GPR IMAGING USING COMPRESSED MEASUREMENTS

A presentation by Prof. James H. McClellan

School of Electrical and Computer Engineering
Georgia Institute of Technology

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Dr. Ali Cafer Gürbüz
Prof. Waymond Scott
Motivation

Currently more than 110 million active landmines worldwide.

Multiple sensing modes
- Metal Detectors: Electro-Magnetic Induction (EMI)
- **Ground Penetrating Radar (GPR) Arrays**
- Seismic Arrays that sense Surface Waves

Imaging over small areas (a few m\(^2\))

Would be good to have...

1. **lower data acquisition times** to move faster
2. mobile (robotic) sensor systems
3. cheaper (analog) hardware and processing
We wish to form an “image” of the subsurface, $b \in \mathbb{R}^n$, from $m$ measurements taken by a scanning sensor.

$$y_k = \langle x, \phi_k \rangle, \quad k = 1, \ldots, m$$

$$y = \Phi x \quad \text{(measurements)}$$

$$x = \Psi b \quad \text{(sensor model, } x \in \mathbb{R}^s)$$

**CS tells us:**
- Possible with $m \ll n$, if the image is sparse
- Use inner products, which could be random sampling
- **Reconstruction requires computation**
Robust Compressive Sensing

Signals are generally noisy. A realistic model for the measurements

\[ y = \Phi x + z \quad \text{and} \quad z_k \sim i.i.d \ N(0, \sigma^2) \]

**Dantzig Selector (Candes and Tao)**

- If the Restricted Isometry Property holds

\[
\hat{b} = \arg\min_b \|b\|_1 \quad \text{s.t.} \quad \|A^T(y - Ab)\|_\infty < \epsilon_N \sigma.
\]

- where \( A = \Phi \Psi \)

- Selecting \( \epsilon_N = \sqrt{2 \log N} \) makes the true \( b \) feasible with high probability.
GPR Antenna System

Lab System

Geometry of GPR Acquisition

TX2  TX1  RX1  RX2

Air  Earth

Tunnel
GPR Imaging via BackProjection (BP)

Raw Data

Surface Removed

BackProjected (BP) Image
A point target model is assumed. Targets don’t interact so superposition is valid.

Time Domain

\[ \zeta_i(t) = A s(t - \tau_i(p)) \]

Stepped Frequency system

\[ \zeta_i(\ell) = \frac{A \sigma}{S(R(p))} e^{j2\pi f_\ell(t - \tau_i(p))} \]

where \( f_\ell = f_0 + \ell \Delta f \) with \( \ell = 0, 1, 2, \ldots, L-1 \)

when \( A e^{-j2\pi(f_0+\ell\Delta f)t} \) is transmitted.
A discrete inverse operator can be created by discretizing the spatial domain target space and synthesizing the GPR model data for each discrete spatial position.

\[
\zeta_i = \sum_{k=1}^{P} b(k) \exp \left[ -j \omega (t - \tau_i(\pi_k)) \right]
\]

\[
\zeta_i(f) = \Psi_i b \quad \text{where} \quad [\Psi_i]_j = \exp \left[ -j \omega (t - \tau_i(\pi_j)) \right]
\]
In CS, linear projections of $\zeta_i$ onto a second set of basis vectors $\phi_{im}$, $m = 1, 2, \ldots M$ are measured. In matrix form for the $i^{th}$ aperture point is

$$\beta_i = \Phi_i \zeta_i = \Phi_i \Psi_i b$$
We use $L$ scan positions and form a “super problem” with the matrices $\Psi = [\Psi_1^T, \ldots, \Psi_L^T]^T$, and $\Phi = \text{diag}\{\Phi_1, \ldots, \Phi_L\}$, and the measurements $\beta = [\beta_1^T, \ldots, \beta_L^T]^T$.

\[
\hat{b} = \arg\min \|b\|_1 \quad \text{s.t.} \quad \beta = \Phi \Psi b
\]

For noisy data we might solve either

\[
\hat{b} = \arg \min \|b\|_1 \quad \text{s.t.} \quad \|A^T(\beta - Ab)\|_\infty < \epsilon_1
\]

or

\[
\min \|b\|_1 \quad \text{s.t.} \quad \|\beta - Ab\|_2 < \epsilon_2
\]

where $A = \Phi \Psi$. 
**Simulation Results**

- Comparison of standard backprojection to CS imaging in terms of generated image quality and number of measurements used for both time/frequency domains.
- Effect of random spatial sampling on the generated images
- Performance in varying noise levels
- Ability to resolve closely spaced targets

**Experimental Results using Lab Data**

- Time-Domain and Stepped Frequency-Domain
- Imaging of a 1" metal sphere in air
- Imaging of multiple buried targets
Compressive Sensing
Compressive Subsurface Imaging with GPR
Simulation & Experimental Results

Time Domain Imaging: Measurements

Target space

Space-Time domain data (SNR = −5 dB) 512 × 30

Compressive Measurements 20 × 30
Time Domain Imaging: Images

Compressive Sensing (20/512 of the data)

Backprojection (All the data)
Frequency Domain Imaging: Measurements

Target space

Space-frequency data (SNR = 0 dB)

100 × 30

Measured Frequencies (black)

20 × 30
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Frequency Domain Imaging: Images

BP w/ all freq. data

BP w/ randomly selected data (20%)

CS w/ randomly selected data (20%)
Random Spatial Sampling & Random Frequencies

Measured (20 x 10) Space-frequency data

BP Result

CS Result
Increased Resolution for CS

Backprojection

Compressive Sensing

Frequency Range: 3–5 GHz and resolution limit is 7.5 cm in air
Behavior in Noise (One Reflector)

Variance of target positions vs. SNR

Variance of created images vs. SNR

BP uses $M = 220$ measurements
Behavior vs. Number of Reflectors

Success vs. # Targets

Success vs. SNR
List of Simulation/Experimental Results

- Experimental Results using Lab Data
  - Time-Domain and Stepped Frequency-Domain
  - Imaging of a 1" metal sphere in air
  - Imaging of multiple buried targets
GPR Air Experiments

Measurement Setup

Space-Time domain data (220 × 70)

Space-Time Compressive Measurements (10 × 70)
Imaging of 1" Metal Sphere: Time-Domain

Compressive Sensing (10 × 70)  Backprojection (220 × 70)
1-inch Metal Sphere: Stepped Frequency

Space-frequency data

Results for random freq. measurements
30 out of 379 frequencies are measured
Buried Target Experiments

Picture of Buried Targets

Burial Map
Backprojection image uses all 379 frequencies.
CS uses only 100 randomly selected frequencies.
Backprojection image uses 128 samples per scan point; CS uses only 15 projections.
3D Subsurface Imaging Results

Backprojection vs Compressive Sensing

3D iso-surface image of the selected region
Questions

jim.mcclellan@ece.gatech.edu