

# EE640 Project #1: Part S: SUPPLEMENTAL SYNTHESIS OF DETERMINISTIC SYSTEM

Face Recognition with LPCCF [1]

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There are 23 target and clutter images which are saved in target.zip and clutter.zip, respectively. The size of each image is  $128 \times 128$  pixels. You can choose any  $N(N = 6)$  images out of the given 23 target images as training images for the filter construction. The  $N$  images should cover all the range. Fig. 1 shows an example of picking 5 target training images. Fig. 2 shows the sample of clutter images.



Figure 1: Training set of target faces



Figure 2: Clutter faces

## The process of creating LPCCF:

In this project, the distortion is side-to-side head rotation. The training images  $\underline{\mathbf{x}}_{t,n}$  have  $d$  elements and are lexicographically represented as  $d \times 1$  vectors, where  $d = 128^2$ . They are normalized to have unit energy such that  $\mathbf{1} = \underline{\mathbf{x}}_{t,n}^T \underline{\mathbf{x}}_{t,n}$ . The training set matrix  $\underline{\mathbf{X}}_t$  has  $N$  columns corresponding to  $N$  distortion of target image. Create a training set of  $N$  rotations for target face and clutter face. The LPCCF mathematical representation is then defined as

$$\mathbf{h}_k = \sum_{n=1}^N \mathbf{x}_{t,n} \exp\left(\frac{-j2\pi kn}{N}\right) = \underline{\mathbf{X}}_t \underline{\phi}_k \quad (1)$$

where the subscript  $N$  is the total number of training maps,  $n$  corresponds to the index of rotation distortion in yaw direction, i.e.,  $\theta$  and  $k = 0, 1, \dots, N-1$  is the filter order. Also,  $\underline{\phi}_k$  is a  $N \times 1$  Fourier vector,  $\underline{\mathbf{X}}_t$  is the  $d \times N$  edge enhanced target training set (try edge enhanced method such as Sobel) and  $h_k$  is the  $d \times 1$  lexicographic form of the  $k^{th}$  order LPCCF.

The idea of LPCCF design is to select a training set which not only sufficiently represent the possible distortions, but also to yield a correlation matrix that is Cyclic Toeplitz (CT). Briefly, a Toeplitz matrix, or a diagonal constant matrix is an  $N \times N$  matrix,  $T_N = t_{i,j}$  where  $t_{i,j} = t_{i-j}$ . The training set can be written as

$$\underline{\mathbf{X}}_t = \begin{bmatrix} \mathbf{x}_{t,0} & \mathbf{x}_{t,1} & \dots & \mathbf{x}_{t,N-1} \end{bmatrix} \quad (2)$$

The training set is desired to approach CT form, hence if we are given a correlation matrix,  $R_{ttA}$  that is approximately of such structure, the complex responses of the LPCC filters at the origin are approximately constant in magnitude and linear in phasor component such that

$$\mathbf{y}_{t,k,n} = \mathbf{x}_{t,n}^T \mathbf{h}_k = \mathbf{x}_{t,n}^T \underline{\mathbf{X}}_t \underline{\phi}_k \approx \lambda_{tt,k} \exp\left(\frac{-j2\pi kn}{N}\right) \quad (3)$$

When all of the training vectors in Eq. 3 are augmented together, the equation becomes

$$\underline{\mathbf{X}}_t^T \mathbf{h}_k = \underline{\mathbf{X}}_t^T \underline{\mathbf{X}}_t \underline{\phi}_k = R_{ttA} \phi_k \approx \lambda_{tt,k} \underline{\phi}_k \quad (4)$$

A family of LPCC filters can be mathematically expressed as

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & \dots & h_N \end{bmatrix} = \underline{\mathbf{X}}_t \underline{\phi} \quad (5)$$

The origin clutter response is

$$\mathbf{y}_{c,k,n} = \mathbf{x}_{c,n}^T \mathbf{h}_k \quad (6)$$

For correlation the peak location should correspond to the origin response. We used  $k=1$  so that the phase response would be non-ambiguous for the full range of the rotation. Without noise the target and clutter magnitude response may look like Fig. 3. The phase response of target and clutter image is shown in Fig. 4.

