Interferometric Array Imaging

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In collaboration with:
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Array data: $P(x_r, t)$ for $(x_r, t)$ a set of receiver locations in $\mathbb{R}^2$ and time in $\mathbb{R}_+$. A three-dimensional parametrization of the data.

Application: Imaging underground structures.

Objective: Use a new form of interferometric imaging that is robust when there is clutter. Provide numerical simulations as well as an adaptive resolution theory.
Outline

- The setup for the numerical simulation of imaging underground structures.

- Synthetic aperture or Kirchhoff migration (time-reversal) imaging and its limitations when there is clutter. Numerical results.

- Incoherent interferometry, what is it, its connection to time reversal and the role of bandwidth. Its uses and its limitations.

- Coherent interferometry, what is it, its uses and its limitations. Numerical results.

- Coherent interferometry for active array or echo-mode imaging.

- Adaptive resolution estimation and a mathematical theory of resolution for coherent interferometry.

- Summary and work in progress
Computational domain $100\lambda_0 \times 100\lambda_0$ with central wavelength $\lambda_0 = 3m$ (at central frequency $f_0 = 1KHz$ and with $c_0 = 3km/sec$), surrounded by a perfectly matched layer (pink).

The array has 185 receiving elements $\lambda_0/2$ apart, for an aperture of $92\lambda_0$. 
The three sources are ultrawideband (more than 100% bandwidth) at a distance of $90\lambda_0$ from the array. The near ones are $6\lambda_0$ apart and the far one is $3\lambda_0$ behind the near ones.

The random fluctuations of the propagation speed, on the right, have 3% STD and a Gaussian correlation (monoscale) function. The correlation length is equal to the central wavelength $\lambda_0 = 3m$.

We use a FINITE ELEMENT TIME DOMAIN CODE (2D or 3D), resolving all scales with 30 – 40 points per wavelength. It takes about two hours on a workstation to produce a set of (2D) synthetic data.

We have well established 3D elastic and electromagnetic FETD codes. The codes are now being ported on a computer cluster that was obtained recently.
The imaging functional for KM at a search point $y^S$ is:

$$I^{KM}(y^S) = \sum_r P(x_r, \tau(x_r, y^S)) = \sum_r \frac{1}{2\pi} \int d\omega \hat{P}(x_r, \omega) e^{-i\omega \tau(x_r, y^S)}$$

where $\tau(x, y) = |x - y|/c_0$ is the travel time in a homogeneous background with propagation speed $c_0$.

It does not work well in clutter.

It is statistically unstable in clutter.

The reason is that KM tries to cancel (by back propagation or time reversal in a homogeneous medium) the random phase of the signals arriving at the array with a deterministic phase using travel times.
Kirchhoff migration imaging results

Down: Homogeneous, 1%, 2%, 3%STD. Across: different realizations.
Incoherent Interferometry

To avoid the random phase problems in Kirchhoff migration imaging we mimic time reversal by computing cross-correlations of data traces, the interferograms, and summing

$$I^{INT}(y^S) = \sum_{x_r, x_r'} P(x_r, \cdot) *_t P(x_r', -\cdot)|_{\tau(x_r, y^S) - \tau(x_r', y^S)}$$

The interferograms are self-averaging. We are doing Differential Kirchhoff Migration on the lag of the interferograms.

In the frequency domain we have

$$I^{INT}(y^S) = \int d\omega \left| \sum_{x_r} \hat{P}(x_r, \omega) e^{-i\omega \tau(x_r, y^S)} \right|^2$$

But this is almost Matched Field Imaging

$$I^{MF}(y^S) = \int d\omega \left| \sum_{x_r} \hat{P}(x_r, \omega) \hat{G}_0(x_r, y^S, \omega) \right|^2, \quad \hat{G}_0(x, y, \omega) = \frac{e^{i\omega \tau(x, y)}}{4\pi|x - y|}$$
The interferograms

\[ P(x_r, \cdot) *_t P(x_{r'}, -\cdot)(t) = \int_{-\infty}^{\infty} P(x_r, s) P(x_{r'}, s - t) ds \]

have a time reversal interpretation (based on the reciprocity of Green’s functions):

A signal is emitted from \( x_{r'} \) and is recorded at the (unknown) source location \( y \). The recorded signal is time-reversed and re-emitted into the medium. The signal received at \( x_r \) is the interferogram above.

