

# EE640 STOCHASTIC SYSTEMS

Spring, 2007

## COMPUTER PROJECT 1

### PART B: ANALYSIS (April 8, 2007)

1. Generate independent identically distributed test vectors. The target vectors are:

$$\begin{aligned} t_{IID1} &= g_1 + 3 \\ t_{IID2} &= g_2 + 3 \\ t_{IID3} &= g_3 + 3 \end{aligned} \tag{1}$$

and the clutter vectors are:

$$\begin{aligned} c_{IID1} &= g_4 + 1 \\ c_{IID2} &= g_5 + 1 \\ c_{IID3} &= g_6 + 1 \end{aligned} \tag{2}$$

2. Histogram: Design a program, or use the MATLAB function hist.m, that will estimate the histogram of an  $N \times 1$  vector of random numbers. Have the program use specified  $M$  bin intervals. Run the program and plot for:

PlotB-1	$u_1, g_1$	histogram
PlotB-2	$s_1, s_2, s_3, s_4, s_5$	histogram
PlotB-1	$t_{IID1}, c_{IID1}$	histogram
PlotB-1	$t_{IID2}, c_{IID2}$	histogram
PlotB-1	$t_{IID3}, c_{IID3}$	histogram
PlotB-1	$s_{intensity}$	histogram

3. Perform tasks 2 and 3 from Project 1S and show graphs.
4. Covariance estimate: estimate covariance matrices ( $3 \times 3$ )

$K_{tIID}$  from  $t_{IID1}, t_{IID2}, t_{IID3}$

$K_{cIID}$  from  $c_{IID1}, c_{IID2}, c_{IID3}$

(In some context, this is called the correlation matrix) i.e.

$$K(m, n) = \frac{1}{(N-1)} \left( \underline{x}_m - \underline{\mu}_m \right)^T \left( \underline{x}_n - \underline{\mu}_n \right) \tag{3}$$

where  $\underline{\mu}_m$  is an  $N \times 1$  vector with all elements equal to the mean value of the vector  $\underline{x}_m$  and  $K(m, n)$  is the  $m^{th}, n^{th}$  element of the matrix  $K$ .

5. Estimate mean vectors ( $3 \times 1$ ) from

$$\underline{t}_{IID1}, \underline{t}_{IID2}, \underline{t}_{IID3}, \quad (4)$$

such that

$$\underline{\mu}_t = \begin{bmatrix} \mu_{t,1} \\ \mu_{t,2} \\ \mu_{t,3} \end{bmatrix} \quad (5)$$

likewise for clutter, use

$$\underline{c}_{IID1}, \underline{c}_{IID2}, \underline{c}_{IID3}, \quad (6)$$

to generate

$$\underline{\mu}_c = \begin{bmatrix} \mu_{c,1} \\ \mu_{c,2} \\ \mu_{c,3} \end{bmatrix} \quad (7)$$

6. Repeat step 4 and 5 for the noisy image responses defined in item 6 of project 1A. The responses should be from type 0 LPCCF, order  $k = 1, 2$  and 3 filters. Note that lexicographic form is used such that the covariance matrix is  $3 \times 3$ .

7. Determine the peak element locations, the centroid element locations of the histograms of  $\underline{b}_{binary}$  and  $\underline{s}_{intensity}$ . The centroid is determined by “simulating” a pdf. For example, let  $x[n]$  be a sequence and you want to approximate  $E\{x[n]\}$ . Let  $h(x)$  be the histogram of  $x[n]$ . First, form a pseudo pdf as

$$f_x(x) = \frac{h(x)}{\sum_{m=1}^M h[m]} \quad (8)$$

where  $m$  is the bin number of a total  $M$  bins in the histogram. The value of  $h(x)$  returns the bin value that contain the value of  $x$ . The centroid is then

$$\mu_x = \sum_{n=1}^N x[n] f_x(x[n]) \quad (9)$$

Determine the time averages of  $\underline{b}_{binary}$  and  $\underline{s}_{intensity}$  and compare with the centroid averages. The should be close.

Optional Method: A conceptually easier technique for implementing the centroid is the following: The approach will first get the fractional value of the estimated bin number. Then that value is linearly mapped to the  $x$  dimension. Given your bin numbers for the histograms  $h[m]$  are equally spaced and vary from 1 to  $M$ . We can map bin values to signal values with  $x_{min} = a * 1 + b$  and  $x_{max} = a * M + b$ . So  $a = (x_{max} - x_{min}) / (M - 1)$  and  $b = x_{min} - a$ . The value  $x_{max}$  and  $x_{min}$  are the center values associated with the end bins. So we find the centroid

$$f_m(m) = \frac{h[m]}{\sum_{m=1}^M h[m]} \quad (10)$$

The centroid is a fractional value

$$\mu_m = \sum_{m=1}^M m f_m(m) \quad (11)$$

and the mean of  $x$  is then  $\mu_x = \mu_m * a + b$ .