cost is almost as inexpensive as the Born approximation.
Outline

- Summary of Accomplishments
- A New 3D Scattering Approximation - The Diagonal Tensor Approximation
- The Rapid Forward Solver Based on the DTA
- The Rapid Inverse Solver Based on the DTA
- Summary and Future Work
Test example: A cube in a 3-layer medium

\[ \begin{align*}
\epsilon_{r1} &= 1.0 \quad \sigma_1 = 0.0 \ \text{S/m} \\
\epsilon_{r2} &= 3.5 \quad \sigma_2 = 0.01 \ \text{S/m} \\
\epsilon_{r3} &= 4.0 \\
\sigma_3 &= 0.1 \ \text{S/m} \\
\text{or} \\
\sigma_3 &= 0.02 \ \text{S/m}
\end{align*} \]

\[ z_1 = 3.25 \ \text{m} \]

\[ z_2 = 2.75 \ \text{m} \]

\[ d = 0.35 \ \text{m} \]

\[ \epsilon_r = 2.0 \]

\[ \sigma = 0.2 \ \text{S/m} \]

D = 1.6 m x 1.6 m x 1.15 m
Test 1: Lower frequency case

$\sigma_3 = 0.1 \text{ S/m. } f = 10 \text{ MHz}$

DTA is the best; the others are worse but still acceptable.
Test 2: Higher frequency case

$\sigma_3 = 0.1 \, \text{S/m}$. $f = 100 \, \text{MHz}$

DTA is the best; the others are substantially worse.
Test 3: Higher frequency and lower host conductivity

$\sigma_3 = 0.02 \text{ S/m. } f = 100 \text{ MHz}$

DTA is the best; the others are not acceptable.
Summary of Errors for Eight Test Cases

The relative errors for computation of the scattered fields

Left: The background conductivity $\sigma_3 = 0.1 \text{ S/m}$
Right: The background conductivity $\sigma_3 = 0.02 \text{ S/m}$
Outline

- Summary of Accomplishments
- A New 3D Scattering Approximation - The Diagonal Tensor Approximation
- The Rapid Forward Solver Based on the DTA
- The Rapid Inverse Solver Based on the DTA
- Summary and Future Work
The Rapid Inverse Solver Based on the DTA

Objective: To rapidly reconstruct dielectric constant and conductivity from the measured field
Formulation

- Consider the data integral equation,

\[ E_{m}^{\text{Sca}}(r) = \int_{D} G_{E}^{mq}(r, r') \cdot \chi(r')E(r')dr', \quad r \in S \]

- Linearize the problem for inverting \( \chi \) via \( E = \tilde{\Gamma}E^{\text{inc}} \)

- For \( M \) measurements and \( N \) image cells,

\[ b = Ax \]

- \( b \) is the \( M \)-dimensional data column vector, \( x \) is \( N \)-dimensional column vector of the contrast
Iterative Inversion Procedure

- Linearize the problem by the previous iteration to form a weighted least-squares problem

\[ F(x) = (b - Ax)\dagger W(b - Ax) + \alpha(x - x_0)\dagger Q(x - x_0) \]

- Minimize this functional by using the CG method

- Within each CG iteration, the reciprocity theorem and FFT are used to speed up the matrix-vector multiply

- Iterate until the contrast converges
DTA Inversion Example

Measurement configuration at 2 GHz
Data: Synthetic data from the exact solver (BCGS-FFT)

SNR: 40 dB

SNR: 20 dB
The Contrast Source Inversion Method requires about 160 minutes.
The DTA requires only about 2 minutes to obtain a comparable result.
The DTA is approximately 80 times faster than the CSI method.
Outline

- Summary of Accomplishments
- A New 3D Scattering Approximation - The Diagonal Tensor Approximation
- The Rapid Forward Solver Based on the DTA
- The Rapid Inverse Solver Based on the DTA
- Summary and Future Work
Summary

Accomplishments to date include
- Exact 3D forward and inverse solvers
- 2D joint EM/seismic inversion method
- A novel diagonal tensor approximation (DTA) for rapid 3D forward and inverse solutions

The DTA provides a highly accurate approximation potentially feasible for real-time field applications

The DTA should be applicable to seismic problems
Future Work

- Use the DTA as a preconditioner in the exact solvers – a seamless combination that can benefit low- and high-contrast problems.
- Inversion of measured data from GA Tech. (Waymond Scott)
- 3D multimodality inversion of EM/seismic data