
Interferometric Imaging in Clutter II

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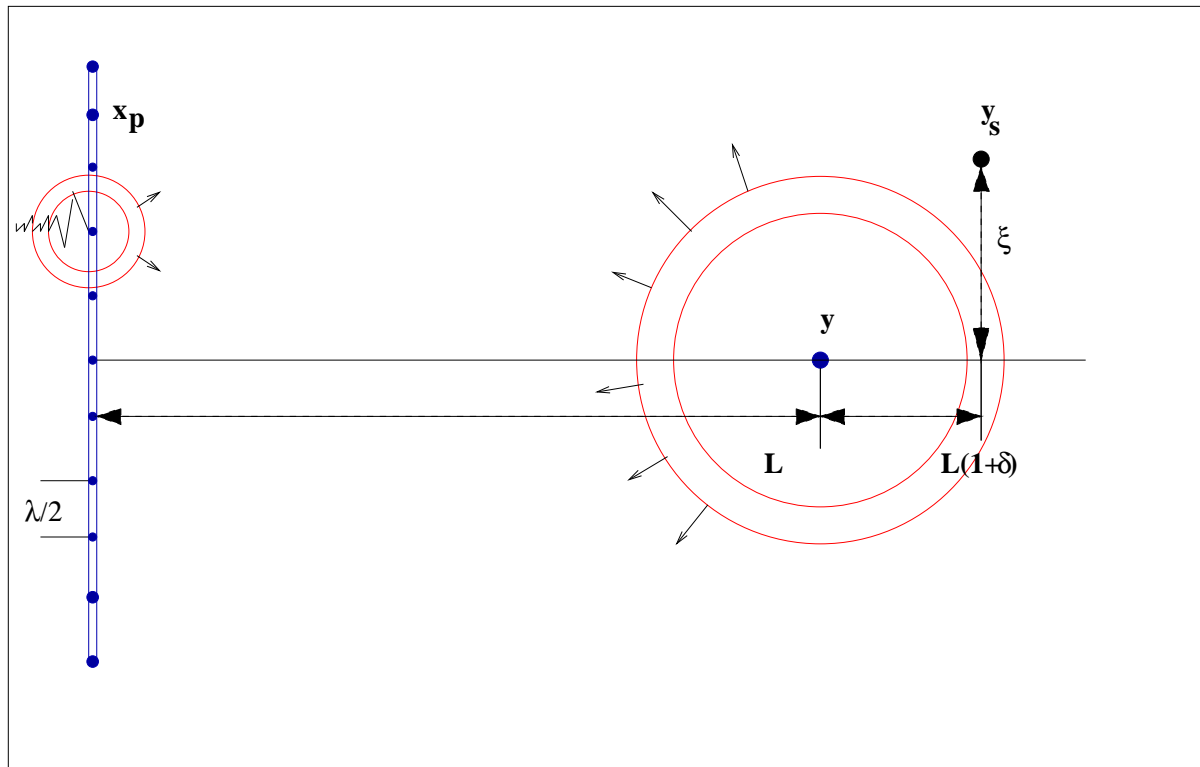
In collaboration with:

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Outline

- Time reversal focusing in a cluttered medium
- Role of clutter in detection and imaging
- Migration (SAI), SVD of response matrix, Interferometry and matched fields
- Implementation: numerical simulations, windowing
- Implementation: understanding the Scott-McClellan experimental setup and exploring the theory

Time Reversal Schematic

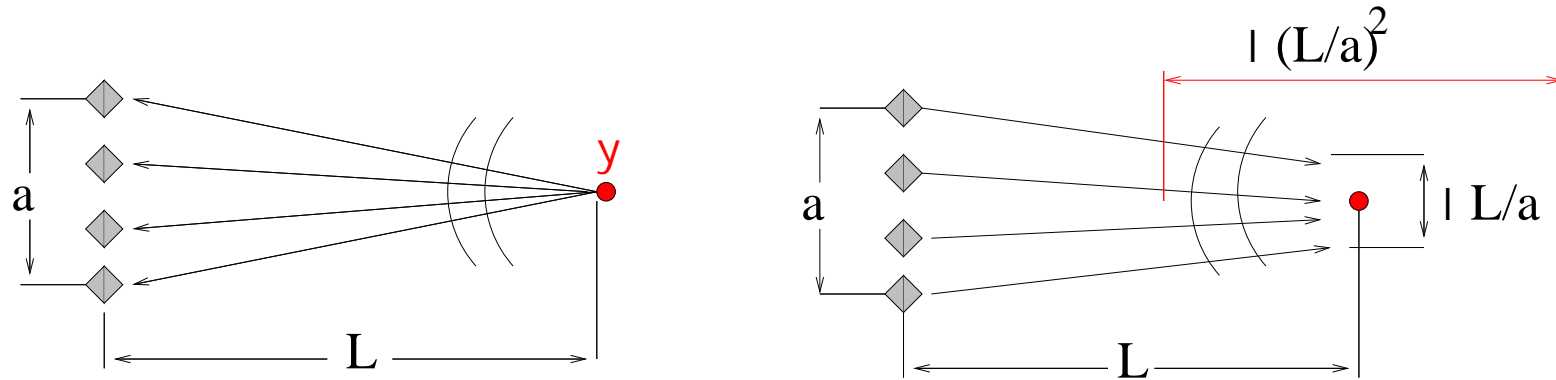


Range: L , Carrier wavelength λ , Array size $a = (N - 1)\lambda/2$.
Source at y , Search point at y_s , Transducers at x_p .