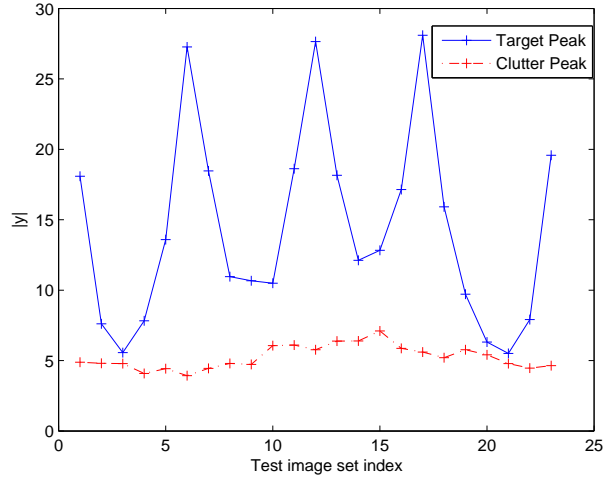


Figure 3: The peak response for target and clutter.

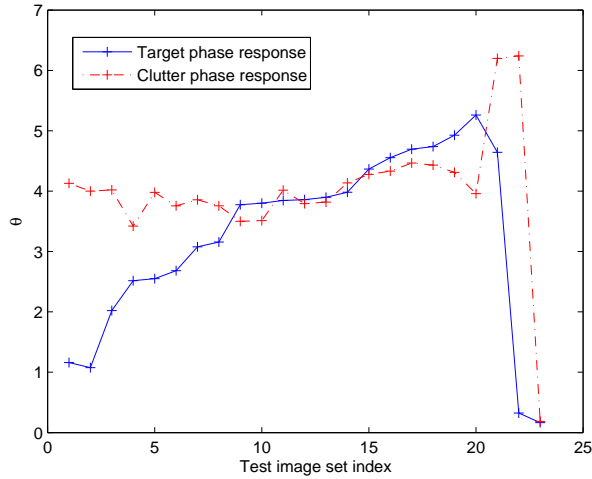


Figure 4: The phase response for target and clutter..

### TYPE 0 LPCCF:

We can constrain the responses using a Type 0 LPCCF defined as

$$\mathbf{h}_k = \underline{\mathbf{X}}_t \alpha_k \quad (7)$$

where the response for all training images is

$$\underline{\mathbf{X}}_t^T \mathbf{h}_k = \underline{\mathbf{X}}_t^T \underline{\mathbf{X}}_t \alpha_k = R_{ttA} \alpha_k = \lambda_{tt,k} \underline{\phi}_k \quad (8)$$

and so  $\alpha_k$  is

$$\alpha_k = R_{ttA}^{-1} \lambda_{tt,k} \underline{\phi}_k \quad (9)$$

The sample Matlab code to do correlation:

```
% correlate x and h
X = fft2(x);
H = fft2(h);
y = ifft2(conj(H).*X);
yr = real(y); % real part
ya = abs(y); % absolute value
yi = imag(y); % imaginary part
yt = angle(y); % phase results
```

### TASKS OR QUESTIONS:

- 1 Show which 6 training images you chose and why. Show Figure of your training set (like Fig. 1)
- 2 Using the test set for target and clutter, reproduce the response curve shapes in Figs 3 and 4 but normalize the test set input signal energy to be unity before correlating.
- 3 Create a type 0 LPCCF from your training set and rerun responses in Figs 3 and 4. Show and compare results and describe differences.

### References

- [1] L. G. Hassebrook, B. V. K. Vijaya Kumar, and L. Hostetler, "Linear phase coefficient composite filter banks for distortion-invariant optical pattern recognition," *Opt. Eng.* **29**, 1033–1043 (1990).