Homework Solutions II

EE 640
Stochastic Systems

HW6A

3.6 By definition, a Martingale is

$E\{X(t_2)|X(t_1)\}=X(t_1) \qquad t_1<t_2$

For a Wiener process w(t), $E\{w(t)\} = 0$

and for any value of t', $0 \le t' \le t$

the moment w(t)-w(t') is Gaussian with zero mean, thus

$$E\{w(t_2)|w(t_1)\}= E\{w(t_1)+w(t_2)-w(t_1)|w(t_1)\}$$
$$= E\{w(t_1)|w(t_1)\}+E\{w(t_2)-w(t_1)|w(t_1)\}$$
$$= E\{w(t_1)\}+0$$

Zero mean and independent

HW6A

3.7

(a) P{X(2)=0} ⇒ n=2, m=0

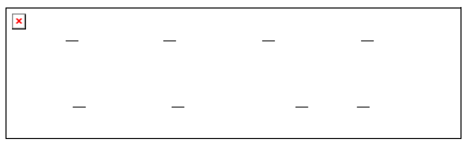
P{X(2)=0} = P{in head 2 tosses}=1/2

(b) P[X(18)=0|X(6)=2]=P[2 tails in seven & eight tosses]=1/2(1/2)=1/4

(c) E{X(10)} = 0, since E{X(10)}=E{J_1+…+J_{10}}

=E{J_1}+…E{J_{10}}=0

(d) E{X(10)|X(4)=4}=4 because it is a Mortingale



HW6A

3.8

$$S(n) = \sum_{k=1}^{n} X(k), \quad X(0) = 0,$$

$$S(n) = S(n-1) + X(n)$$

$$E\{S(n) \mid S(n-1)\} = E\{S(n-1) + X(n) \mid S(n-1)\}$$

$$= S(n-1) + E\{X(n) \mid S(n-1)\}$$

$$= S(n-1) + E\{X(n)\} = S(n-1) + 0 = S(n-1)$$

↑ Since X(n) and S(n-1) are independent

HW6B

3.13

$$(a)\quad Z(t)=a+bX(t)+cY(t)$$

$$R_{zz}(\tau)=E\{[a+bX(t)+cY(t)][a+bX(t+\tau)+cY(t+\tau)]\}$$

$$=a^2+b^2R_{xx}(\tau)+c^2R_{yy}(\tau)$$

$$(b)\quad Z(t)=aX(t)Y(t)$$

$$R_{zz}(\tau)=E\{[aX(t)Y(t)][aX(t+\tau)Y(t+\tau)]\}$$

$$=a^2R_{xx}(\tau)R_{yy}(\tau)$$

HW6B

3.15

(a) Yes

(b) Not even, no

(c) Yes

(d) Yes

HW7

3.17

$$R_{xx}(\tau) = \exp(-\alpha|\tau|) \qquad S_{xx}(f) = \frac{2/\alpha}{1+(2\pi f/\alpha)^2} = \frac{2\alpha}{\alpha^2+(2\pi f)^2}$$

$$R_{xx}(\tau) = \frac{\sin(1000\tau)}{1000\tau} \qquad S_{xx}(f) = \frac{\pi}{1000} rect\left(\frac{f}{1000/\pi}\right)$$

$$R_{xx}(f) = \frac{1}{4}e^{-|\tau|}[\cos\tau + \sin|\tau|]$$

$$S_{xx}(f) = \frac{1}{8}\left[\frac{2}{1+4\pi^2\left(f-\frac{1}{2\pi}\right)^2} + \frac{2}{1+4\pi^2\left(f+\frac{1}{2\pi}\right)^2}\right] - \frac{1}{8j}\left[\frac{2}{1+4\pi^2\left(f-\frac{1}{2\pi}\right)^2} - \frac{2}{1+4\pi^2\left(f+\frac{1}{2\pi}\right)^2}\right]$$

$$= \frac{1/4(1+j)}{1+4\pi^2\left(f-\frac{1}{2\pi}\right)^2} + \frac{1/4(1-j)}{1+4\pi^2\left(f-\frac{1}{2\pi}\right)^2}$$

HW7

3.19

$$S_{xx}(f) = \frac{1}{T}\left[R_{XX}(0) + 2\sum_{k=1}^{\infty} R_{XX}(k)\cos(2\pi fkT)\right]; T=1$$

