Bernoulli Trials & Binomial Probability

Lecture 3

EE 640
Stochastic Systems

Outline

- Bernoulli Trials
  - Considers the presence or absence of sampled events combinatorics
- Binomial Coefficient
  - Sampling with replacement
  - Sampling without replacement
- Binomial Probability
- Multinomial Coefficient
- Multinomial Probability

Sampling with Replacement

Assume *n* possible outcomes such as numbered balls 1 through *n*. We sample r balls and replace the ball each time we sample. The total combination is n^r.

[Ex] Let n=2, r=3,

We predict $2^3 = 8$ combinations

1: 1 1 1
2: 1 1 2
3: 1 2 1
4: 1 2 2
5: 2 1 1
6: 2 1 2
7: 2 2 1
8: 2 2 2

Sampling without Replacement

First sample, n balls are available; Second sample, n- balls, ..., r^{th} sample, n-r+1 balls

Number of possible ordered samples is

$n(n-1)(n-2)\ldots(n-r+1) = n!/(n-r)!$

[Ex] Consider a group of 4 numbered balls, what different combinations can you select 2 balls

Solution

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

n!/(n-r)! = 4!/(4-2)! = 3x4 = 12

n (n-1) = 12: number of possible ordered samples, w/o replacement

n (n-1)/2 = 6: unique combination, order doesn't count

n^2 = 16: with replacement, number of possible ordered samples

Binomial Coefficient

For a combination subset size r from a set size n without replacement, we have

n!/(n-r)!/r! unique combination

Let $C_r^n = \binom{n}{r}$ = n!/(n-r)!/r! = Binomial Coefficient

[Ex] n = 4, r = 2

4!/2!/2! = 4x3x2/2/2 = 6

Bernolli Trials and Binomial Distribution

Outcome is binary Like tossing a coin

Let A_k represent the event of getting k "1"s in n trials, where the outcomes are either "0" or "1".

Then $P(A_k) = \binom{n}{k} p^k q^{n-k}$

where $P(a_1) = p$

and $P(a_0) = q = 1 - p$

a_1: getting a "1" in one trail;

a_0: getting a "0" in one trial

When np<<1, it can be approximated by Poisson distribution

Example

[Ex] 5 missiles are fired at a ship. It takes two to destroy the ship.

q = prob. that the missile misses the ship = 0.9

p = prob. That the missile hits the ship = 1 – q = 0.1

E : the ship doesn't sink

P(E) = P(0 or 1 missile hits the ship)?

Solution

P(E) = P(0 or 1 missile hits the ship)

$$= \sum_{K=0}^{1} \binom{n}{k} p^k q^{n-k}$$

$$= \binom{5}{0} (0.1)^0 (0.9)^5 + \binom{5}{1} (0.1)^1 (0.9)^4$$

$= 5!/5!/0!\ 0.9^5 + 5!/4!/1!0.1(0.9)^4 = 0.95+0.5\ (0.9)^4$

Multi-nomial Coefficient

Let r_1, ... r_k be a set of non-negative number such that $r_1 + r_2 + \ldots r_k = n$

The number of partitioned combinations from a population size n is

$n!/r_1!/r_2!\ldots/r_k!$ Multinomial coefficient

Example

[Ex] $n = 5$, unique balls, $r_1 = 3$, $r_2 = 2$

then

$$5!/3!/2! = 1x2x3x4x5/(1x2x3)/2 = 10$$

group1	1 2 3	2 3 4	3 4 5	4 5 1	5 1 2
group2	4 5	5 1	1 2	2 3	3 4

group1	2 4 5	2 3 5	1 3 5	1 3 4	1 2 4
group2	1 3	1 4	2 4	2 5	3 5

Multi-nomial Probability

For a test of n trials, in which there are k possible outcomes for each trial with probabilities of $[p_1, p_2, \ldots, p_k]$, the probability of having $[r_1\ r_2\ \ldots\ r_k]$, where $r_1 + r_2 + \ldots r_k = n$, different outcomes will be

$$n!/r_1!/r_2!\ldots/r_k! \quad p_1^{r1}\ p_2^{r2}\ldots p_k^{rk}$$