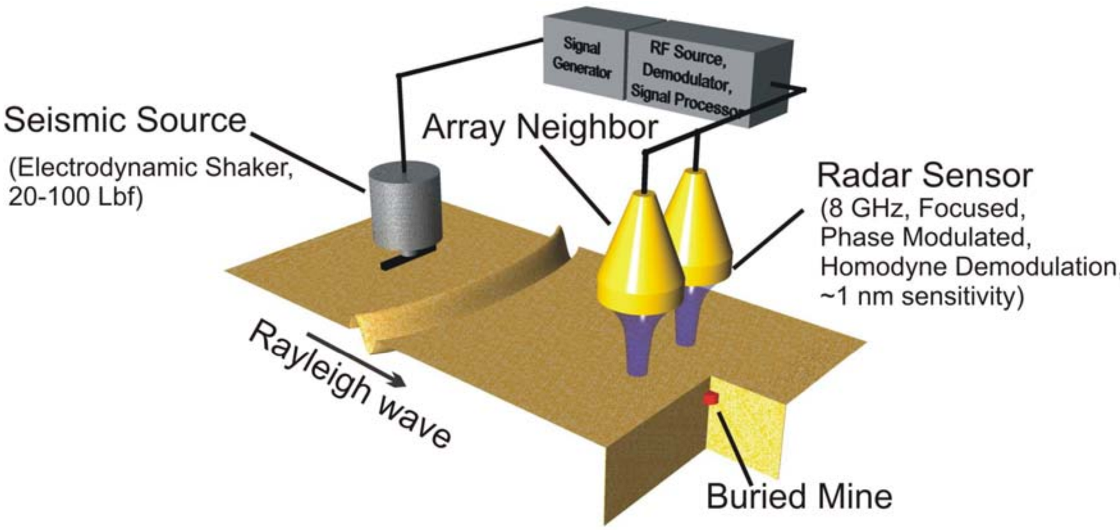
Optimal Experiments with Acoustic-Seismic Sensors

James McClellan and Waymond R. Scott, Jr.
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0250
jim.mcclellan@ece.gatech.edu
404-894-8325

Outline

- Introduction
  - Seismic System
  - Spectrum Analysis of Seismic Waves
- Location with Acoustic/Seismic Arrays
  - Maneuvering Array (3x10)
  - Cumulative Array strategy for imaging
    - Optimal Experiments
  - Examples