Remote sensing regime: $\lambda \ll a \ll L$.

Random medium: Correlation length $l \ll L$, fluctuation strength $\sigma \ll 1$.

Resolution in time reversal



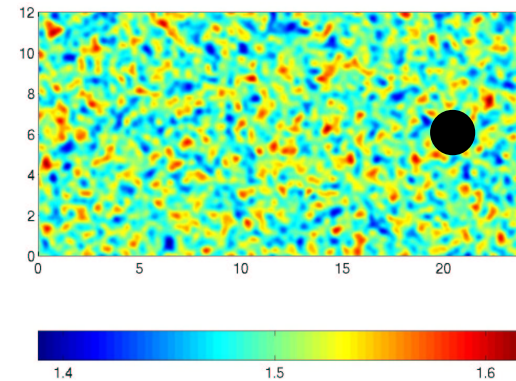
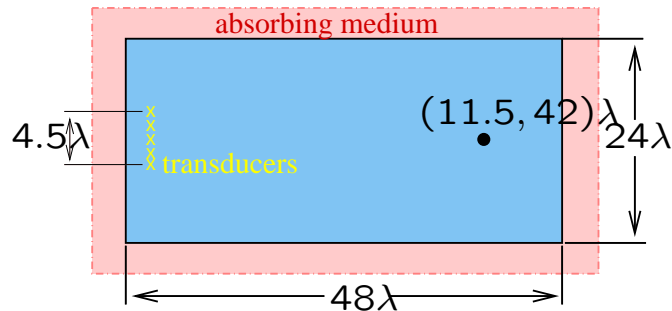
Cross-range (Rayleigh) resolution: $\frac{\lambda L}{a}$

Range resolution: $\frac{\lambda L^2}{a^2}$

- What happens in a randomly inhomogeneous medium?

SUPER-RESOLUTION

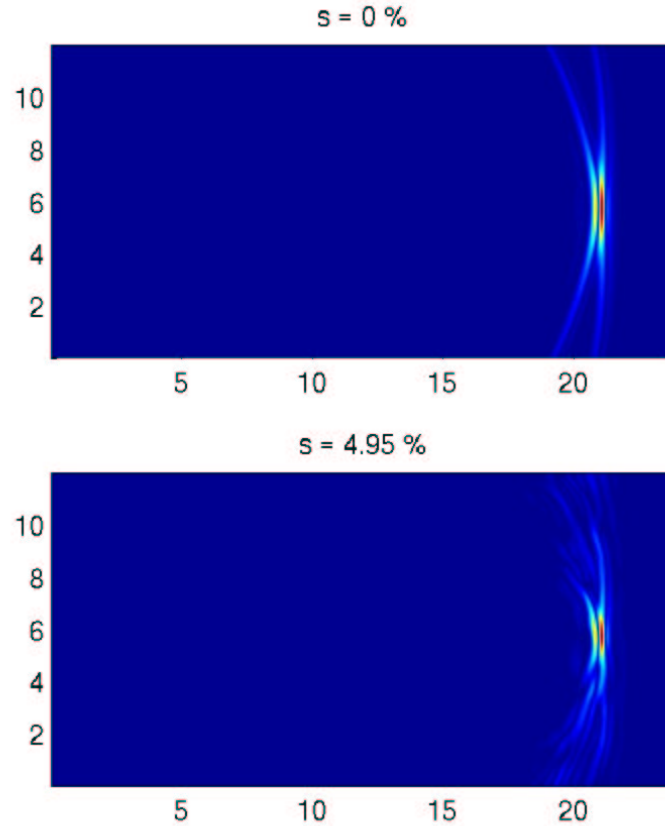
Setup for numerical simulations



On the left, the dimensions of the problem are in terms of the central wavelength $\lambda = 0.5\text{mm}$. The medium is infinite in all directions so in the numerical computations an absorbing layer surrounds the domain.

