Imaging for distributed sensors networks

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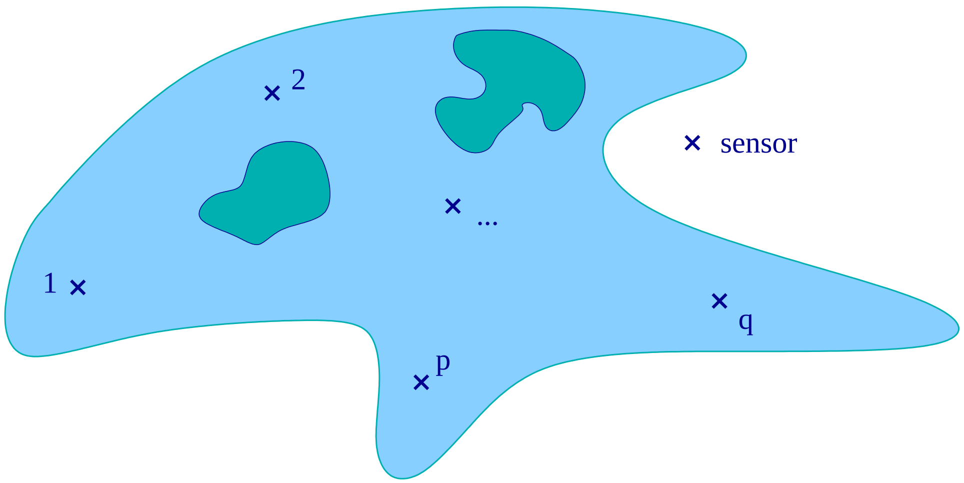
Position of the problem

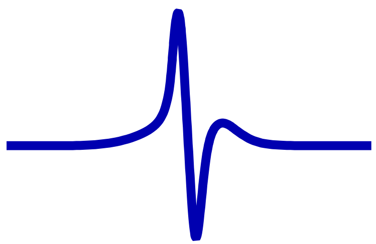
We considere a *complex medium*:



We have some knowledge of this medium in the following sense:

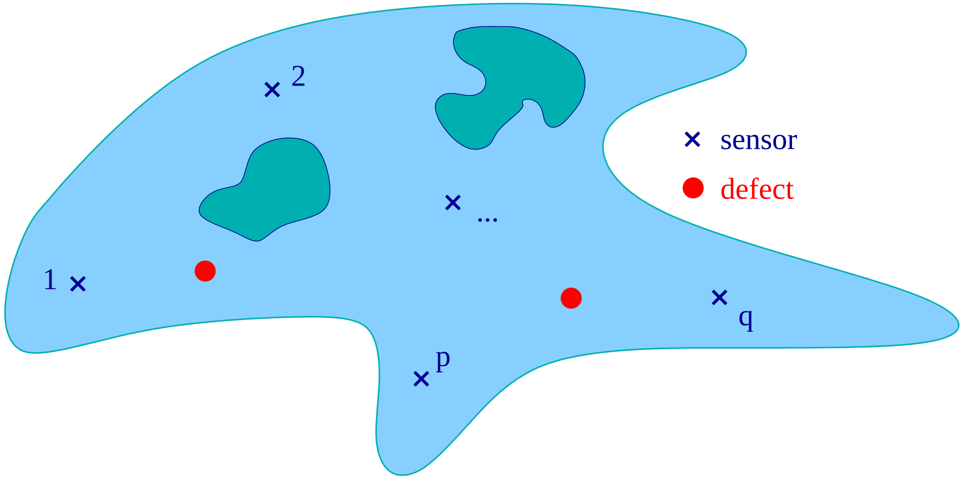
For a given set of N distributed sensors, the Response Matrix of the medium is gathered:

$$\left.\begin{array}{l}\textbf{Sensor } p \textbf{ is firing}\\ \textbf{Echos are measured at all sensors } q = 1, \ldots N\end{array}\right\} \rightarrow P^0_{pq}(t)$$

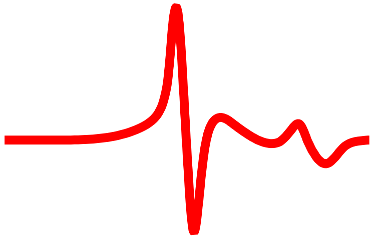


Position of the problem

Suppose now that there is an *object* in the medium:



The **Response Matrix with object** $P^d_{pq}(t)$ is gathered with *the same* set of N distributed sensors



The **difference Response Matrix** $P_{pq}(t) = P^d_{pq}(t) - P^0_{pq}(t)$ contains the echos coming from the objects only.

