

# EE640 STOCHASTIC SYSTEMS

Spring, 2007

## COMPUTER PROJECT 1

### PART C: DETECTION and DISCRIMINATION (April 8, 2007)

1. Fisher Discriminant: Perform a fisher discriminant measure [1] (Rayleigh quotient) on each of the target/clutter vector pairs:

$$\begin{aligned}
 & \underline{t}_{IIDi}, \underline{c}_{IIDi} \quad i = 1, 2, 3 \\
 J_i &= \frac{(\mu_{ti} - \mu_{ci})^2}{\sigma_{ti}^2 + \sigma_{ci}^2} \quad i = 1, 2, 3 \tag{1}
 \end{aligned}$$

where

$$\begin{aligned}
 \sigma_{ti}^2 &= \frac{1}{N-1} \sum_{m=1}^N (\underline{t}_{Ii}[m] - \mu_{ti})^2 \\
 \sigma_{ci}^2 &= \frac{1}{N-1} \sum_{m=1}^N (\underline{c}_{Ii}[m] - \mu_{ci})^2
 \end{aligned}$$

2. Use the results of step 1 for the magnitude response to the noisy images defined in item 6 of project 1A. The magnitude responses should be from type 0 LPCCF, order  $k = 1, 2$  and 3 filters.
3. Mathematically and in words describe a MLR test for 3 correlated random variables. **Assume equal covariances and reduce to the linear form.** Using vector pairs (target/clutter pair) with the highest  $J_i$  value first, form a MLR discriminator for one, two and three r.v.s. Use the average of the target and clutter covariances to get a common Covariance Matrix. For each of these discriminators, plot the target data response and clutter data response on the same graph. Estimate minimum probability of error from graphs and plot the three MPE values. Assume  $P(\text{target}) = P(\text{clutter}) = 1/2$ . (a) Perform this process for the independent data vectors but don't make any assumptions about the Covariance matrices being diagonal. Show the covariance matrices for the 3 variable cases. (b) Repeat for the type 0 LPCCF noisy responses.
4. (a) Form a  $M = 64$  long filter from a segment of a noise sequence and correlate with the original sequence. Plot correlation and note peak location. At peak location in output correlation determine the performance measures1, Signal-to-Noise Ratio,  $SNR = \text{peak}^2 / \text{variance}$  and also Peak-to-Average Correlation Energy,  $PACE = \text{peak}^2 / (\text{average squared value of output correlation})$ . The correlation is given in terms of convolution as:

$$y[n] = x[n] \otimes h^*[-n] \tag{2}$$

where in the frequency domain this is

$$Y[k] = X[k]H^*[k] \tag{3}$$

The entire process is implemented in MATLAB by  $y = \text{ifft}(\text{fft}(x) .* \text{conj}(\text{fft}(h)))$ .  
Solve this problem for the following two cases:

The signal sequence  $x[n]$  is  $\underline{b}_{binary}$  and the filter is

$$h[n] = \begin{cases} \underline{b}_{binary}[n + m_1] & \text{for } n = 1, 2, \dots, M \\ 0 & \text{for } n = (M + 1), (M + 2), \dots, N. \end{cases} \quad (4)$$

where  $m_1$  is chosen arbitrarily between 1 and  $N - M$ .

(b) Repeat 4(a) for  $x[n]$  is  $\underline{s}_{intensity}$ .

## References

- [1] B. V. K. Vijaya Kumar and L. G. Hassebrook, "Performance Measures for Correlation Filters," *Applied Optics*, **29**, 2997–3006 (1990).