On the right is a typical realization of the random sound speed $c(\mathbf{x})$. The target is shown as a large black dot \bullet . The units in the horizontal and vertical axes are mm and, in the color bar, km/s. The standard deviation is $s = 4.95\%$

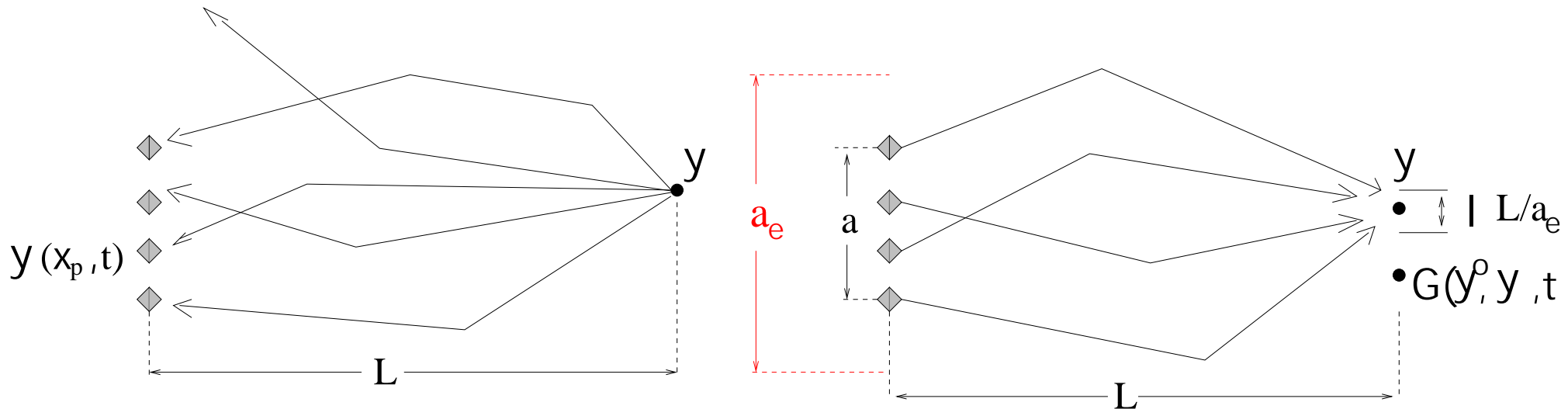
Super-resolution.



Top **deterministic**. Bottom **random**. s = standard deviation of the fluctuations of sound speed.

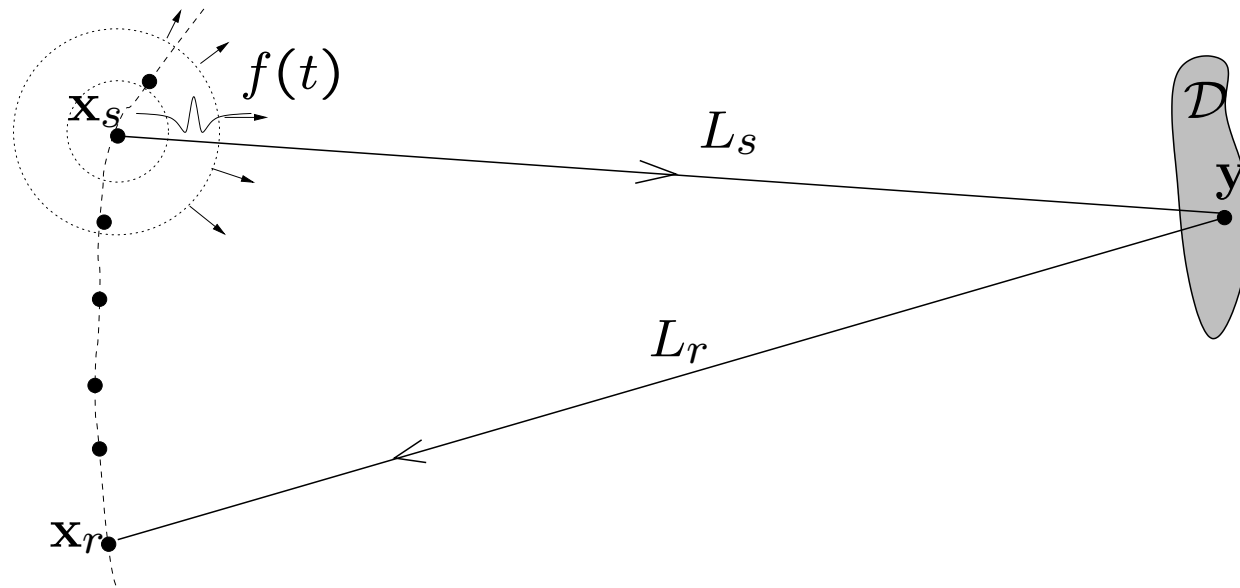
Multipathing causes super-resolution

- A source at y radiates onto the array. The recorded signal is time reversed (**conjugated in Fourier domain**) and sent into the medium (denoted by $\Gamma(y, y^o, t)$)



- **Super-resolution**: In a random medium, because of multipathing the **effective** aperture $a_e \gg a$ or, $\frac{\lambda L}{a_e} \ll \frac{\lambda L}{a}$.
- **Self-Averaging**: Nature does the averaging (ergodic theory) in the time domain (**broadband**).