Some references

There is a very important litterature in array imaging:

- Migration in geophysics: Claerbout (1976), Bleistein *et al.* (2001),...
- Synthetic Aperture Radar: Skolnik (1970), Curlander (1991), Cheney (2001),...
- Music algorithm: Schmidt (1979), Devaney (2000),...
- Interferometry: Zebker (1986), Borcea *et al.* (2003),...

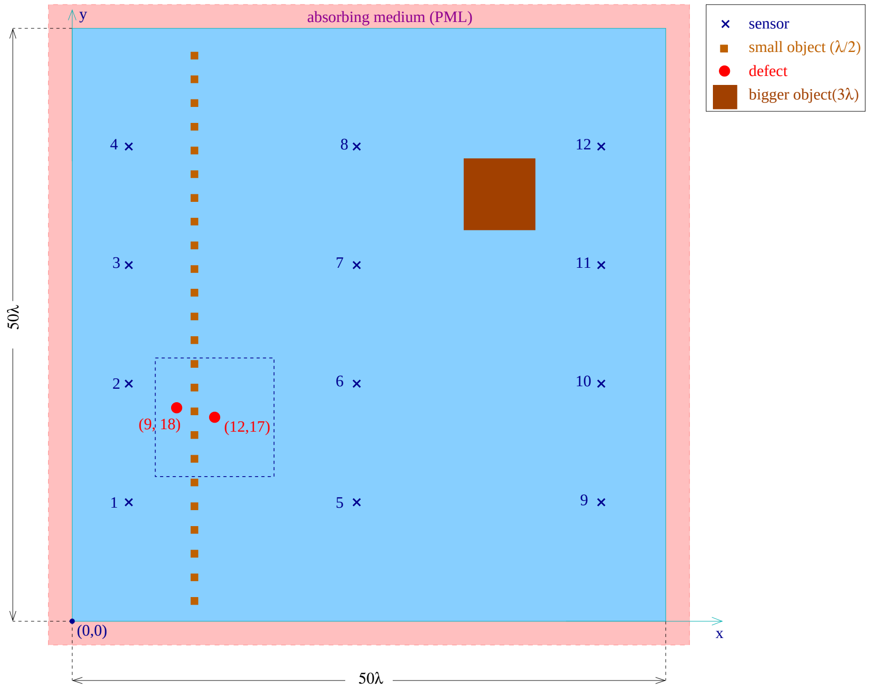
Relatively few references in distributed sensor imaging:

- Analysis of Lamb wave propagation and triangulation: Lemistre & Balagas (2001), Giurgiutiu *et al.* (2000)
- Beamforming: Chang *et al.* (2003)
- Pattern recognition with neural networks, Bishop (1995)

Outline

1. Numerical Setup
2. Kirchhoff Migration
3. Time reversal imaging
4. Separation of the defects by Singular Value Decomposition

Numerical setup



Wave equation in 2 dimension in free space. Computational domain 50λx50λ, with central wavelength $\lambda = 1$cm the reference wave speed $c_0 = 3500$m.s^{-1}

The medium contains 25 scatterers with Dirichlet Boundary conditions

12 sensors (4 rows of 3) located at $x_p, \quad p = 1, \ldots N$

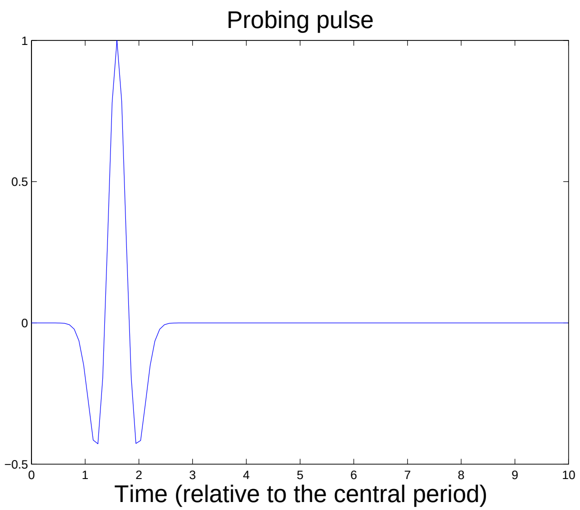
We want to image 2 pointlike defects with Dirichlet Boundary conditions

