EE640 STOCHATIC SYSTEMS
EXAM II, 2007

NAME:____

DO ALL 3 problems.

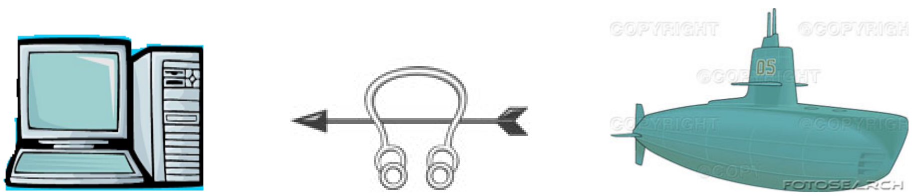
Problem	Points	Score
1	33	
2	35	
3	32	
Total	100	

Open Book and Open Notes. No communicating with other students. No cell phones or other electronic communications devices. Write down all your final answers on this document in an elegant way.

GOOD LUCK AND REMEMBER THAT NO ONE CAN JUDGE YOU JUST BY YOUR GRADE

1. SCHWARTZ INEQUALITY.

In a sonar system, the position of a submarine, X, can be detected with a constant decaying factor α. Besides, the measurement, Y, is also corrupted by an additive noise, W. That is $Y = \alpha X + W$. Assume that the signal-to-noise ratio $\frac{E\{X^2\}}{E\{W^2\}} = \frac{1}{\beta}$ and the cross-correlation ratio between signal and noise $\frac{E\{XW\}}{E\{X^2\}} = \rho$.



Y $\alpha(\bullet) + W$ X

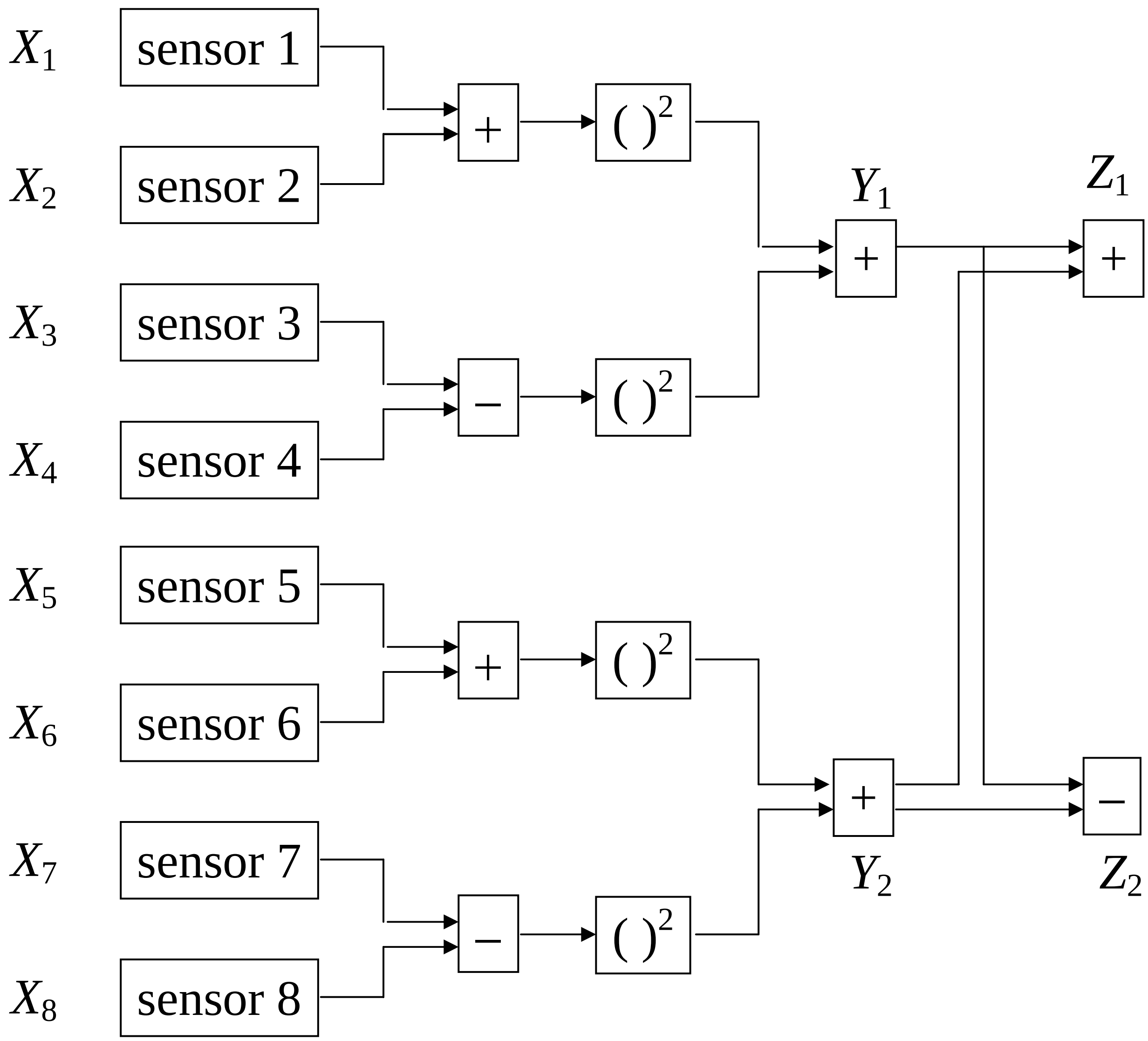
(1) Design an estimation gain λ to process Y, such that the minimum mean square error, $E\left\{(\alpha X - \lambda Y)^2\right\}$, can be achieved.