Acquisition of the data for imaging



Array data: $P(\mathbf{x}_s, \mathbf{x}_r, t)$ for $(\mathbf{x}_s, \mathbf{x}_r, t)$ a set of source-receiver locations in $R^2 \times R^2$ and time in R_+ . A five-dimensional parametrization of the data.

Different data **acquisition geometries:** Synthetic aperture imaging (zero-offset, large linear apertures, broadband), Ultrasonic imaging arrays (many sources and receivers, broadband signals), etc.

Look carefully at resolution and noise issues.

Structure of the inverse problem

Primary goal: To reconstruct the extended reflectivity $\varrho(\mathbf{y})$, $\mathbf{y} \in \mathcal{D}$. This is often the contrast in propagation speed in the scatterer.

Secondary goal: To reconstruct the background propagation velocity $c(\mathbf{y})$: velocity estimation. This may involve interfaces that are mostly known up to a few undetermined parameters. Needed primarily for an accurate reconstruction of the reflectivity using arrival times. Needed in the Scott-McClellan experiments.

Secondary goal: To characterize the overall properties of the clutter, that is, the random fluctuations in the propagation speed. Needed for a good statistical stabilization of the data and for assessing blurring.

Synthetic aperture imaging or zero offset migration

Same source/receiver locations (zero offset). Sum over all sources the **migrated**, or **back-propagated**, weighted data

$$I^{KM}(\mathbf{y}^S) = \sum_{\mathbf{x}_s} \frac{y_3^S}{|\mathbf{y}^S - \mathbf{x}_s|} P(\mathbf{x}_s, \mathbf{x}_s, 2\tau(\mathbf{x}_s, \mathbf{y}^S))$$

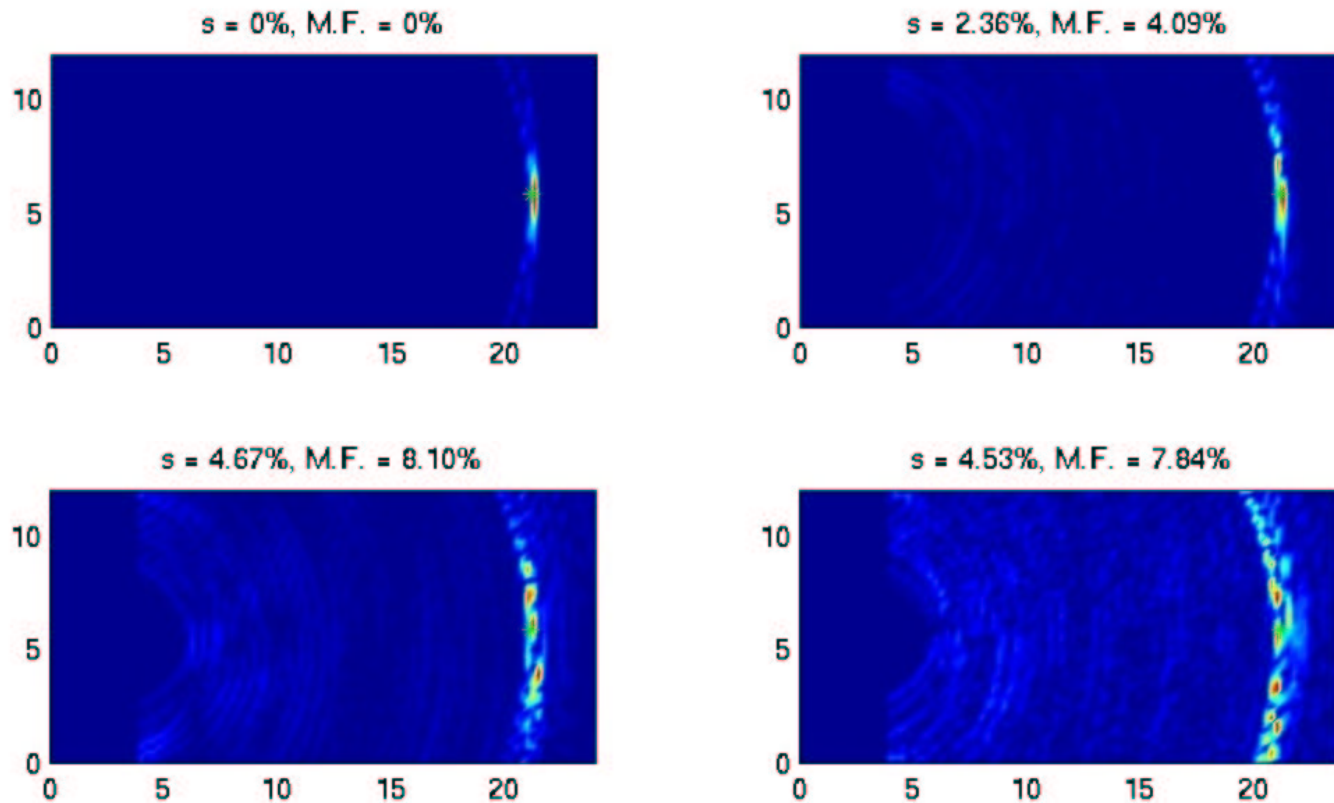
Here $2\tau(\mathbf{x}_s, \mathbf{y}^S)$ is the **travel time** from \mathbf{x}_s to the **search point** \mathbf{y}^S and back. This is the **Kirchhoff Migration** imaging functional, whose Fourier form is