$a)\ S_{XX}(f) = [1+0] = 1,\ R_{XX}(0) = 1$

$b)\ S_{XX}(f) = R_{XX}(0) + 2\sum_{k=1}^{\infty} e^{-\alpha k}\cos(2\pi fkT) = 1 + 2\sum_{k=1}^{\infty} e^{-\alpha k}\cos(2\pi fkT)$

$c)\ S_{XX}(f) = 1 + 2(-1/2)\cos(2\pi f) = 1 - \cos(2\pi f)$

HW7

3.23

$$S_{XX}(f) = 10\exp(-f^2/10000)$$

$$B_{eff} = \frac{1}{2}\frac{\int_{-\infty}^{\infty} S_{XX}(f)df}{\max[S_{XX}(f)]} = \frac{1}{2}\frac{\int_{-\infty}^{\infty} 10\exp(-f^2/10000)df}{10} = \frac{1}{2}2\frac{1}{2(1/1000)}\sqrt{\pi} = 50\sqrt{\pi}$$

$$B_{rms}^2 = \frac{\int_{-\infty}^{\infty} f^2 S_{XX}(f)df}{\int_{-\infty}^{\infty} S_{XX}(f)df} = \frac{\int_{-\infty}^{\infty} f^2 10\exp(-f^2/10000)df}{\int_{-\infty}^{\infty} 10\exp(-f^2/10000)df} = \frac{2\pi(3/2)/(2\times10^{-6})10}{10^{-2}\sqrt{\pi}} = 5000$$

$$B_{rms} = \sqrt{5000}$$

HW7

3.23

$$S_{XX}(f) = \frac{100}{[1+(2\pi f/100)^2]^2}, \quad S_{XX}(0) = 100,$$

$$R_{XX}(\tau) = \frac{(100)^2}{4}e^{-|100\tau|}[1+|100\tau|] \quad R_{XX}(0) = 2500$$

$$B_{eff} = \frac{R_{XX}(0)}{2S_{XX}(0)} = \frac{2500}{2\times100} = 12.5$$

HW7

3.41

$$a) E\{<\mu_X>_n\} = \frac{1}{N}\sum_{i=1}^{N} E\{X(i)\} = \mu_X$$

$$Var\{<\mu_X>_n\} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} E\{X(i)X(j) - \mu_X^2\}$$

$$Let\ l = j - i,\ then$$

$$E\{X(i)X(i+l)\} - \mu_X^2 = R_{XX}(l) - \mu_X^2 = C_{XX}(l)$$

$$Var\{<\mu_X>_n\} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{l=1-i}^{N-i} C_{XX}(l) = \frac{1}{N^2}\sum_{i=1-N}^{N-1}(N - |i|)C_{XX}(l)$$

$$= \frac{1}{N}\sum_{i=1-N}^{N-1}\left(1 - \frac{|i|}{N}\right)C_{XX}(l)$$

HW7

3.41

$$E\{<R_{XX}(k)>_n\} = \frac{1}{N}\sum_{i=1}^{N} E\{X(i)X(i+k)\} = R_{XX}(k)$$

$$Var\{<R_{XX}(k)>_n\} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} E\{X(i)X(i+k)X(j)X(j+k) - R_{XX}^2(k)\}$$

$$Let\ Z(i) = X(i)X(i+k) \quad Thus\ \mu_Z = R_{XX}(k)$$

$$Var\left\{<R_{XX}(k)>_m\right\} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} E\{Z(i)Z(j) - \mu_Z^2(k)\}$$

similarly

$$Var\left\{<R_{XX}(k)>_n\right\} = \frac{1}{N}\sum_{i=1-N}^{N-1}\left(1 - \frac{|i|}{N}\right)C_{ZZ}(i)$$

$$b)\ For\ ergodicity\ of\ the\ mean\ \lim E\{\langle\mu_X\rangle_T\} = \mu_X$$

$$and\ \lim_{N\to\infty}\frac{1}{N}\sum_{i=1-N}^{N-1}\left(1 - \frac{|i|}{N}\right)C_{XX}(i) = 0$$