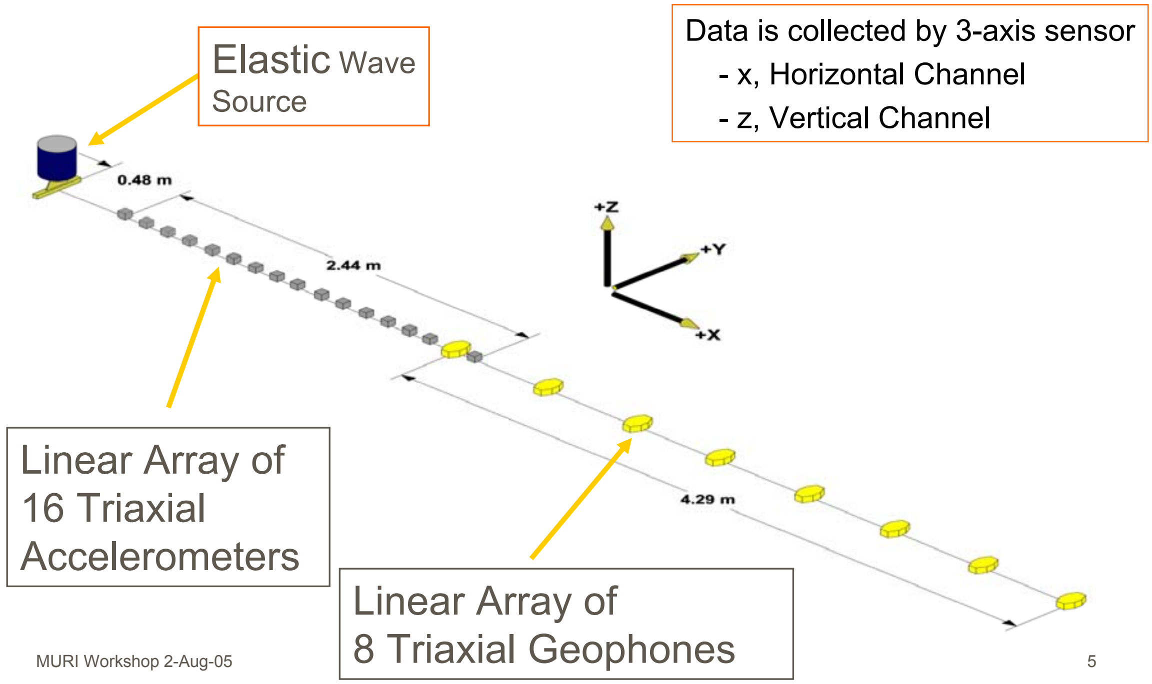
Prototype Seismic Mine Detection System



Data Movies

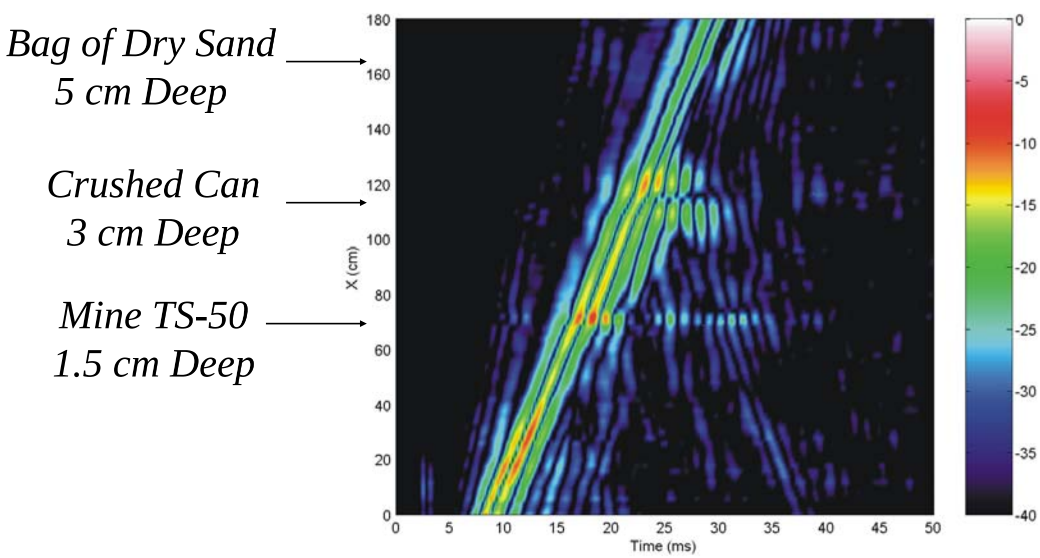
- All wave types
- Only the reflected wave
  - After k-domain processing

Typical Surface Sensor Arrays Used in Experimental Model & at Field Test Sites



Seismic Sensor

Pseudo Color Graph; y = 20 cm



Extract Reflected Waves

- Surface Waves: Velocity vs. Frequency
  - Parametric Model for surface waves
  - IQML (Prony, Steiglitz-McBride)
  - Multi-Channel Modeling
- Need **robust** method to work with **field** data
  - Extract various types of waves
    - Individual modes
  - Exploit data collected from **tri-axial** sensors
  - Polarization can be used to identify different wave types

Frequency-Domain Model

- 2-D Fourier transform of sensor data s (**x**, t) represents signal as propagating plane waves

$$s(x,t) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(k,\omega) e^{j(kx-\omega t)} dk d\omega$$

- Slowness = k / ω
- Velocity = ω / k
- k = wave-number
- ω = temporal frequency

Constant velocity is a ray from the origin

Parametric Model for <u>One</u> Channel

STEPS:

- Take 1-D Fourier transform over time, s(**x**,t)->S(**x**,ω)

$$S(x,\omega) = \frac{1}{2\pi}\int S(k,\omega)e^{j(kx)}dk$$

- **<u>ARMA</u>** Modeling is done across x to obtain a space-frequency (k , ω) model,

$$S(x,\omega) \approx \sum_{p=1}^{P} a_p(\omega)e^{jk_p(\omega)x}$$

- Estimate a(ω) and k(ω) by IQML (aka St-McB/Prony)

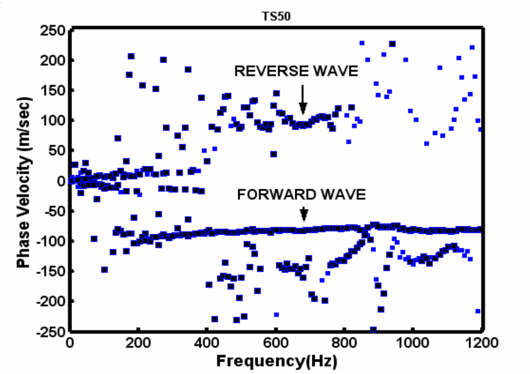
Results from Vector IQML

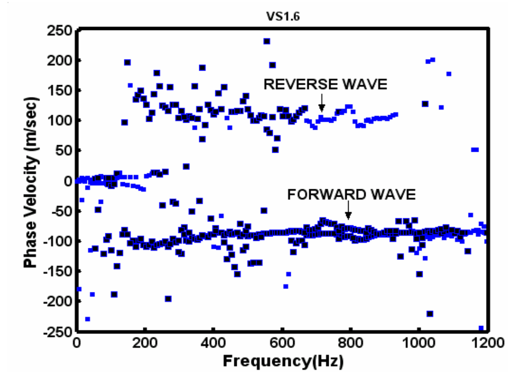
- k(ω) vs. ω becomes **<u>velocity</u>** vs. Ω

- **<u>Extract</u>** individual modes from (k , ω) and reconstruct them in time domain from the model parameters, i.e., Sum across frequency

$$s_z(\mathbf{x},t) = \sum_i A_z(\omega_i)e^{(\alpha(\omega_i)\mathbf{x}+j(\omega_i t+k(\omega_i)\mathbf{x}))}$$