$$I^{KM}(\mathbf{y}^S) = \frac{1}{2\pi} \int d\omega \sum_{\mathbf{x}_s} \frac{y_3^S}{|\mathbf{y}^S - \mathbf{x}_s|} \overline{\widehat{P}(\mathbf{x}_s, \mathbf{x}_s, \omega)} e^{2i\omega\tau(\mathbf{x}_s, \mathbf{y}^S)}$$

If the array is large and the bandwidth is also large then $I^{KM}(\mathbf{y}^S)$ reconstructs well the **reflectivity** $\varrho(\mathbf{y}^S)$ in **deterministic homogeneous media**. This is the **fundamental theorem of synthetic aperture imaging**.

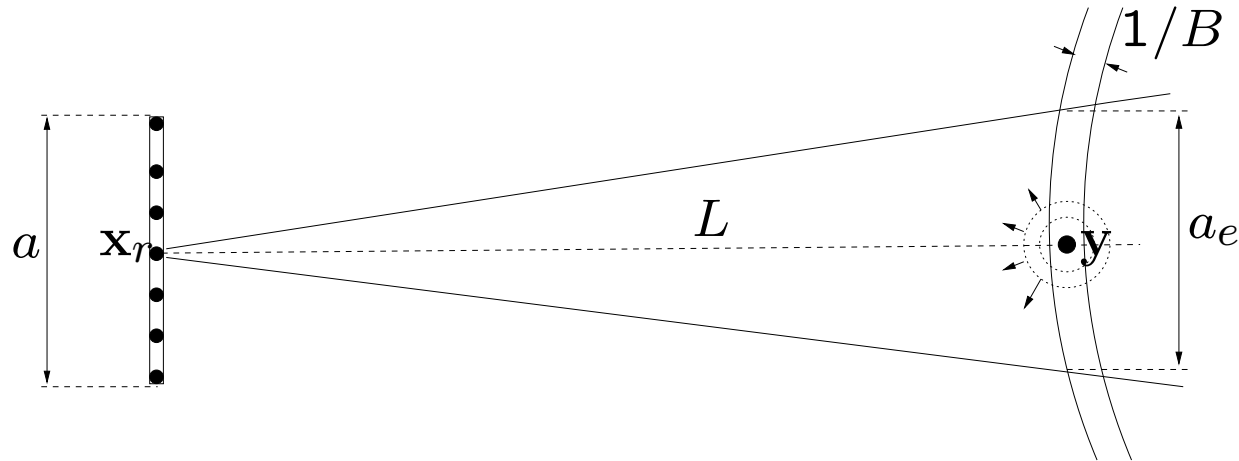
Note that **non-zero offset** data, if available, are not needed here.

SAI in clutter



Imaging a passive point scatterer with the SAI functional I^{KM} . The standard deviation is s . Note the statistical **instability** of the imaging functional.

Small scatterer in clutter: back-propagation



A source at \mathbf{y} emits a pulse received at \mathbf{x}_r . Recorded data is:

$$\hat{P}(\mathbf{x}_r, \omega) = \hat{f}_B(\omega - \omega_0) \hat{G}(\mathbf{x}_r, \mathbf{y}, \omega)$$

If we knew the Green's function (travel time) of the medium then we would consider the imaging functional

$$\Gamma^{TR}(\mathbf{y}^S) = \int d\omega \sum_{\mathbf{x}_r} \overline{\hat{P}(\mathbf{x}_r, \omega)} \hat{G}(\mathbf{x}_r, \mathbf{y}^S, \omega)$$

This is **TIME REVERSAL**: Spot size around the source location is $\frac{\lambda_0 L}{a_e}$, with $a_e \gg a$ the **effective** aperture. **Super-resolution**.