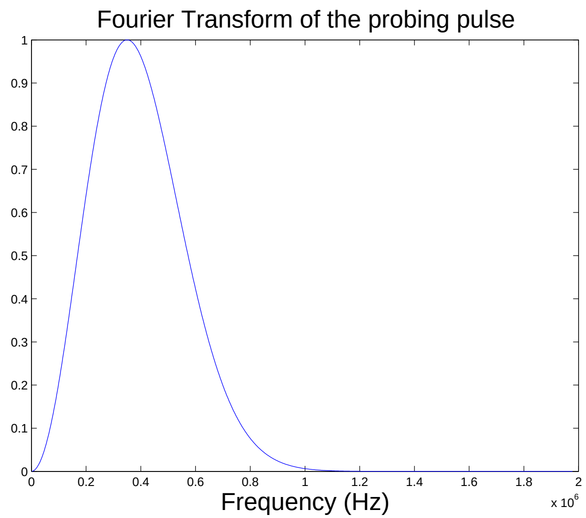
About the numerical resolution

We use a Finite Element Time Domain code to solve the 2D wave equation. The space resolution is 32 points per wavelength. Perfectly matched layered (PML) are used to simulate the propagation in free space. It takes about 1:30 hour on a workstation to produce a synthetic Response Matrix.

The probing pulse is ultrawideband: second derivative of a gaussian with central frequency $\nu = 350\text{kHz}$ and $\approx 130\%$ bandwidth:

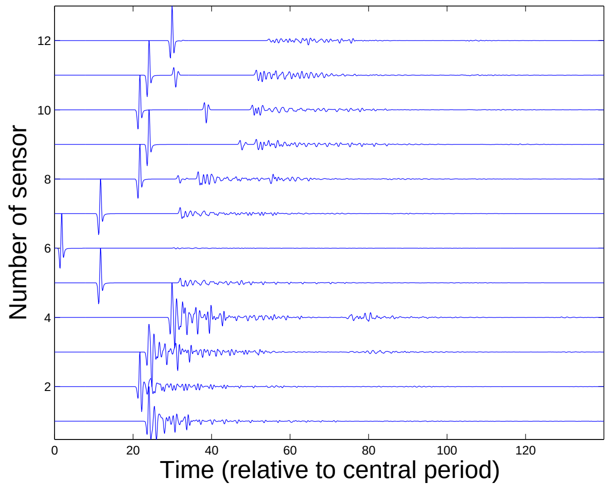
$$f(t) = (2(\sqrt{2}\pi\nu t - t_0)^2 - 1)\exp(-(\sqrt{2}\pi\nu t - t_0)^2)$$



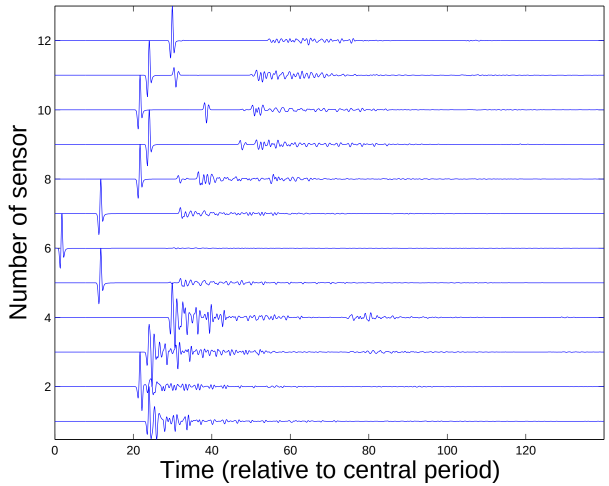


The probing pulse in the time domain (left) and its Fourier Transfom (right)

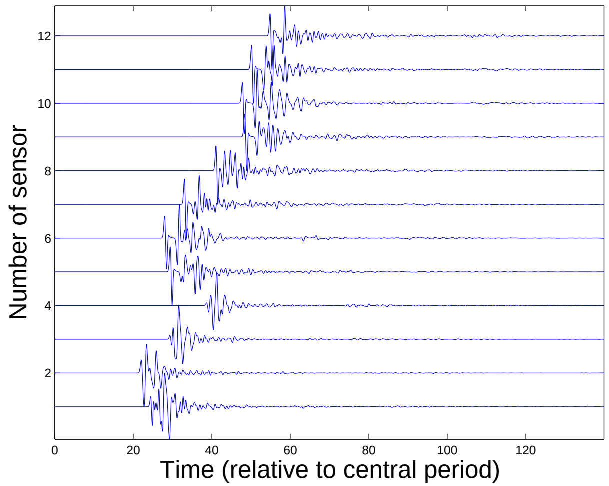
Traces



Background



With objects



Differences

Sensor # 6 is firing. Normalized traces measured at all sensors.

X-axis: time, Y-axis: number of sensor

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2. **Kirchhoff Migration**

3. **Time reversal imaging**

4. **Separation of the defects by Singular Value Decomposition**

Kirchhoff Migration

The only knowledge on the background that is used is the reference velocity c_0

With traces measured when sensor # p is firing, compute for any search point y^S:

$$I_p^{KM}(y^S) = \sum_{q=1}^{N} P_{pq}(\tau_p(y^S) + \tau_q(y^S))$$

where $\tau_q(y^S) = |x_q - y^S|/c_0$ is the travel time from sensor #q to y^S.