HW8A

4.3

$LTIVC$

$Show: R_{YY}(k) = R_{YX}(k) * h(k)$

$$R_{yy}(k) = E\{Y(n)Y(n+k)\} = E\left\{Y(n)\sum_{m=-\infty}^{\infty} h(m)X(n+k-m)\right\}$$

$$= \sum_{m=-\infty}^{\infty} E\{Y(n)X(n+k-m)\}h(m) = \sum_{m=-\infty}^{\infty} R_{YX}(k-m)h(m)$$

$$= R_{YX}(k) * h(k)$$

HW8A

4.4 (a)

$$Y(n) = \frac{1}{k}\sum_{i=1}^{k} X(n-i)$$

$$h(n) = \begin{cases} 1/k & n = 1,2,\cdots,k \\ 0 & elsewhere \end{cases}$$

$$H(f) = F\{h(n)\} = \frac{1}{k}\sum_{n=1}^{k} e^{-j2\pi nf} = \frac{1}{k}\frac{e^{-j2\pi f}[1-e^{-j2\pi kf}]}{1-e^{-j2\pi f}}$$

$$= \frac{1}{k}\frac{e^{-j\pi(k+2)f}}{e^{-j\pi f}}\frac{[e^{j\pi kf} - e^{-j\pi kf}]}{e^{j\pi f} - e^{-j\pi f}}$$

$$= \exp[-j\pi f(k+1)]\frac{1}{k}\frac{\sin c(kf)}{\sin c(f)}$$

HW8A

4.4 (b)

$X(n)$ *is stationary*

$$R_{XX}(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \qquad S_{XX}(f) = 1$$

$$S_{YY}(f) = |h(n)|^2 S_{XX}(f) = \frac{1}{k^2} \frac{\sin c^2(kf)}{\sin c^2(f)}$$

$$E\{Y^2(n)\} = R_{YY}(0) = \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} h(m_1)h(m_2)R_{XX}(0-m_1, 0-m_2)$$

$$= \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} h(m_1)h(m_2)\delta(-m_1+m_2) = \sum_{m_1=1}^{k} \frac{1}{k^2} = \frac{1}{k}$$

HW8A

4.6 (a)

LTIVC System

input is stationary, zero mean, Gaussian

$$E\{X(n)\} = 0,\ E\{X^2(n)\} = 1,\ R_{XX}(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \qquad S_{XX}(f) = 1$$

$Y(n)$ *is Gaussian, a linear combination of Gaussian*

$$E\{Y(n)\} = E\left\{\sum_{i=0}^{k} h(i)X(n-i)\right\} = 0$$

$$E\{Y^2(n)\} = E\left\{\left[\sum_{i=0}^{k} h(i)X(n-i)\right]^2\right\} = \sum_{i=0}^{k} h^2(i)$$

HW8A

4.6 (b), (c)

$$R_{YY}(n) = R_{XX}(n) * h(n) * h(-n) = h(n) * h(-n)$$

$$= \sum_{i=-\infty}^{\infty} h(n-i)h(-i) = \sum_{i=-\infty}^{\infty} h(n+i)h(i)$$

$$R_{YY}(n) = \begin{cases} \sum_{i=0}^{k-|n|} h(|n|+i)h(i) & -k \le n \le k \\ 0 & otherwise \end{cases}$$

$$S_{YY}(f) = F\{R_{YY}(n)\} = F\left\{\sum_{i=0}^{k} h(n+i)h(i)\right\}$$

$$= \sum_{n=-k}^{k} \sum_{i=0}^{k-|n|} h(|n|+i)h(i)e^{-j2\pi nf}$$

HW8B

4.8

$$Y(t) = a_1 Y(t-T) + a_2 Y(t-2T) + \ldots + a_m Y(t-mT) +$$
$$X(t) + b_1 X(t) + \ldots + b_n X(t-nT)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + b_1 z^{-1} + \ldots + b_n z^{-n}}{1 - a_1 z^{-1} - \ldots - a_m z^{-m}}$$

$$H(f) = H(z)\Big|_{z=e^{j2\pi f}} = \frac{1 + \sum_{k=1}^{n} b_k e^{-j2\pi fkT}}{1 - \sum_{k=1}^{m} a_k e^{-j2\pi fkT}}$$

$$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$$

HW8B

4.12

$$Y(t)=\frac{1}{T}\int_{t-T}^{t}X(\alpha)d\alpha, \quad H(t)=\begin{cases}\frac{1}{T} & 0\le t\le T\\ 0 & elsewhere\end{cases}$$

$$H(f)=\exp[-j2\pi f(T/2)]\sin c(fT)=\exp[-j\pi f]\sin c(fT)$$

$$X(t)\sim stationary,\ zero\ mean,\ white\ Gaussian,\ S_{XX}=\frac{\eta}{2}$$

$$S_{YY}(f)=|H(f)|^2 S_{XX}(f)=\frac{\eta}{2}\sin c^2(fT)$$

$$E\{Y^2(t)\}=\int_{-\infty}^{\infty}S_{XX}(f)df=\frac{\eta}{2}\int_{-\infty}^{\infty}\sin c^2(fT)df=\frac{\eta}{2}\int_{0}^{T}\frac{1}{T^2}dt=\frac{\eta}{2T}$$