The interferogram decorrelates rapidly in a random medium as the distance between \( x_r \) and \( x_{r'} \) increases.

The decorrelation distance \( \Delta_d \) is related to the effective aperture in time reversal, which is essentially independent of any physical aperture. It can be estimated from the array data directly.
Interferometric imaging is motivated by the fact that focusing in real time reversal improves when there is clutter (super resolution), in a statistically stable way. There is, however, loss of energy, again because of the clutter.

Across: Homogeneous, 1%, 3%STD, for an array of size $10\lambda_0$. The focusing strength is normalized to have peak equal to one.
Limitations of incoherent interferometry

- Incoherent interferometric imaging is effective only if the decoherence frequency $\Omega_d$ is very small, essentially zero relative to the bandwidth. Frequency coherence is not used!

- Incoherent interferometric imaging gets depth resolution only by triangulation, which needs large arrays. Sometimes an arrival time analysis can give depth resolution but there is no robust way to do this.

- Incoherent interferometry gets direction of arrival (angular) resolution from which the decoherence length $\Delta_d$ can be estimated. Broadband is essential here (Inverse Problems, vol 19, (2003), pp. 5139-5164).
To combine good random phase cancellation, which KM does not have but INT of MF has, with exploitation of residual coherence effects we introduce the Coherent Interferometric functional:

\[
I^{CINT}(y^S) = \int \int |\omega_1 - \omega_2| \leq \Omega_d d\omega_1 d\omega_2 \sum \sum |x_r - x'_r| \leq \Delta_d \frac{\hat{P}(x_r, \omega_1)\hat{P}(x'_r, \omega_2)}{P(x_r, \omega_1)P(x'_r, \omega_2)} e^{-i(\omega_1 \tau(x_r, y^S) - \omega_2 \tau(x'_r, y^S))}
\]

Using sum and difference variables: \( x_r = \bar{x} - \bar{x}/2 \), \( x'_r = \bar{x} + \bar{x}/2 \) and \( \omega_1 = \bar{\omega} - \bar{\omega}/2 \), \( \omega_1 = \bar{\omega} + \bar{\omega}/2 \) we rewrite it as

\[
I^{CINT}(y^S) = \int \int d\bar{\omega} \sum \sum \frac{\hat{P}(\bar{x} - \bar{x}/2, \bar{\omega} - \bar{\omega}/2)\hat{P}(\bar{x} + \bar{x}/2, \bar{\omega} + \bar{\omega}/2)}{P(\bar{x} - \bar{x}/2, \bar{\omega} - \bar{\omega}/2)P(\bar{x} + \bar{x}/2, \bar{\omega} + \bar{\omega}/2)} e^{i\bar{\omega}[\tau(\bar{x} + \bar{x}/2, y^S) - \tau(\bar{x} - \bar{x}/2, y^S)]} e^{i\bar{\omega}/2[\tau(\bar{x} - \bar{x}/2, y^S) + \tau(\bar{x} + \bar{x}/2, y^S)]}
\]
Using the space-wavenumber, time-frequency Wigner function of the data

\[ W_D(\bar{x}, p; \bar{t}, \bar{\omega}) = \int_{|\tilde{\omega}| \leq \Omega_d} d\tilde{\omega} \sum_{|\tilde{x}| \leq \Delta_d} \hat{P}(\bar{x} - \frac{\tilde{x}}{2}, \bar{\omega} - \frac{\tilde{\omega}}{2}) \hat{P}(\bar{x} + \frac{\tilde{x}}{2}, \bar{\omega} + \frac{\tilde{\omega}}{2}) e^{i(p \cdot \bar{x} + \bar{t} \bar{\omega})} \]

and simplifying, the coherent interferometric functional is

\[ I^{CINT}(y^S) = \int d\tilde{\omega} \sum_{\bar{x}} W_D(\bar{x}, \bar{\omega} \nabla \bar{x} \tau(\bar{x}, y^S); \tau(\bar{x}, y^S), \bar{\omega}) \]