Spectrum Analysis via Prony





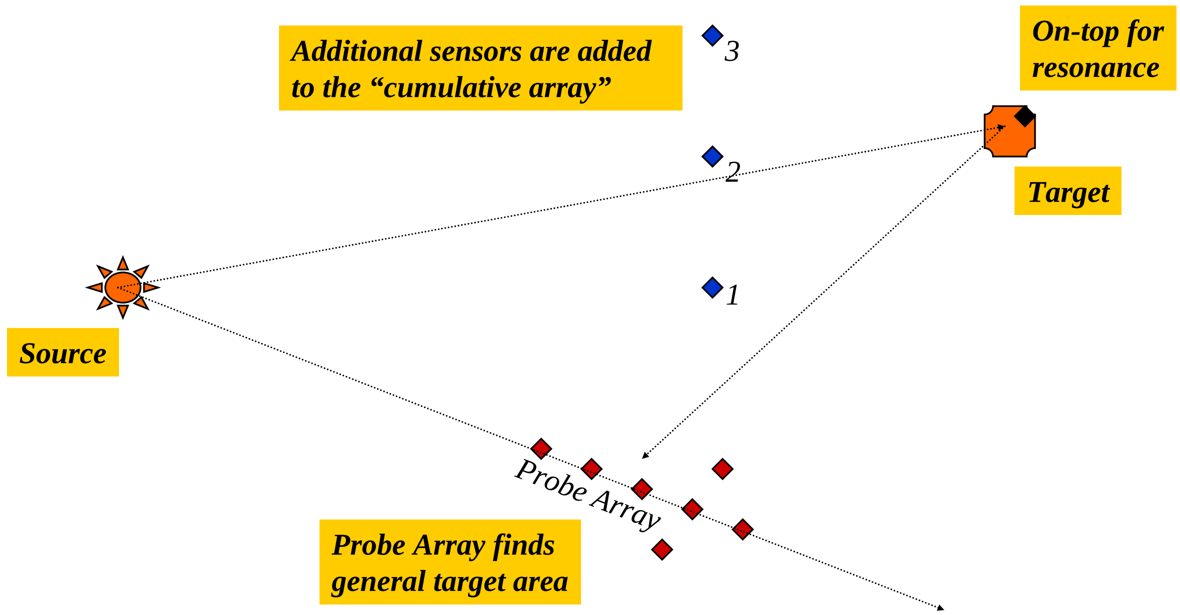
TS-50 at 1cm

VS1.6 at 5cm

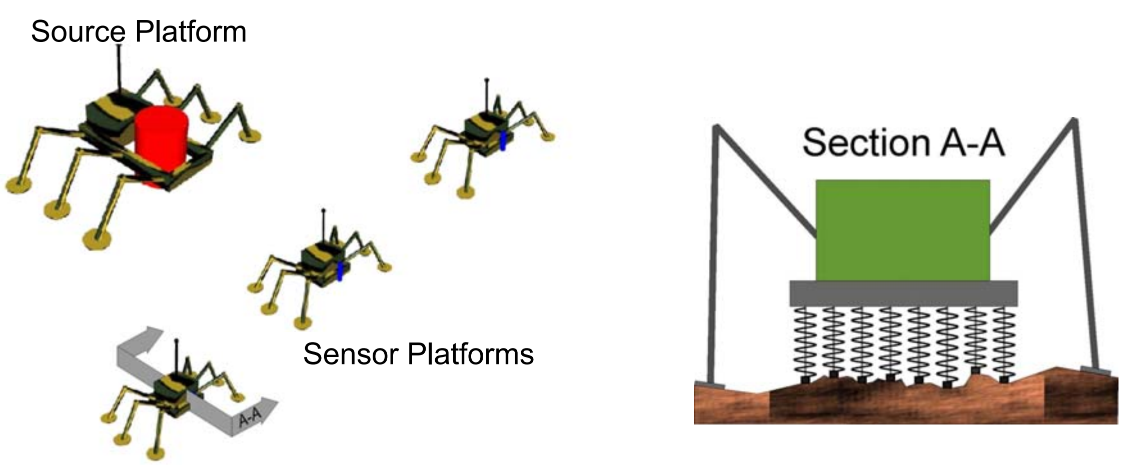
Home in on a Target

- Problem Statement:
  - Use a small number of source & receiver positions to locate targets, i.e., landmines
    - Minimize the number of measurements
- Three phases
  1. Probe phase: use a small 2-D array (rectangle or cross)
     - Find general target area by imaging with reflected waves
  2. Adaptive placement of additional sensors
     - Maneuver receiver(s) to increase resolution
     - Use "Theory of Optimal Experiments"
  3. On-top of the target
     - End-game: Verify the resonance