HW8B

4.17

$$S_{YY}(f)=\frac{(2\pi f)^2+1}{(2\pi f)^4+13(2\pi f)^2+36}=\frac{[1+j2\pi f][1-j2\pi f]}{[9+(2\pi f)^2][4+(2\pi f)^2]}$$

$$=\frac{[1+j2\pi f][1-j2\pi f]}{[3+j2\pi f][3-j2\pi f][2+j2\pi f][2-j2\pi f]}$$

$$=S_{XX}(f)|H(f)|^2=\frac{\eta}{2}|H(f)|^2$$

$$H(f)=\sqrt{\frac{2}{\eta}}\frac{[1+j2\pi f]}{[3+j2\pi f][2+j2\pi f]}$$

HW9A

5.32

$$P(4) = [0.3\ 0.3\ 0.4]\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}^4$$

HW9A

5.34

$$P(1) = [0.7\ 0.2\ 0.1]\begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix} = [0.72\ 0.145\ 0.085]$$

$$P(2) = [0.72\ 0.145\ 0.085]\begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix} = [0.7245\ 0.192\ 0.0835]$$

$$P(4) = [0.7245\ 0.192\ 0.0835]\begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.1 \end{bmatrix}^2 = [0.726\ 0.191\ 0.083]$$

$$\pi_1 = 0.8\pi_1 + 0.5\pi_2 + 0.6(1 - \pi_1 - \pi_2)$$

$$\pi_2 = 0.15\pi_1 + 0.3\pi_2 + 0.3(1 - \pi_1 - \pi_2)$$

$$[\pi_1 \quad \pi_2 \quad \pi_3] = [0.72797 \quad 0.19080 \quad 0.08123]$$

HW9A

5.35

$$From\ eq.5.81$$

$$P(2) = \frac{1}{r+q}\begin{bmatrix} r+q(1-r-q)^2 & q-q(1-r-q)^2 \\ r-r(1-r-q)^2 & q+r(1-r-q)^2 \end{bmatrix}$$

$$P(4) = \frac{1}{r+q}\begin{bmatrix} r+q(1-r-q)^4 & q-q(1-r-q)^4 \\ r-r(1-r-q)^4 & q+r(1-r-q)^4 \end{bmatrix}$$

$$P = \lim_{n\to\infty} P(n) = \frac{1}{r+q}\begin{bmatrix} r & q \\ r & q \end{bmatrix}$$

HW9B

7.30

$$K(1) = \frac{10}{10+1} = \frac{10}{11}$$

$$\hat{S}(2) = 0.9\left\{\frac{1}{11}\hat{S}(1) + \frac{10}{11}X(1)\right\} = .9\left\{\frac{1}{11}\times 0 + \frac{10}{11}\times 1\right\} = 0.818$$

$$P(2) = (0.9)^2\left(\frac{1}{11}\right)(10) + 1 = 1.736$$

$$K(2) = \frac{1.736}{2.736} = 0.635$$

$$\hat{S}(3) = 0.9\{0.365\hat{S}(1) + 0.635X(1)\} = 0.9\{0.365\times 0.818 + 0.635\times 1.1\} = 0.897$$

$$P(3) = (0.9)^2(0.365)(1.736) + 1 = 1.513$$

$$K(3) = \frac{1.513}{2.513} = 0.602$$

HW9B

7.31

$$\varphi = \frac{\sigma_W^2 + \sigma_V^2(a^2-1) + \sqrt{[\sigma_W^2 + \sigma_V^2(a^2-1)]^2 + 4\sigma_W^2\sigma_V^2}}{2}$$

$$= \frac{1 + 1\times(0.81-1) + \sqrt{(0.81)^2 + 4}}{2} = 1.484$$

$$K = \frac{P}{P+\sigma_V^2} = \frac{1.484}{1.484+1} = 0.597$$

HW9B

7.35

$$S_{XX} = S_{SS} + S_{NN} = \frac{(2\pi f)^2 + 4 + 4(2\pi f)^2 + 4}{[(2\pi f)^2+1][(2\pi f)^2+4]}$$

$$H(f) = \frac{S_{XS}}{S_{XX}} = \frac{S_{SS}}{S_{XX}} = \frac{(2\pi f)^2 + 4}{5(2\pi f)^2 + 8}$$