(2) Find an upper bound of λ using Schwartz inequality.

(3) If we use $\hat{X} = \frac{\lambda}{\alpha} Y$ to estimate the submarine position, then what is the normalized estimation error, $\frac{E\left\{\left(X - \hat{X}\right)^2\right\}}{E\left\{X^2\right\}}$?

2. FUNCTION OF RANDOM VARIABLES.

In a multiple sensor system, all the measurements, X_1, X_2, X_3, X_4, X_5, X_6, X_7 and X_8, are independent Gaussian variables and have the identical distribution $N(0, 1)$.



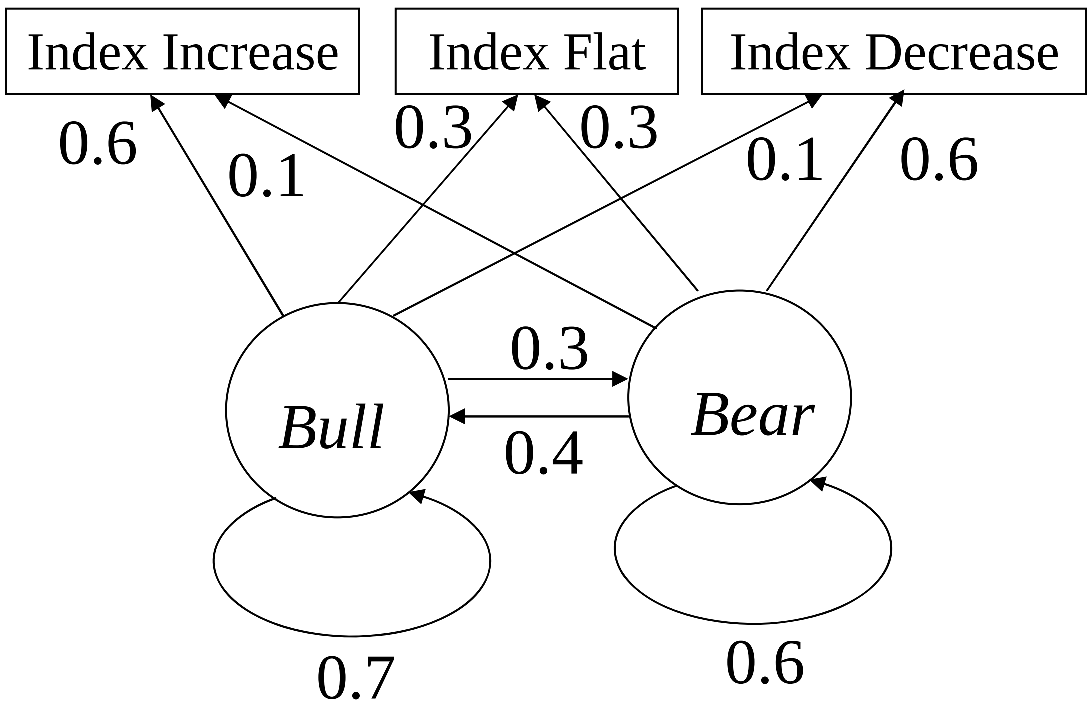
(1) What is the joint distribution of Y_1 and Y_2?

(2) What is the joint distribution of Z_1 and Z_2?

(3) What are the Tchebycheff and Chernoff bounds of the $P(Z_1 > a \mid |Z_2| < b)$ when $a > b > 0$?

3. BAYES RULE.

In the stock market, there are two types of market, Bull and Bear. The probabilities of a market type changing from Bull to Bear and staying in its course are 0.3 and 0.7 respectively. The probabilities of a market type changing from Bear to Bull and staying in its course are 0.4 and 0.6 respectively. For a Bull market, the probabilities of an increase, a flat and a decrease in stock index are 0.6, 0.3, and 0.1 respectively. For a Bear market, 0.1, 0.3 and 0.6 respectively.



(1) What are the steady-state probabilities of having a Bull market and a Bear market?

(2) When the stock index decreases, what is the probability of having a Bull market and what is the probability of having a Bear market?

(3) On Monday, the market is Bull and the index increases; On Tuesday, the index still increases. What is the probability of the market changing from Bull to Bear?

(4) The stock index has been flat for two days. What is the probability of having a bull market for two days?