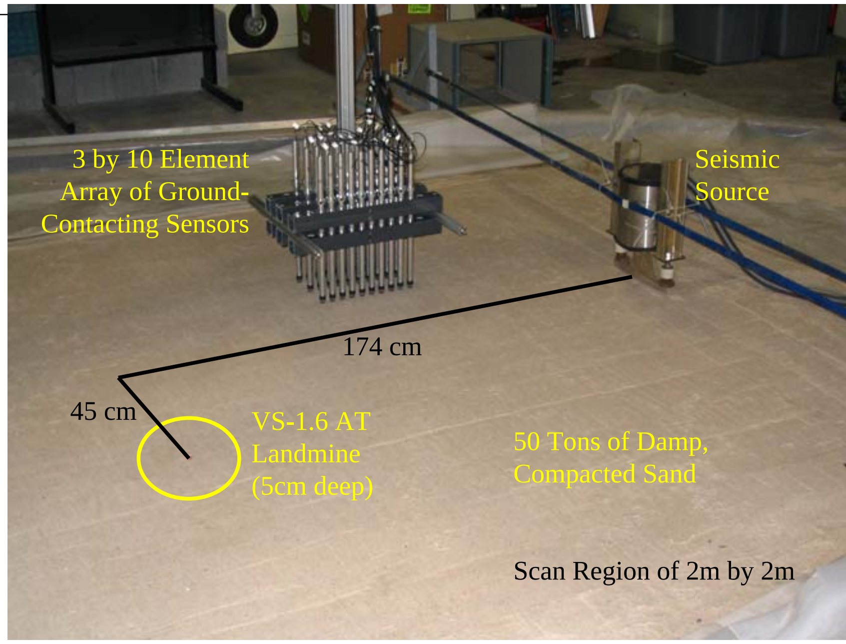
Adaptive Sensor Placement



Ground Contacting Seismic Sensor Array Deployed on a Small Robotic Platform

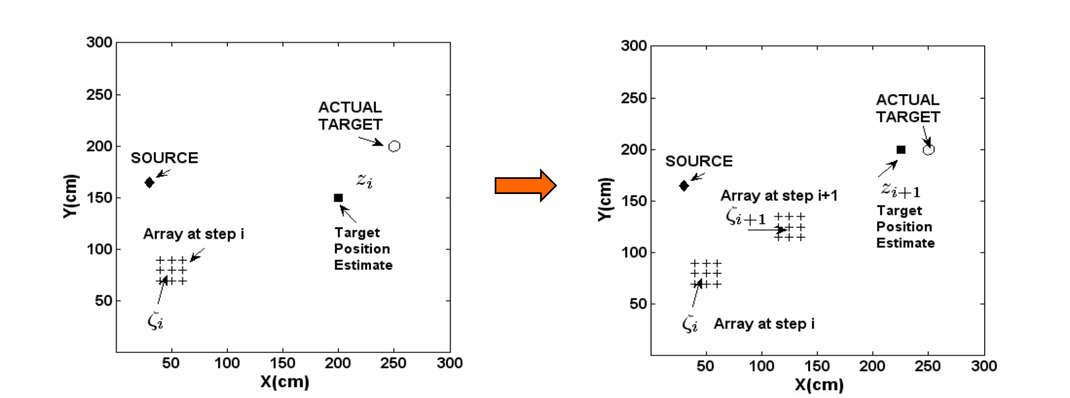


Experimental Setup of Seismic Landmine Detection Measurements



Choose Next Sensor Position

- Want to formulate an **optimal maneuvering strategy**
  - Instead of adding **One Sensor**, move the **Whole Array** to the next optimal position
  - Append the array data at the new position with previous data
  - Estimate the target location
  - Find the “best” next sensor position



Steps in Optimal Maneuver

- Wave Separation by Prony
- Imaging algorithm
  - Data Model
  - ML solution for target position estimates
  - Performance bounds for position estimates
- Next optimal Array Position
  - D-optimal Design
  - Constrained Optimization

Data Model

$$\mathbf{y}(\omega) = \mathbf{A}(\zeta, \mathbf{z}, \omega)\mathbf{s}(\omega) + \mathbf{n}(\omega)$$

$\mathbf{y}(\omega) \in \mathcal{C}^{P\times 1}$ is the array output vector

$\mathbf{n}(\omega) \in \mathcal{C}^{P\times 1}$ is a complex additive noise

$\mathbf{s}(\omega) \in \mathcal{C}^{K\times 1}$ is the signal vector

$\mathbf{A}(\zeta, \mathbf{z}, \omega)$ is the propagation (steering) vector matrix whose elements are given by 2-D Green's function

$$g(r, r', \omega) = \frac{i}{4} H_0^{(1)} \left(\frac{\omega}{v(\omega)} |r - r'| \right)$$

where ζ = array center, and $\mathbf{z}$ = target position in 2-D

Target Position Estimate

The maximum likelihood estimate can be reduced to a cost function that depends on target position only

$$J(\mathbf{z}) = \sum_{k=1}^{N} \left\| \left[I - \mathbf{a}(\omega_k) \left[\mathbf{a}(\omega_k)^H \mathbf{a}(\omega_k) \right]^{-1} \mathbf{a}^H(\omega_k) \right] \mathbf{y}(\omega_k) \right\|^2$$

The best choice for target position $(\mathbf{z})$ is:

$$\mathbf{z} = \arg\min_{\mathbf{z}} J(\mathbf{z})$$

- The Cramer-Rao lower bound (CRLB) provides a lower bound for the variances of the unbiased estimators
- CRLB requires the inverse of the Fisher information matrix

Fisher Information Matrix for Position Estimate

$$F_{i,j}(\mathbf{z},\zeta,\omega) = E_y \left\{ \frac{\partial^2 L(\mathbf{z},\zeta,\omega)}{\partial \mathbf{z}_i \partial \mathbf{z}_j} \right\}$$

$$= -\frac{2}{\sigma^2} \sum_{\omega=1}^{N} \Re \left\{ \left(\frac{\partial \mathbf{a}(\mathbf{z},\zeta,\omega)}{\partial z_i} \right)^H \frac{\partial \mathbf{a}(\mathbf{z},\zeta,\omega)}{\partial z_j} \right\}$$

where $E_y\{\cdot\}$ denotes the expected value.

- The elements of the steering vector are given in terms of the 2-D Green's function.
- The partial derivative of the steering vector is calculated with respect to target coordinates for a fixed array center.

Theory of Optimal Experiments

- The theory of optimal experiments uses various measures of the Fisher information matrix to produce decision rules
  - Determinant
  - Trace
  - Maximum value along the diagonal
- **D-optimal Design** uses the determinant

X. Liao and L. Carin, "Application of the Theory of Optimal Experiments to Adaptive Electromagnetic-Induction Sensing of Buried Targets," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 26 , no.8, pp. 961−972, August 2004.