HW10A

6.2 (a)

Toss Coin 8 *times*

H_0 : *fair coin*, $P(Head) = 0.5$

H_1 : *unfair coin*, $P(Head) = 0.4$

Assume $P(H_0) = 0.5$, *then* $P(H_1) = 0.5$

$f(x|H_1) \sim B(8, 0.4) \quad f(x|H_0) \sim B(8, 0.5)$

$$\frac{f(x|H_1)}{f(x|H_0)} = \frac{\binom{8}{x} 0.4^x 0.6^{8-x}}{\binom{8}{x} 0.5^x 0.5^{8-x}} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1/2}{1/2} = 1$$

$$\Rightarrow 0.4^x 0.6^{8-x} \underset{H_0}{\overset{H_1}{\gtrless}} 0.5^8 \Rightarrow x \underset{H_0}{\overset{H_1}{\gtrless}} 3.5$$

HW10A

6.2 (b)

Find : P_e

$P_e = P(H_0)P(D_1|H_0) + P(H_1)P(D_0|H_1)$

$P(D_1|H_0) = P(3 \text{ or less heads } | H_0)$

$= P(x=3|H_0) + P(x=2|H_0) + P(x=1|H_0) + P(x=0|H_0)$

$= \binom{8}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 + \binom{8}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 + \binom{8}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^7 + \binom{8}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^8$

$= 0.3633$

$P(D_0|H_1) = P(4 \text{ or more heads } | H_1)$

$= P(x=4|H_0) + P(x=5|H_0) + \ldots + P(x=8|H_0)$

$= \binom{8}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^4 + \binom{8}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^3 + \ldots + \binom{8}{8}\left(\frac{2}{5}\right)^8\left(\frac{3}{5}\right)^0$

$= 0.4059$

$P_e = 0.3633\,(0.5) + 0.4059\,(0.5) = 0.3846$

HW10A

6.6

$H_1 : Y = 2 + N \quad N \sim N\left(0, \frac{1}{9}\right)$

$H_0 : Y = N \qquad P(H_0) = P(H_1) = 0.5$

$C_{00} = 0, C_{11} = 1, C_{01} = C_{10} = 2$

Find decision rule that minimize $\overline{C}$

$$\frac{f(y|H_1)}{f(y|H_0)} = \exp[18(y-1)] \underset{H_0}{\overset{H_1}{\gtrless}} \frac{0.5(C_{10} - C_{00})}{0.5(C_{01} - C_{11})} = \frac{2}{1}$$

$$\Rightarrow 18(y-1) \underset{H_0}{\overset{H_1}{\gtrless}} \ln 2 \Rightarrow y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\ln 2}{18} + 1 = 1.0385$$

HW10A

6.11

$N - P$ *decision rule* $\quad P_F = 0.001$

$f(y|H_1) \sim N(-2,4) \quad f(y|H_0) \sim N(2,4)$

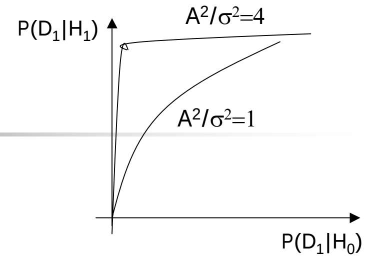
$P_F = P(D_1 | H_0) = 0.001$

$P_F = Q\left(\frac{2 - T_h}{2}\right) = 0.001 \quad \frac{2 - T_h}{2} = 3.1 \quad T_h = -4.2$

$P_D = 1 - P_M = P(D_1 | H_1) = Q\left(\frac{2.2}{2}\right) = Q(1.1) = 0.1357$

Not a reasonable rule!