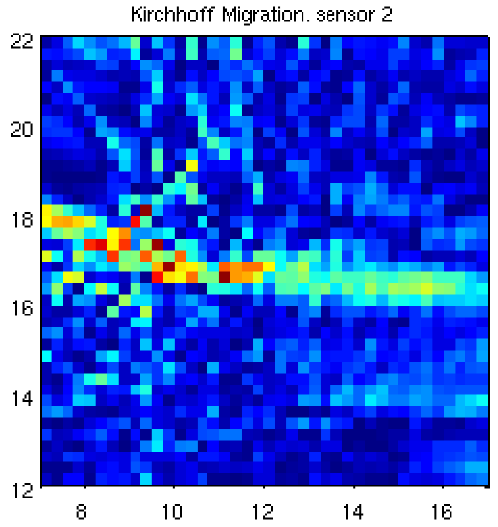
Important result: for a single target at y^* in a homogeneous medium, a large array and large bandwidth, one has $I^{KM}(y^S) \approx \delta(y^S - y^*)$. *Is there a similar result for distributed sensors ?*

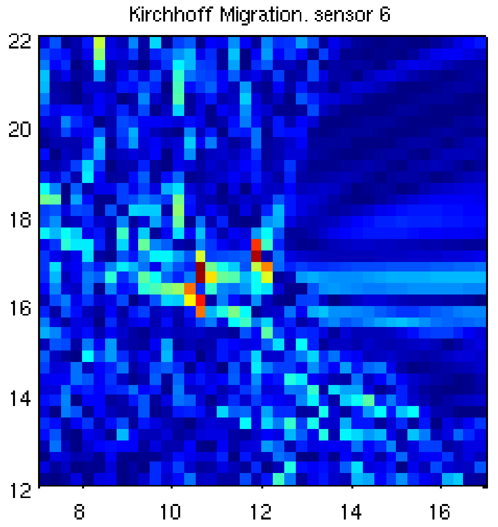
With arrays: Range resolution is controled by the bandwidth and cross range resolution is limited by the aperture of the array.

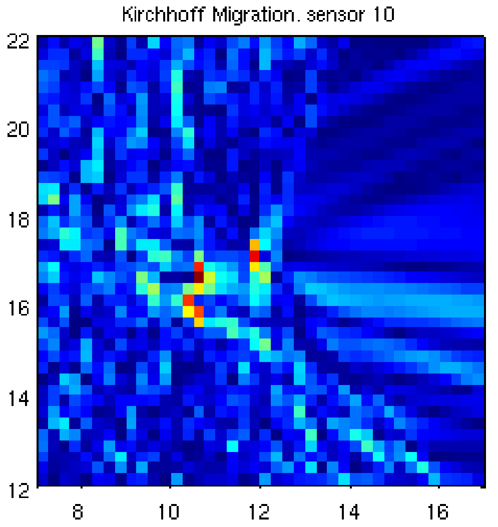
With sensors: resolution limited by the bandwidth of the probing pulse and uniformity of distribution of sensors around the target

Kirchhoff Migration Images

Images with illumination #2, #6 and #10 (second row of sensors)







Highly dependent on the illumination (large spot, only 1 target, ghosts,. . .)

Kirchhoff Migration is unstable with data that have lot of delay spread and is thus unreliable

Increasing the number of sensors improves the result

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A Time Reversal Imaging algorithm with echo-mode data

For imaging, we back propagate *numerically* the data in an idealized medium.

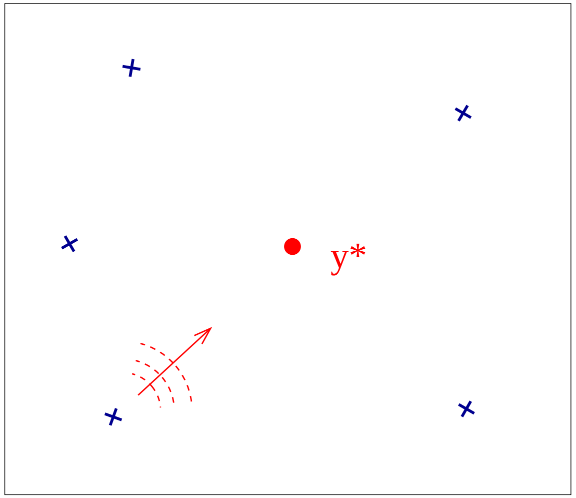
We assume here that the actual background is perfectly known, to see what best can be achieved with TR imaging.