Next Optimal Position

Let q represent the determinant of Fisher information matrix. The logarithmic increase by doing the estimation at step $i+1$ is given by:

$$\begin{aligned} \delta_q(\zeta) &= \ln q(\{\zeta_1, ..., \zeta_i, \zeta_{i+1}\}) - \ln q(\{\zeta_1, ..., \zeta_i\}) \\ &= \ln \left| I + F_{i+1} B_i^{-1} \right| \end{aligned}$$

where $|\cdot|$ stands for determinant, I is the identity matrix, $B_i = \sum_{j=1}^{i} F_j$, and F represents the Fisher information matrix.

To achieve the maximum information gain, the next optimal array center is obtained from:

$$\zeta_{i+1} = \arg\max_{\zeta} \ln \left| I + F_{i+1} B_i^{-1} \right|$$

Unique Problems for Seismic

- Target position is estimated from reflected seismic energy
  - Source is nearby
  - Need separation between forward and reverse waves
    - Array position has to be between source and targets always
  - Landmines reflections are not "omni-directional"
- Next array position has to be **Constrained**
  - "Between" source and estimated target position
  - Two ways to implement constrained optimization
    - Circle constraint, or Penalty Function

Constrained optimization for next array position

- **Limited Movement:** The next optimum sampling point is located on a circle (or half circle) of radius r from the previous array center ζ_i

$$\zeta_{i+1} = \arg\max_{\zeta} \ln \left| I + F_{i+1} B_i^{-1} \right|$$

Subject to: $(x - \zeta_{xi})^2 + (y - \zeta_{yi})^2 = r^2$

- **Penalty function:**

$$\Psi(\zeta) = \ln \left| I + F_{i+1} B_i^{-1} \right| - \mu \sqrt{(\zeta_{i+1} - \zeta_i)^T \Sigma^{-1} (\zeta_{i+1} - \zeta_i)}$$

where $\mu \geq 0$ is a scaling factor balancing the relative importance of the two terms in the cost function.

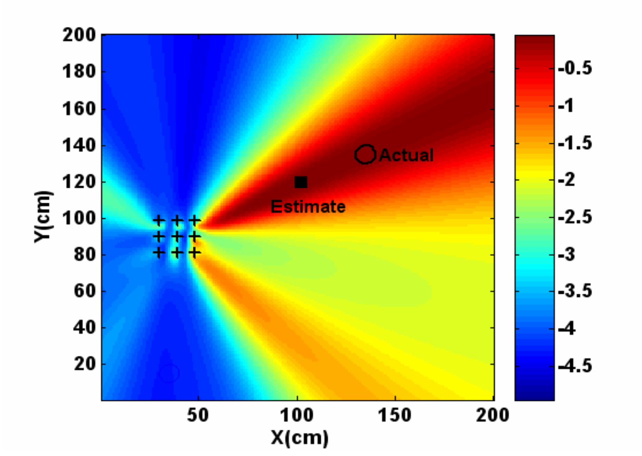
The matrix Σ is a diagonal; its elements penalize movement of the array from its previous position.

Experimental Data Setup

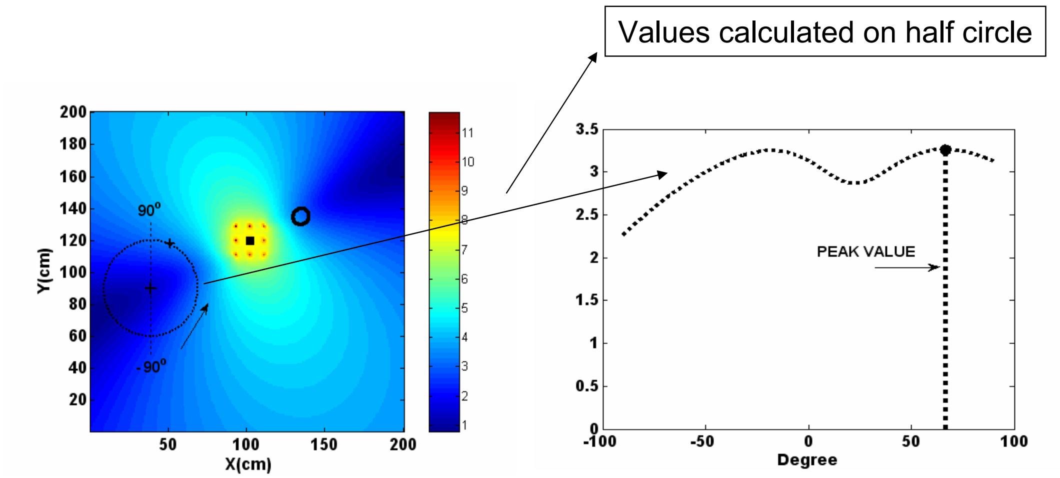
- Single TS-50 landmine is buried at 1cm
- Reflected waves are separated at each position by using a line array of 15 sensors (3cm apart)
  - Actual receiver is a single sensor (Synthetic Array)
  - Measurements are grouped into line arrays
- From each line array, three sensors at equal distance positions are chosen
  - With three line arrays, an array of nine sensors is available for 2-D imaging
- Inverse of the cost function is plotted for imaging algorithm

First Iteration

ACTUAL TARGET : (135,135) ± 5
ESTIMATE (102,120)



