Adaptive Methods in Coherent Interferometric Array Imaging

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Outline

- Coherent Interferometry (CINT) based on the decoherence frequency and length for the detection and imaging of structures in clutter.

- Numerical simulations to assess algorithms for imaging structures in clutter with passive and active arrays.

- Resolution theory for CINT.

- Introduction of adaptive CINT or ACINT.

- The ACINT and Deblurring research program.
Passive and Active Array data

Active array data: $P(x_s, x_r, t)$ for $(x_s, x_r, t)$ a set of source-receiver locations.

Passive array data: $P(x_r, t)$.

Different data acquisition geometries: Synthetic aperture imaging (zero-offset, large linear apertures, broadband), Ultrasonic imaging arrays (many sources and receivers, broadband signals).
Coherent interferometric imaging

We introduce the Coherent Interferometric imaging functional:

\[
I^{CINT}(y^S; X_d, \Omega_d) = \int \int_{|\omega_1-\omega_2| \leq \Omega_d} d\omega_1 d\omega_2 \sum \sum |x_r-x'_r| \leq X_d \\
\hat{P}(x_r, \omega_1) \hat{P}(x'_r, \omega_2) e^{-i(\omega_1 \tau(x_r,y^S) - \omega_2 \tau(x'_r,y^S))}
\]

The smoothing parameters are the Decoherence Length \(X_d\), \(0 < X_d \leq a\) and the Decoherence Frequency \(\Omega_d\), \(0 < \Omega_d \leq B\). Here \(a\) is the array size and \(B\) the bandwidth.

They are determined Adaptively as the image is formed.

If we take \(X_d = a\) and \(\Omega_d = B\), which means that there is no smoothing, then the CINT functional is just the Kirchhoff migration functional squared.
Computational domain $100\lambda_0 \times 100\lambda_0$ with central wavelength $\lambda_0 = 3m$ (at central frequency $f_0 = 1KHz$ and with $c_0 = 3km/sec$), surrounded by a perfectly matched layer (pink).

The array has 185 receiving elements $\lambda_0/2$ apart, for an aperture of $92\lambda_0$. 
The three sources are ultrawideband (more than 100% bandwidth) at a distance of $90\lambda_0$ from the array. The near ones are $6\lambda_0$ apart and the far one is $3\lambda_0$ behind the near ones.

The random fluctuations of the propagation speed, on the right, have 3% STD and a Gaussian correlation (monoscale) function. The correlation length is equal to the central wavelength $\lambda_0 = 3m$.

We use a FINITE ELEMENT TIME DOMAIN CODE (2D or 3D), resolving all scales with 30 – 40 points per wavelength. It takes about two hours on a workstation to produce a set of (2D) synthetic data.

We also have well established and tested 3D elastic and electromagnetic FETD codes.
Passive and active array data

Traces recorded on the array

Left figures: Passive array

Right figures: Active array

Top figures: Homogeneous medium.

Bottom figures: Random medium with standard deviation \( s = 3\% \).
Coherent interferometric imaging results I

Coherent Interferometry images in random media with $s = 3\%$.

Left Figures: $X_d = a$, $\Omega_d = B$ (Kirchhoff Migration, no smoothing)

Middle Figures: $X_d = X^*_d$, $\Omega_d = \Omega^*_d$ (Adaptively selected optimal smoothing)

Right Figures: $X_d < X^*_d$, $\Omega_d < \Omega^*_d$ (Too much smoothing)
Coherent interferometric imaging results II

Coherent Interferometry : Effect of $X_d$ on the image resolution.

The value of $\Omega_d$ is fixed and $X_d$ decreases from left to right. The optimal is in the middle.
Coherent Interferometry: Effect of $\Omega_d$ on the image resolution.

The value of $X_d$ is here fixed and $\Omega_d$ decreases from left to right. The optimal is in the middle.
Comments on the CINT results

- Without smoothing there is no statistical stability of the image: Different realizations of the random medium give different images. Smoothing, especially in frequency, gives stable but blurred images.

- Statistical stability of the image is very important because it allows further processing with deblurring methods. We have used Level Set Deblurring methods successfully, provided that we have a good estimate of the amount of blurring.

- The optimal decoherence frequency $\Omega^*_d$ is not known and it is determined adaptively, as explained in a slide below. So is the decoherence distance $X^*_d$. 
A resolution theory can be developed based on several assumptions about the random medium and the propagation regime.

Such assumptions are NOT used in the numerical simulations.

- With the paraxial approximation, the white noise limit, and a high frequency expansion we reduce all theoretical calculations to the use of one relatively simple formula obtained from the random Schrödinger equation: a second order moment formula.

- One other regime where analytical results can be obtained: Layered media. In no other regime do we have, or expect, analytical results.
• The main results are: (a) the resolution in range is $c_0/\Omega_d$ with $c_0$ the homogeneous propagation speed, and (b) the angular or direction of arrival resolution is $(k_0X_d)^{-1}$ with $k_0$ the central wavenumber. We assume here that the SNR at the array is high (essentially infinite).