Interferometric imaging

Mimic time reversal by computing cross-correlations of data traces, the **interferograms**

$$I^{INT}(\mathbf{y}^S) = \sum_{\mathbf{x}_r, \mathbf{x}_{r'}} P(\mathbf{x}_r, \cdot) *_t P(\mathbf{x}_{r'}, -\cdot) |_{\tau(\mathbf{x}_r, \mathbf{y}^S) - \tau(\mathbf{x}_{r'}, \mathbf{y}^S)}$$

The interferograms are **self-averaging**. We are doing **Differential Kirchhoff Migration** on the lag of the interferograms.

In the frequency domain we have

$$I^{INT}(\mathbf{y}^S) = \int d\omega \left| \sum_{\mathbf{x}_r} \hat{P}(\mathbf{x}_r, \omega) e^{-i\omega\tau(\mathbf{x}_r, \mathbf{y}^S)} \right|^2$$

But this is almost **Matched Field Imaging**

$$I^{MF}(\mathbf{y}^S) = \int d\omega \left| \sum_{\mathbf{x}_r} \overline{\hat{P}(\mathbf{x}_r, \omega)} \hat{G}_0(\mathbf{x}_r, \mathbf{y}^S, \omega) \right|^2$$

Interferometric or Matched field imaging

This is because

$$\hat{G}_0(\mathbf{x}_r, \mathbf{y}^S, \omega) = \frac{e^{i\omega\tau(\mathbf{x}_r, \mathbf{y}^S)}}{4\pi|\mathbf{x}_r - \mathbf{y}^S|}$$

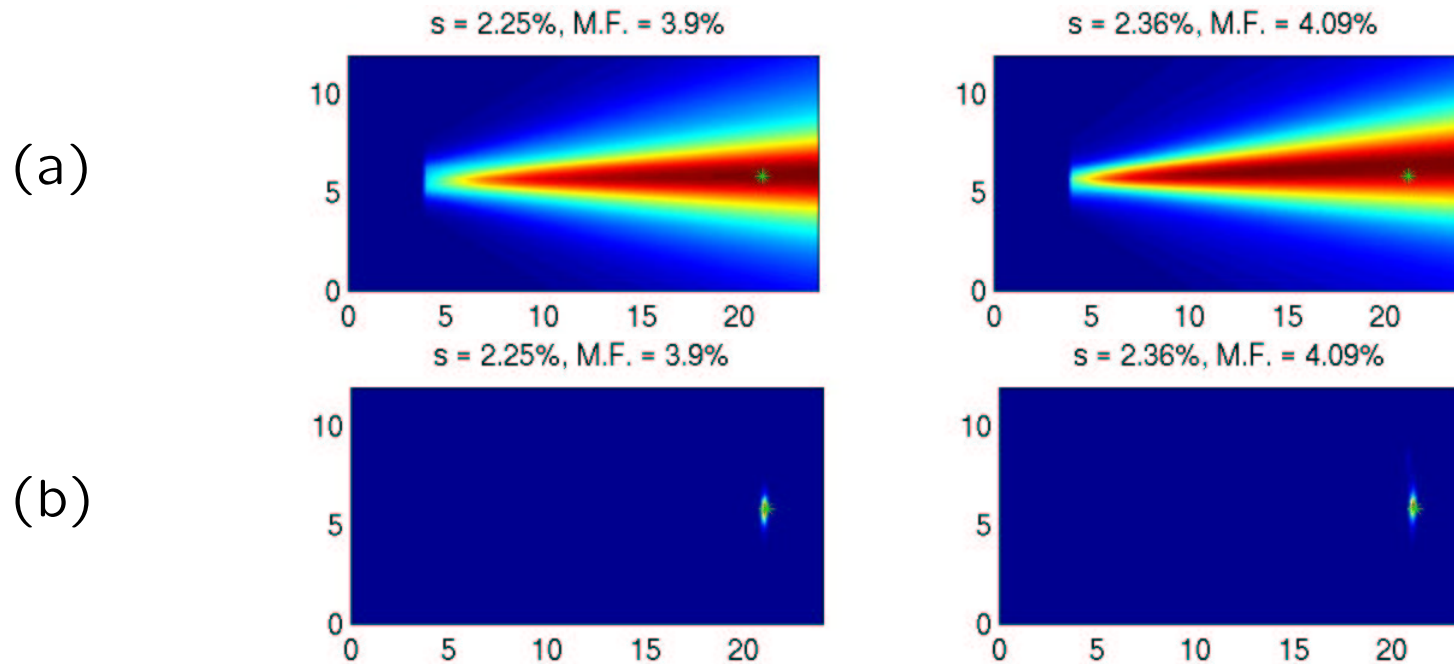
So to image, we use the Green's function of the homogeneous reference medium because **we do not know the random Green's function** $\hat{G}(\mathbf{x}_r, \mathbf{y}, \omega)$. We back-propagate with the best available information: in a homogeneous medium. But we back-propagate **interferograms** of the data.