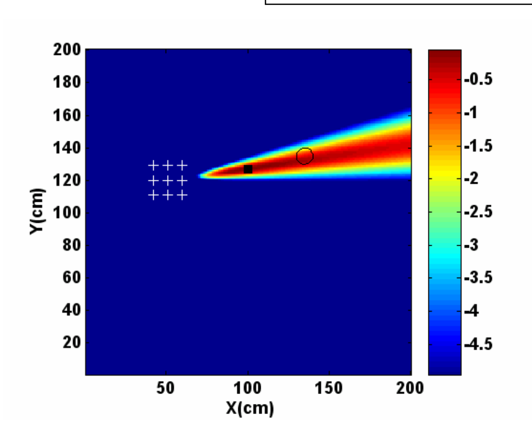
Next array position values calculated on half circle of radius=30cm



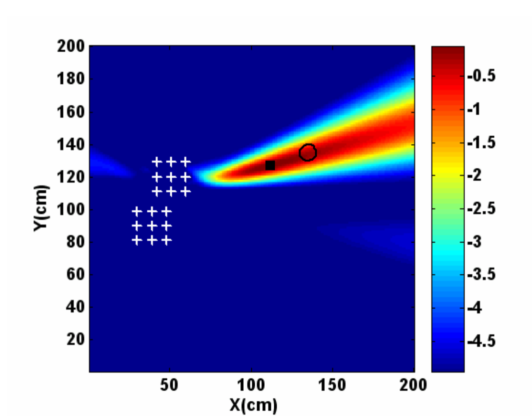
$$\zeta_{i+1} = \arg\max_{\zeta} \ln \left| I + F_{i+1} B_i^{-1} \right|$$

Second Iteration

ACTUAL TARGET : (135,135) ± 5
ESTIMATE: ALONE (100,127), CUM(112,127)