HW10B



6.12

$P(H_0) = P(H_1) = 0.5$

$f(y|H_1) \sim N(A, \sigma^2) \quad f(y|H_0) \sim N(-A, \sigma^2)$

$ROC\ curves\ i.e.\ P(D_1|H_1)\ vs\ P(D_1|H_0) \quad for\ A^2/\sigma^2 = 1\ A^2/\sigma^2 = 16$

$$P(D_1|H_1) = 1 - Q\left(\frac{A - T_h}{\sigma}\right) \quad P(D_1|H_0) = Q\left(\frac{T_h + A}{\sigma}\right)$$

$A^2/\sigma^2 = 1 \Rightarrow A = \sigma$

$$P(D_1|H_1) = 1 - Q\left(1 - \frac{T_h}{A}\right) \quad P(D_1|H_0) = Q\left(1 + \frac{T_h}{A}\right)$$

$A^2/\sigma^2 = 16 \Rightarrow A = 4\sigma$

$$P(D_1|H_1) = 1 - Q\left(4 - \frac{4T_h}{A}\right) \quad P(D_1|H_0) = Q\left(4 + \frac{4T_h}{A}\right)$$

HW10B

6.14 (a)

$S_1(t) = 4\sin(2\pi f_0 t) \quad 0 \le t \le T \quad T = 1ms \quad t_0 = 10/T$

$S_0(t) = -4\sin(2\pi f_0 t) \quad 0 \le t \le T \quad P(S_1) = P(S_0) = 0.5$

$Y = S + N \quad S_{NN}(f) = 10^{-3}\ W/Hz$

Decision rule that minimize P_e

$$\int_0^T y(t)[S_1(t) - S_0(t)]dt \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln \frac{P(H_0)}{P(H_1)}$$

$$\int_0^T y(t)[8\sin(2\pi f_0 t)]dt \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\mu_1 + \mu_2}{2} = \frac{-4\sin(2\pi f_0) + 4\sin(2\pi f_0)}{2} = 0$$

HW10B

6.14 (b)

$$Z(T) = \frac{1}{T}\int_0^T Y(t)\sin(2\pi f_0 t)\,dt = \int_0^T Y(t)\frac{1}{T}\sin[2\pi f_0(T-t)]\,dt = Y(t) * h(t)$$

$$h(t) = \sin(2\pi f_0 t)\,rect\left(\frac{t}{T}\right)$$

$$H(f) = \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] * \sin c(fT) = \frac{1}{2}\left[\sin c(f-f_0)T + \sin c(f+f_0)T\right]$$

$$Z = \frac{1}{T}\int_0^T \left[\pm 4\sin(2\pi f_0 t) + N(t)\right]\sin(2\pi f_0 t)\,dt$$

$$E\{Z\} = \frac{1}{T}\int_0^T \pm 4\sin(2\pi f_0 t)\sin(2\pi f_0 t)\,dt = \frac{\pm 4}{T}\frac{T}{2} = \pm 2$$

$$Var\{Z\} = \text{var}\left\{\frac{1}{T}\int_0^T N(t)\sin(2\pi f_0 t)\,dt\right\} = \text{var}\{N(t) * h(t)\}$$

$$= \int_{-\infty}^{\infty} S_{nn}(f)\,|H(f)|^2\,df = \eta\int_{-\infty}^{\infty}\frac{1}{4}\{\sin c^2[(f-f_0)T] + \sin c^2[(f+f_0)T]\}\,df$$

$$= \frac{\eta}{2}\int_{-\infty}^{\infty}\sin c^2(fT)\,dt = \frac{\eta}{2}\int_{-\infty}^{\infty} rect^2\left(\frac{t}{T}\right)dt = \frac{\eta}{2}\frac{1}{T} = \frac{\eta}{2T}$$

HW10B

6.18

$$\overline{C} = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} C_{ij} P(D_i \mid H_i)P(H_j) = \sum_{i=0}^{M-1}\sum_{\substack{j=0\\ j\neq i}}^{M-1} P(D_i \mid H_i)P(H_j) = P_e$$

$$C(H_i \mid y) = \sum_{\substack{j=0\\ j\neq i}}^{M-1} P(H_j \mid y) = 1 - P(H_i \mid y)$$

$$\min imize\ P_e \Leftrightarrow \max imize\ P(H_i \mid y) \Leftrightarrow \min imize\ C(H_i \mid y)$$

$$\min imize\ \overline{C} \Leftrightarrow \min imize\ C(H_i \mid y)$$