• Note that in a deterministic medium $\Omega_d = B$, the bandwidth, and $X_d = a$, the array aperture. Loss of resolution is directly tied to space and frequency decoherence. ($0 \leq \Omega_d \leq B, \ 0 \leq X_d \leq a$.)

• We estimate $\Omega_d$ and $X_d$ optimally as the image is formed using a Total Variation criterion, as explained below.
Starting from first principles and in a carefully controlled scaling limit we find that

\[ I^{CINT}(y^S, \Omega_d, X_d) \approx \int_{A_{pr}} d\bar{x} \int_{B_{ndw}} d\bar{\omega} |\hat{f}_B(\bar{\omega} - \omega_0)|^2 \]

\[ e^{-\left(\nabla_\bar{x}|\bar{x} - y^S|^2 - \nabla_\bar{x}|\bar{x} - y|^2\right)/2\sigma_{DoA}^2} e^{-\left(|\bar{x} - y^S|^2 - |\bar{x} - y|^2\right)/2\sigma_R^2} \]

Here: \( \sigma_{DoA} = \frac{c_0}{\omega_0 X_d} \), \( \sigma_R = \frac{c_0}{\Omega_d} \) are the direction of arrival and range resolution, respectively, \( \hat{f}_B \) is the Fourier transform of the base-band pulse (depending on the bandwidth \( B \)), and \( \omega_0 \) is the central frequency. We have assumed here that there is only one source at location \( y \).

The decoherence length and frequency, \( X_d \) and \( \Omega_d \), are related to the statistical properties of the random medium in an explicit manner (not shown here).
Comments on the theoretical CINT formula

- Although the theoretical formula is derived for a particular class of random media, it has a general and rather intuitive form as the integral of a Gaussian point spread function.

- The theoretical results are extended to distributed sources and to the active-array case with distributed reflectivity.

- The CINT theory is consistent with our previous incoherent interferometric results, which correspond to the case $\Omega_d = 0$.

- The theory can be used to further process the image by deblurring.
Adaptive Selection of $X_d$ and $\Omega_d$

For CINT to be effective we need to be able to determine the decoherence length and frequency adaptively. We do this as follows.

The imaging functional is

$$I^{CINT}(y^S) = \int \int d\omega_1 d\omega_2 \sum \sum \hat{F}(x_r, \omega_1, y^S) \hat{F}(x'_r, \omega_2, y^S) \leq \Omega_d \quad \sum \sum |x_r - x'_r| \leq X_d$$

with $\hat{F}(x_r, \omega, y^S) = \hat{P}(x_r, \omega) e^{-i\omega \tau(x_r, y^S)}$

Given the box space-frequency box size, we calculate it by

$$I^{CINT}(h_x, h_\omega, y^S) \approx \int_{Aprt} dx \int_{Bndw} d\omega_1 \int \int_{N(\omega_1, x)} dx' d\omega_2 \hat{F}(x_r, \omega_1, y^S) \hat{F}(x'_r, \omega_2, y^S)$$
The adaptive CINT algorithm

To select $h_x$ and $h_\omega$ we minimize an appropriate norm of the normalized image

For $h_x, h_\omega \in [0, a] \times [0, B]$, the normalized image is

$$S(h_x, h_\omega, y^S) = \frac{\sqrt{I(h_x, h_\omega, y^S)}}{\max_{y^S \in D} \sqrt{I(h_x, h_\omega, y^S)},}$$

where $I = I^{CINT}$ and the norm used is

$$\|S\|_D = \|S\|_{L^1(D)} + \|\nabla S\|_{L^1(D)}$$

$$= \int_D dy^S \left( |S(h_x, h_\omega, y^S)| + |
abla S(h_x, h_\omega, y^S)| \right)$$

The decoherence frequency and length are directly related to the optimal window size ($\Omega_d = 2h_\omega, X_d = 2h_d$) The total variation norm performs well because it preserves features of the image, that is, it does not smooth excessively.
The ACINT and Deblurring research program

- Complete a full length adaptive CINT (ACINT) paper with some theory and many numerical simulations. Show in some detail how the ACINT algorithm can improve the quality of the images dramatically.

- When coupled with the theoretical results and a good estimate of the amount of blurring then ACINT imaging can be followed by Level Set Deblurring which enhances resolution greatly.

- Use the ACINT algorithm with the Gatech GPR data. The non-adaptive CINT algorithm is expected to be quite inadequate with these data.

- Extend the CINT theory and algorithms to near-field imaging and to imaging with distributed sensors.
Concluding remarks

• Coherent interferometry, which is the backpropagation of local cross-correlations of traces, is an effective way to deal with cluttered environments in array imaging and remote sensing.

• Adaptive estimation of the space-frequency decoherence in the array data addresses very well the difficult issue of implementing coherent interferometry and makes the CINT algorithm much more robust.

• Adaptive CINT image formation combined with deblurring and adaptive illumination is our current research program.