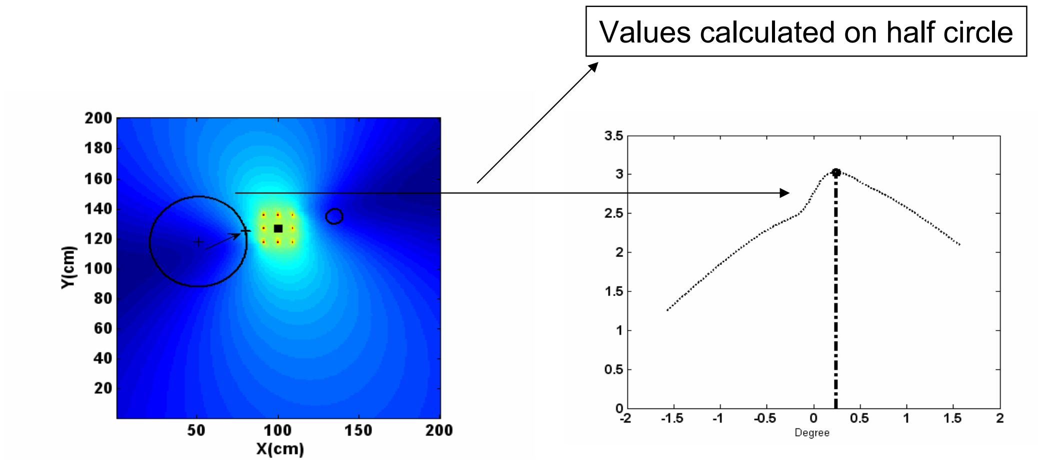


ALONE



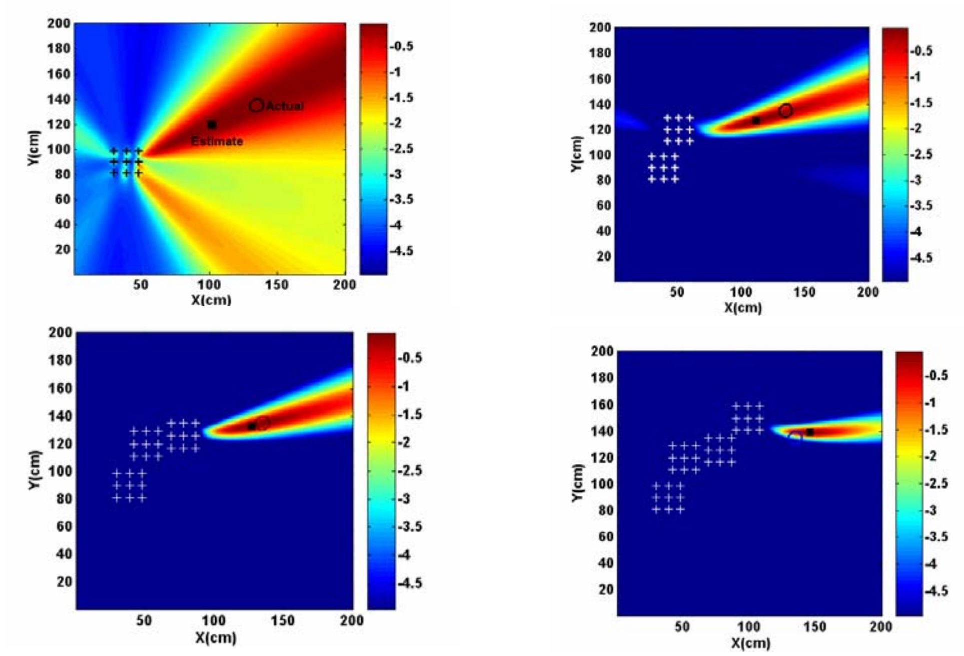
CUMULATIVE

Next Array Position



$$\zeta_{i+1} = \arg\max_{\zeta} \ln \left| I + F_{i+1} B_i^{-1} \right|$$

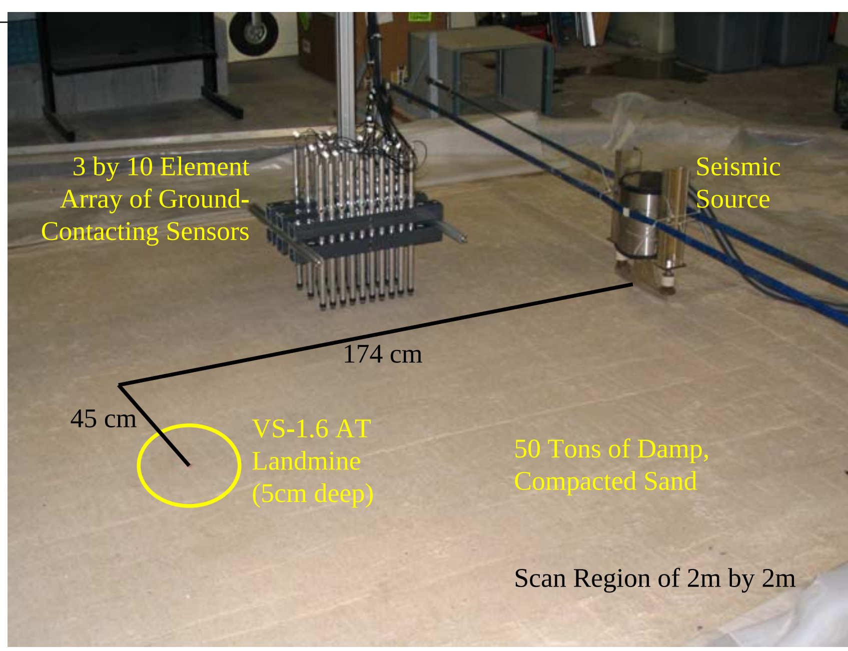
Four Iterations: Cumulative Imaging



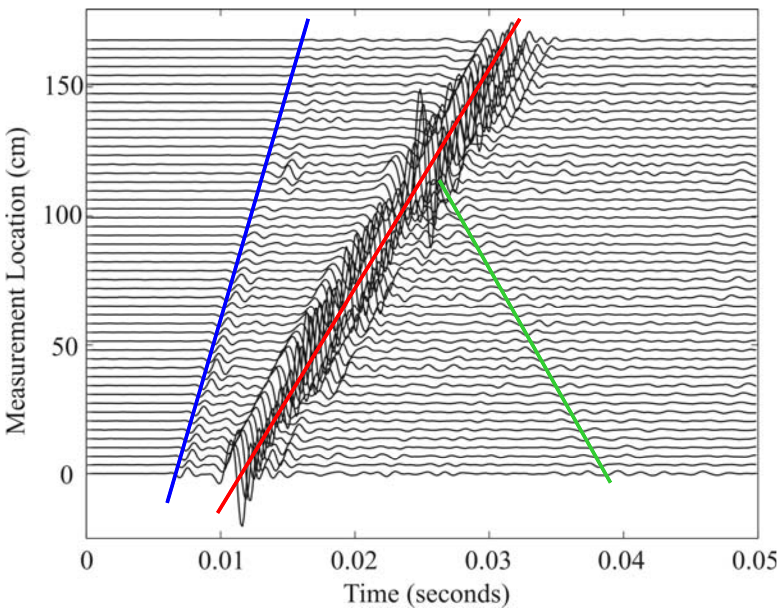
Implementation

- 2D array (3 X 10):
  - Three lines having 10 sensors each
- Sensors are ground contacting accelerometers
- LabView is used to control the movement of array and seismic source firing and interface with MATLAB
- Processing algorithms are implemented in MATLAB
- Target is a VS1.6 mine buried at 5cm

Experimental Setup of Seismic Landmine Detection Measurements



Seismic Detection of VS-1.6 AT Landmine Buried 5cm Deep in Experimental Model



Landmine (22.2 cm diameter) buried 110 cm from first measurement location in 171 cm linear scan.

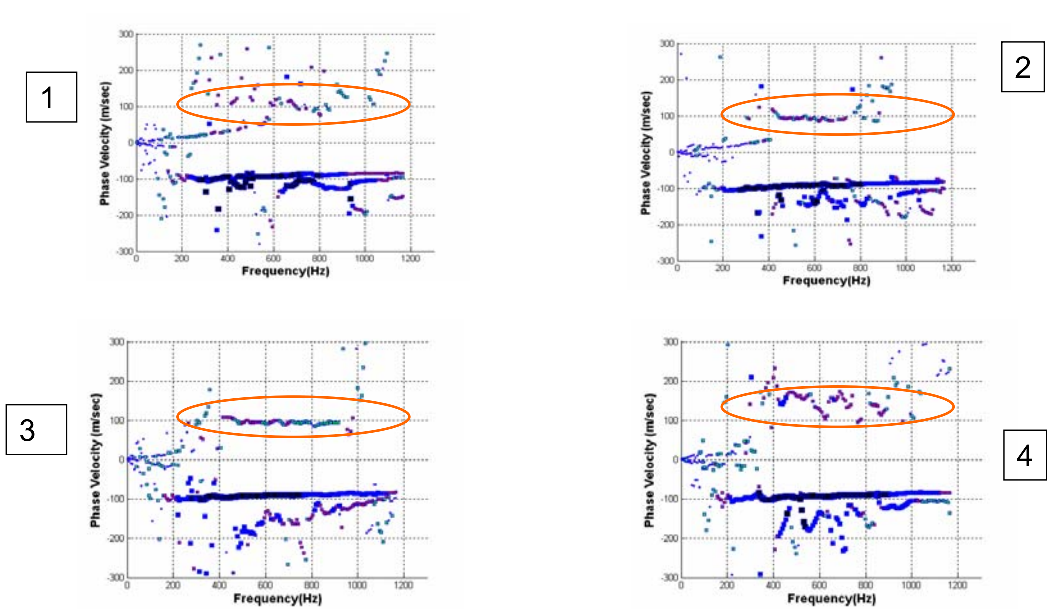
5 measurements with center line of sensor array along a line over the burial location.