For each illumination $\#p = 1, \ldots N$, the back propagated field $u_p(y,t)$ is solution of:

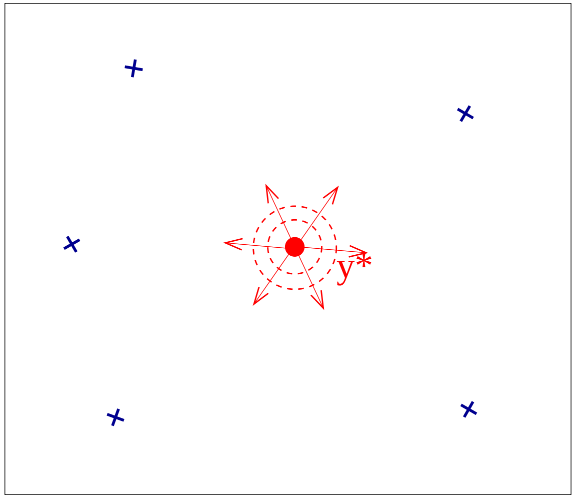
$$\begin{cases} \dfrac{\partial^2 u_p}{\partial t^2} - c_0^2 \Delta u_p = \displaystyle\sum_{q=1}^{N} \delta(x_q, y) P_{pq}(T - t) & \text{in } \mathbb{R}^2 \setminus \Omega, \\ u_p(y,t) = 0, & \text{on } \delta\Omega, \end{cases}$$

where Ω is the set of all objects in the background. T is the recording duration.

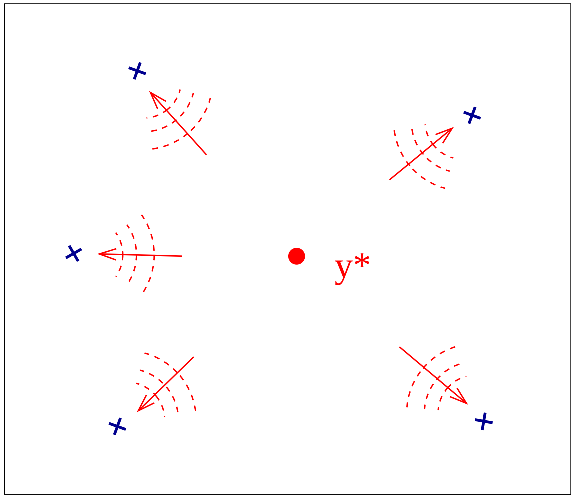
Time reversal with echo mode data : forward propagation



(a) sensor p sends a pulse at time $t = 0$

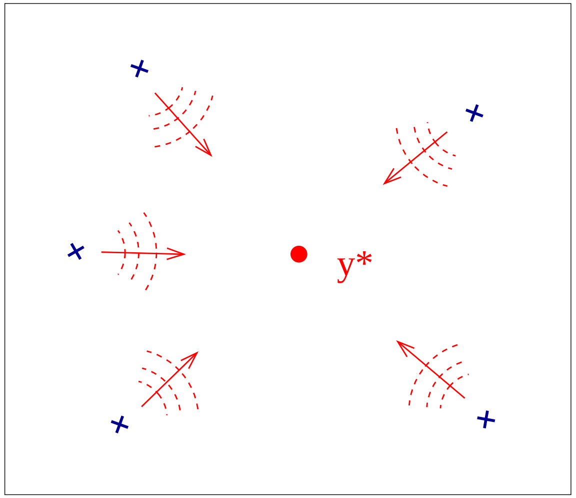


(b) target y^* starts emiting later on at unknown time $t = t^*$ as a secondary source