This is the imaging functional that we use in the results shown next, where we also vary the decoherence frequency \( \Omega_d \) and the decoherence distance \( \Delta_d \).
Coherent interferometric imaging results I

Down: 0%, 1%, 3% STD. Across: $\Omega_d = 0, 10, 20\%$. All: $\Delta_d = 20\%$
Down: 0%, 1%, 3% STD. Across: $\Omega_d = 0, 10, 20\%$. All: $\Delta_d = 40\%$
No $\tilde{\omega}$ averaging. Three realizations at 3% STD. Top: $\Omega_d = 10\%$ $\Delta_d = 20\%$. Bottom: $\Omega_d = 10\%$ $\Delta_d = 40\%$. No statistical stability when bandwidth is not used fully.
Coherent interferometric imaging results IV

Sum of three $\bar{x}$’s. Three realizations at 3% STD. Top: $\Omega_d = 10\% \Delta_d = 20\%$. Bottom: $\Omega_d = 10\% \Delta_d = 40\%$. Some statistical stability when array is not used fully.
A resolution theory has been developed based on several assumptions about the random medium and the propagation regime. No assumptions such as these are, however, used in the numerical simulations.

- With the paraxial approximation, the white noise limit, and a high frequency expansion we reduce all theoretical calculations to the use of one relatively simple formula obtained from the random Schrödinger equation.

- The main results are: (a) the resolution in range is $c_0/\Omega_d$ with $c_0$ the homogeneous propagation speed, and (b) the angular or direction of arrival resolution is $(k_0 \Delta_d)^{-1}$ with $k_0$ the central wavenumber. We assume here that the SNR at the array is high (essentially infinite).
Active or echo-mode array imaging

In the echo-mode numerical simulations we use a configuration of three (Dirichlet) scatterers of size equal to one central wavelength $\lambda_0$, at the same location as the sources in the passive mode simulations.

Three array elements act as sources of illumination, one at a time: one central and two side elements.

We show calculations with 1% STD of fluctuations. In echo-mode the random medium is, however, twice as long as in direct mode imaging so its effect is stronger.

We use the coherent interferometric imaging functional $I^{CINT}(y^S)$ for each illumination.
Active array imaging schematic

Array data: $P(x_s, x_r, t)$ for $(x_s, x_r, t)$ a set of source-receiver locations in $R^2 \times R^2$ and time in $R_+$. Up to a five-dimensional parametrization of the data is possible.

In the numerical simulations we use only three source locations $x_s$, one at a time.
Active or echo-mode imaging results I

Across: central, right side, left side illumination. Top: Kirchhoff migration with 1% STD. Bottom: Coherent interferometry with 1% STD, $\Omega_d = 20\%$ and $\Delta_d = 40\%$. 
Active or echo-mode imaging results II

Down: $x_s =$ center, right and left. Across: $\Omega_d = 1\%, 10\%, 20\%; \Delta_d = 20\%$. 
Active or echo-mode imaging results III

Down: $x_s =$ center, right and left. Across: $\Omega_d = 1\%, 10\%, 20\%; \Delta_d = 40\%$. 
We have introduced a new imaging technique, coherent interferometry, that is remarkably effective when imaging in a cluttered environment.

Imaging underground structures with active or passive arrays should benefit substantially from this methodology.

Coherent interferometry is fully adaptive and model independent. It does need high SNR at the array.
• Near term items: Complete the passive array theory, the echo-mode simulations and the echo-mode theory. Consider the SNR issues carefully. Refine the estimation and effective use of the decoherence frequency and distance. Use a robust space-wavenumber, time-frequency analysis with the multidimensional, adaptive local cosine transform.

• Longer term items: Apply the theory to GPR. Use optimal illumination in echo-mode to improve SNR. Do 3D simulations. Use real data.