We can calculate **approximately** the form of this imaging functional. If $\mathbf{y} = (0, L)$ and $\mathbf{y}^S = (\xi^S, L + \eta^S)$ then

$$I^{INT} \approx C e^{-\frac{|\xi^S|^2}{2(L+\eta^S)^2} \left(\frac{L}{a_e(L)}\right)^2}$$

This gives the **cross-range** in the **cone with slope** a_e/L . **Range** is **lost** and must be estimated with **windowing** of interferograms, and partly by **triangulation**.

INT or MF Imaging without and with arrival time analysis



In (a) imaging with the matched field functional for two realizations of the random medium with standard deviation $s = 2.25\%$ (left) and $s = 2.36\%$ (right). In (b) we combine the matched field functional with arrival time estimation for the same realizations of the random medium.

Note the statistical **stability** of the imaging functionals.

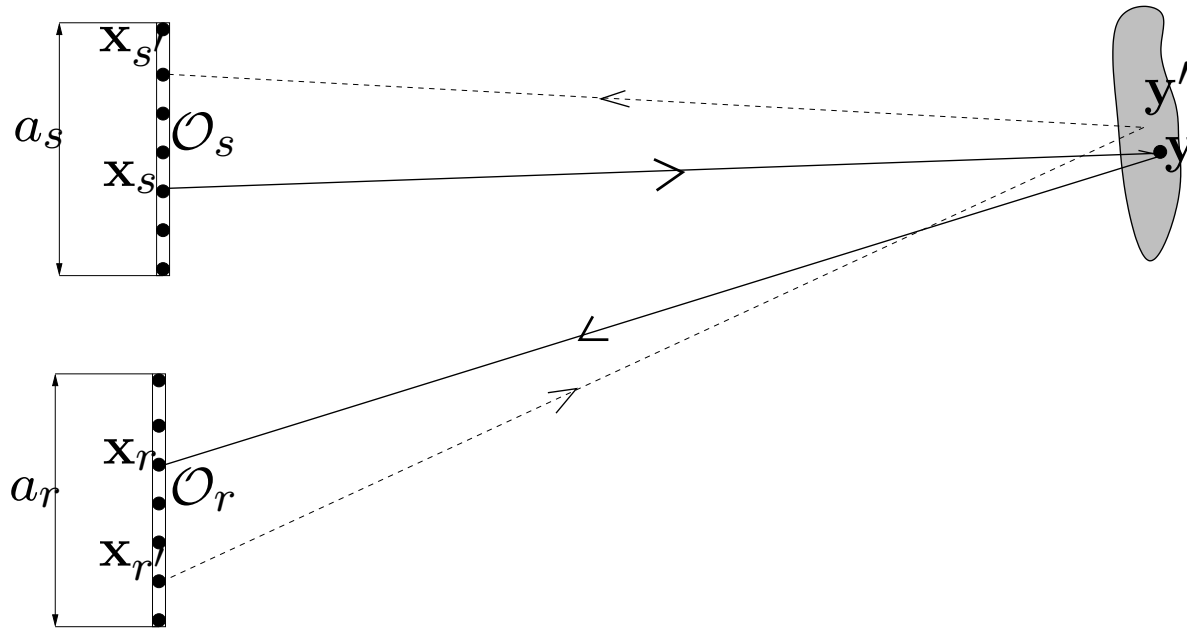
Windowed interferometry (matched fields)

- Needed to recover range information
- Relatively easy when clean arrival times can be picked from time traces
- In dispersive and random media there is a delicate trade-off between **delay spread** and **depth resolution**: windows must be narrow for depth resolution but broad enough for statistical stability
- We have a theory for windowing only for randomly layered media
- Windowing is a priority research direction

Loss of resolution from clutter: how can we control it?

- We need to use large arrays
- One possibility when data is sparse: Time-domain SVD (interferometry) with broad-band data
- Another approach: pre-processing array data to stabilize clutter effects. **Coarse-grain** the data with a **Local Data Covariance (LDC)** reduction
- **TRANSFORM ARRAY IMAGING IN CLUTTER TO A DETERMINISTIC IMAGE DEBLURRING PROBLEM.** This is computationally intensive.