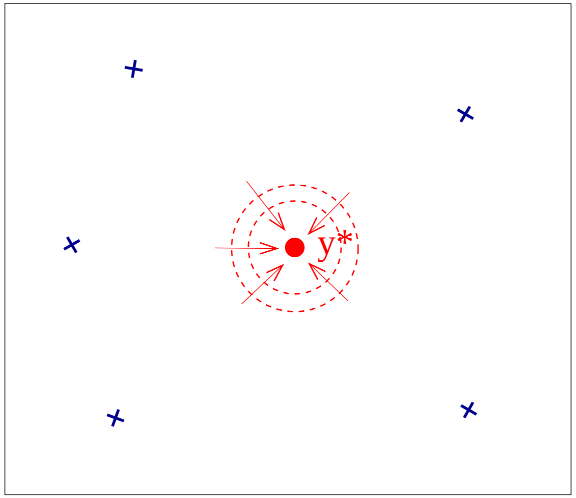


(c) The traces are recorded at all sensors

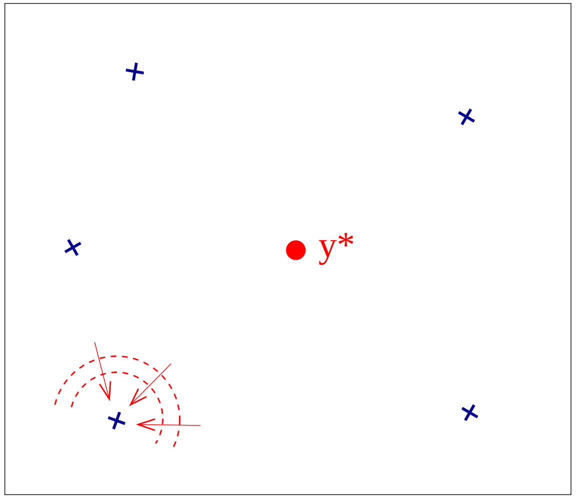
Time reversal with echo mode data : Backward propagation



(d) All sensors reemit the time reversed traces



(e) The back propagated field focuses first on the target y^* at unknown time $t = t^*$



(f) In a second time, the back propagated field focuses on the illuminating sensor p at time $t = 0$

Time reversal imaging with echo mode data

$u_p(y,t)$ focuses on the defects because they act as secondary sources, so we can make an image by taking a snapshot of the field $u_p(y,t)$ at the refocusing time

But we do *not* know at what time the refocusing occurs, because we do not know where the target is. So the refocusing time will be characterized as the minimum of a sparsity norm of the field:

– Shannon entropy

– Total Variation norm (TV norm)

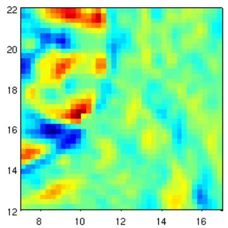
Definition of Shannon entropy

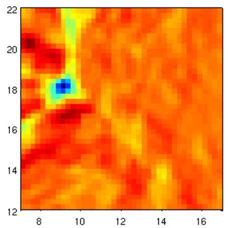
Let u_{ij} denote the space discretized field u_p at time t, containing N_p pixels. Given a number of gray levels N_c, the histogramm of frequencies of colors of the image u_{ij} is:

$$h_c = \sum_{i,j} \delta_c(u_{ij})$$

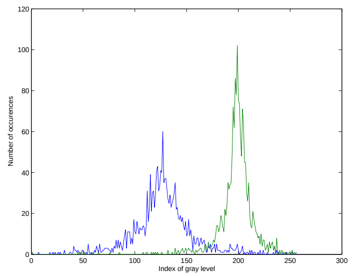
where δ_c is the counting function of the gray level c.

Example:
the histogram of





are given by



The entropy is defined by: $S(u_{ij}) = -\sum_c \frac{h_c}{N_p} \log_2 \frac{h_c}{N_p}$

The Shannon entropy is a measure of the information needed to encode a discrete image. Its penalizes images that have a high speckle level.

Definition of the TV norm

It is the norm of the space of functions of bounded variation. Roughly speaking this space contains functions that are integrable as well as their gradient.

For a discrete image u_{ij}, the TV norm is defined by:

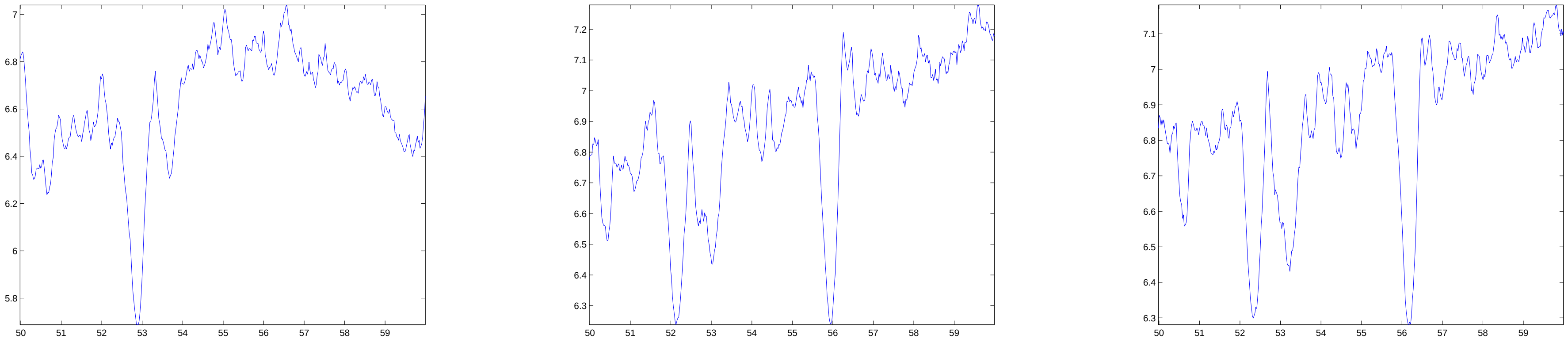
$$TV(u_{ij}) = h^2 \sum \left(|\tilde{u}_{ij}| + |\nabla_h \tilde{u}_{ij}| \right),$$

where $\tilde{u}_{ij} = u_{ij} / \max(|u_{ij}|)$ and ∇_h is a low order finite difference approximation of the gradient with space step h.