Compressional, Rayleigh Surface, and Reflected Waves.

Resonance of Landmine-Soil System.

Interactions of incident waves with buried landmine evident in measured data.

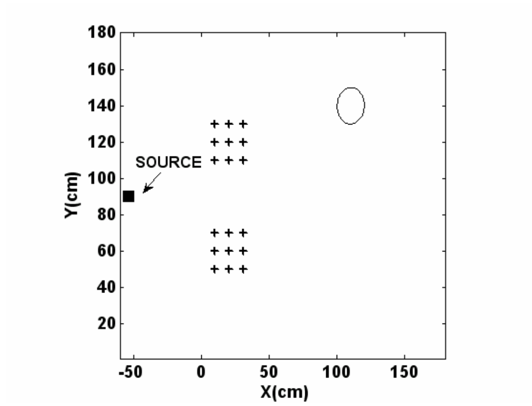
Wave Separation at each iteration: along top-most line array



Demo in Sandbox

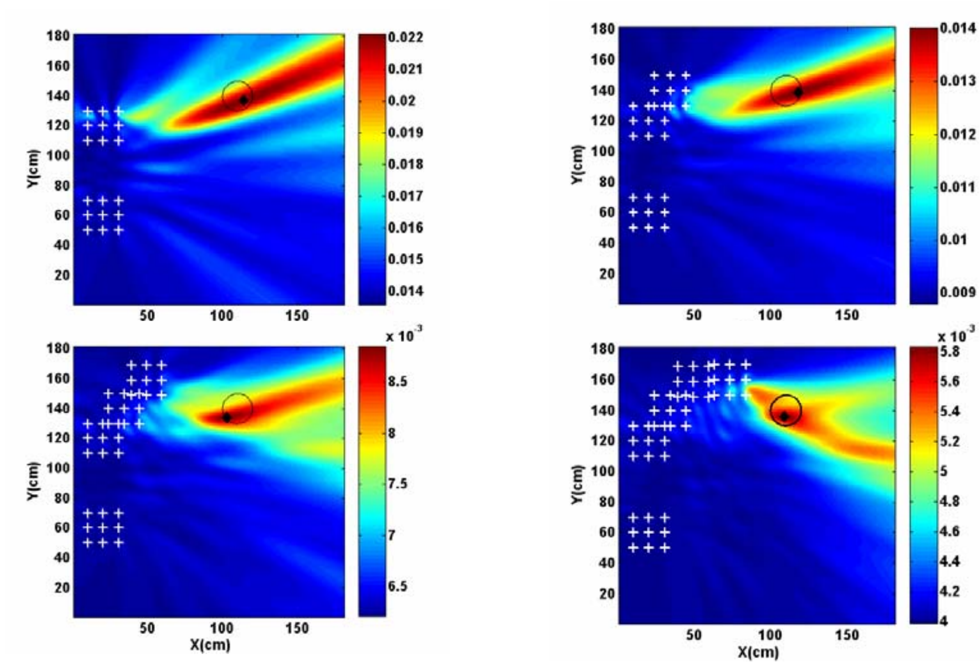
- Target is localized in four iterations
  - “Bracket” start
- Each iteration takes 10 secs. for processing and 30 secs. for data acquisition
- Total time for four iterations is 2 minutes
- Typical raster scan takes a few hours to locate the target with a large number of measurements

EXPERIMENTAL SETUP



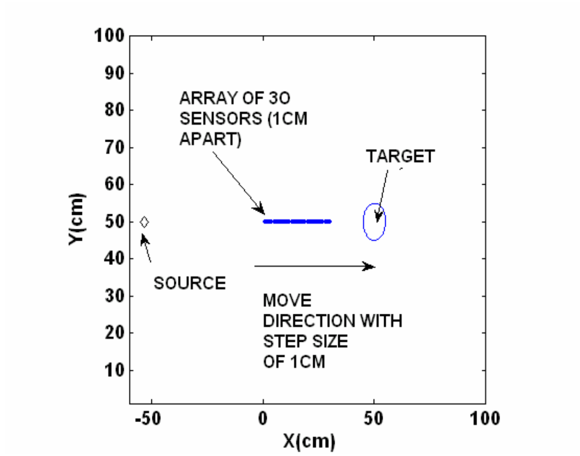
MOVIE of Experiment

Four Iterations: bracket start



End-game Detection

- We can further scan in line with the last estimated target position
- Extract the reverse wave and try to locate the exact location of resonance
- Movies shown the extracted wave with the line array of 30 sensors, which is moved toward the target with 1cm increment
- Target center is (50,50) and the starting location and position of target is shown



Movie: VS-1.6 (5cm)

Extracted reverse wave as array moves near and above the target