Local Data Covariances I



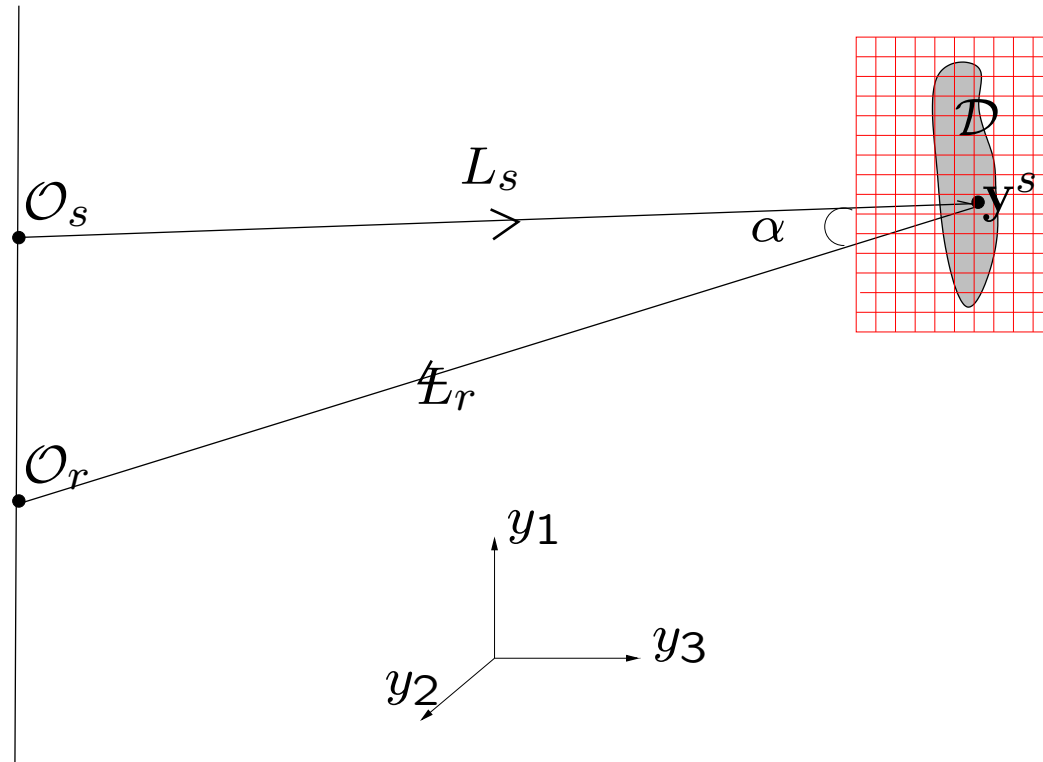
Construct the **local matched field functional** of the data for small arrays around $\mathcal{O}_s, \mathcal{O}_r$. That is, compute

$$I^{LDC}(\mathcal{O}_s, \mathcal{O}_r, \mathbf{y}^S) = \sum_{s,s';r,r'} \frac{1}{2\pi} \int d\omega e^{-i\omega(\tau_{sr}(\mathbf{y}^S) - \tau_{s'r'}(\mathbf{y}^S))} \cdot \hat{P}(\mathbf{x}_s, \mathbf{x}_r, \omega) \overline{\hat{P}(\mathbf{x}'_{s'}, \mathbf{x}'_{r'}, \omega)}$$

Asymptotic form of the coarse-grained LDC functional

In a **remote sensing regime** $a_s, a_r \ll L$, in random media the coarse-grained LDC imaging functional is

$$I^{LDC}(\mathcal{O}_s, \mathcal{O}_r, \mathbf{y}^S) \approx C(\mathcal{O}_s, \mathcal{O}_r; f) \cdot \int_{\mathcal{D}} d\mathbf{y} (\varrho(\mathbf{y}))^2 e^{-\frac{(y_2 - y_2^S)^2}{2} \left(\frac{1}{(a_e^s)^2} + \frac{1}{(a_e^r)^2} \right) - \frac{(y_1 - y_1^S)^2}{2(a_e^s)^2} - \frac{((y_1 - y_1^S) \cos \alpha - (y_3 - y_3^S) \sin \alpha)^2}{2(a_e^r)^2}}$$



Comments on the LDC reduction to deblurring

- The LDC reduction (local MF's) **does not image** the reflectivity $\varrho(\mathbf{y})$. It reduces in a precise manner the **imaging-in-clutter** to an **image-deblurring with noise** problem.

$$I^{LDC}(\mathcal{O}_s, \mathcal{O}_r, \mathbf{y}^S) = K_{s,r}^{\ominus} * \varrho^2(\mathbf{y}^S) + N_{s,r}$$

- For LDC coarse-graining we need some prior estimate of the effective apertures (clutter effects). **The theory tells us: how to estimate effective apertures, how to collect good array imaging data in clutter, and how to do a good LDC coarse-graining**
- The **key to resolution enhancement** in deblurring is having **multiple views of the reflectivity from the coarse-grained source-receiver locations \mathcal{O}_s and \mathcal{O}_r** , a subset of R^4 . Role of windowing here?

Conclusions

- The Local Data Covariance coarse-graining is a controlled and precise method for minimizing the effects of clutter when highly resolved array imaging data is available. LDC with windowing and/or use of SVD needs to be explored, simulated and understood. Apply in the Scott-McClellan experimental setup.
- Other research directions: Explore imaging problems that are in-between clean media (coherent array data) and very cluttered media (with only incoherent array data after LDC coarse-graining). Such an intermediate theory can be used with the GPR data of Q. Liu