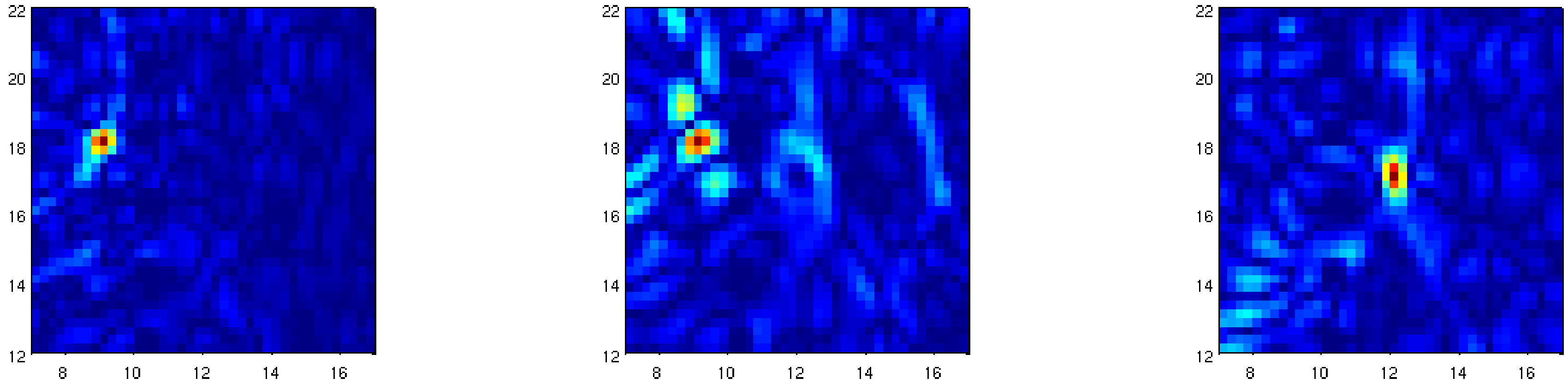
The TV norm is widely used in image processing (debluring,...) for its good ability to detect or preserve sharp contours of objects

It penalizes images with lot of oscillations

Time reversal images using optimal entropy stoping



Entropy versus time – illumination #2, #6 and #10 (second row of sensors)



Backpropagated field at minimum of entropy

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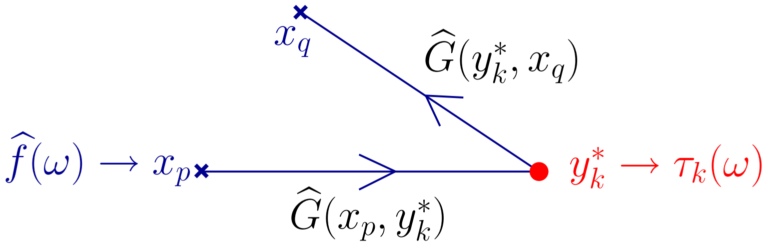
4. **Separation of the defects by Singular Value Decomposition**

A model for the Response Matrix

Assume there are M isotropic pointlike targets located at y_k^*. If we neglect multiple reflections, a model of the Response Matrix in the frequency domain is given by:

$$\widehat{P}_{pq}^{mod}(\omega) = \widehat{f}(\omega) \sum_{k=1}^{M} \tau_k(\omega) \widehat{G}(x_p, y_k^*, \omega) \widehat{G}(y_k^*, x_q, \omega),$$

$\widehat{G}(x_p, y_k^*, \omega)$ is the Transfert Function (or Green Function) from x_p to y_k^* of the actual background and $\tau_k(\omega)$ is the scattering amplitude of the k^{th} target.



Let $\widehat{g}_k(\omega)$ denote the illuminating vector comming from the target y_k^*:

$$\widehat{g}_k(\omega) = \left[\widehat{G}(y_k^*, x_1, \omega), \widehat{G}(y_k^*, x_2, \omega), \ldots, \widehat{G}(y_k^*, x_N, \omega) \right]^T$$

Then rewrite $\widehat{P}$ as:

$$\widehat{P}^{mod}(\omega) = \widehat{f}(\omega) \sum_{k=1}^{M} \tau_k(\omega) \widehat{g}_k(\omega) \widehat{g}_k^T(\omega),$$

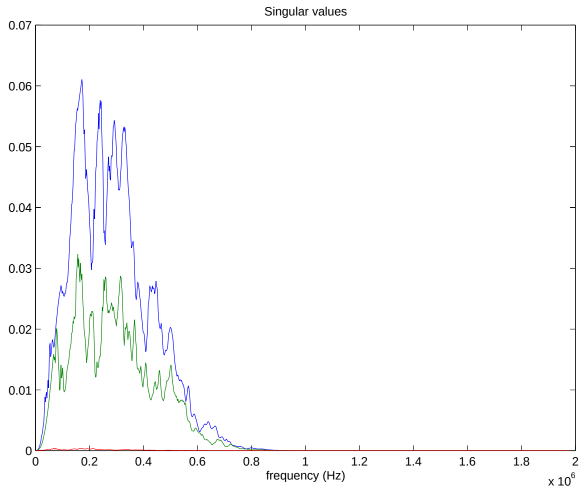
Singular Value Decomposition of the Response Matrix

$$\widehat{P}(\omega) = \sum_{k=1}^{N} \sigma_k^2(\omega) \widehat{U}_k(\omega) \widehat{V}_k^H(\omega),$$

$(\widehat{U}_k(\omega))$ is an orthonomal basis of eigenvectors of the hermitian matrix $\widehat{P}(\omega)\widehat{P}^H(\omega)$ associated to the real positive eigenvalues $\sigma_k^2(\omega)$.

By comparison with the model, the number of leading singular values gives the number of targets:

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M \quad >> \quad \sigma_{M+1} \approx \ldots \approx \sigma_N \approx 0.$$



The first 3 singular values of the response matrix $\widehat{P}(\omega)$ versus frequency

Separation of the targets using the SVD

We say that the targets are well resolved if:

$$\widehat{g}_k^H(\omega)\widehat{g}_l(\omega) \approx 0, \quad \forall k \neq l.$$

then the leading singular vectors are proportional to the illuminating vectors:

$$\widehat{U}_k(\omega) \approx e^{i\phi_k(\omega)} \frac{\widehat{g}_k(\omega)}{|\widehat{g}_k(\omega)|},$$

So in principle, the SVD transforms an echo-mode problem into an active source problem.

But the singular vector $U_k(t)$ looks incoherent in the time domain, because of the arbitrary phase $\phi_k(\omega)$.

Separation of the targets using the SVD

To get rid of the arbitrary phase, we construct N different versions of the eigenvectors by projecting each column of the Response Matrix into it:

$$\widehat{U}_k^{(p)}(\omega) = [\widehat{U}_k^H(\omega)\,\widehat{P}^{(p)}(\omega)]\,\widehat{U}_k(\omega), \qquad p = 1, \ldots N,$$

One has:

$$\widehat{U}_k^{(p)}(\omega) \approx \widehat{f}(\omega)\widehat{\tau}_k(\omega)\widehat{G}(x_p, y_k^*, \omega)g_k(\omega)$$

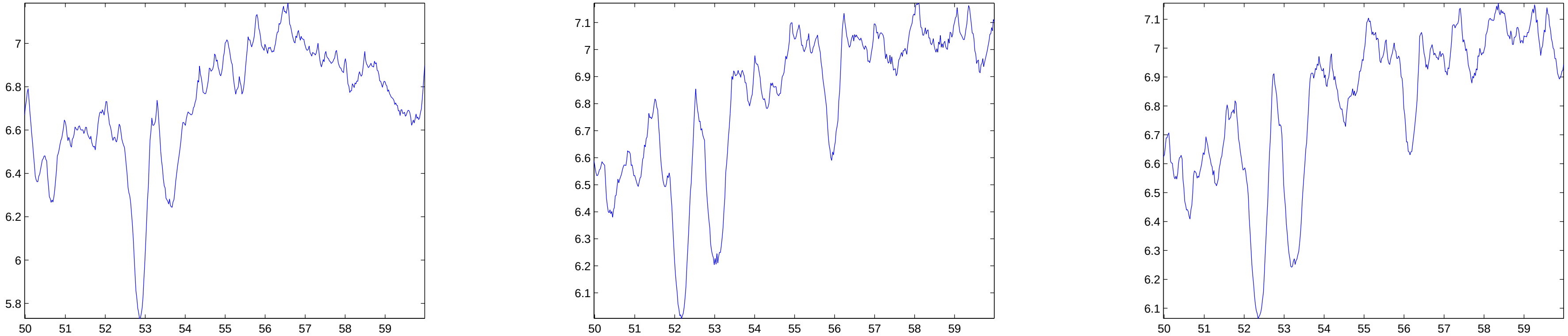
So $\widehat{U}_k^{(p)}(\omega)$ corresponds to the column obtained when the k^{th} defect is alone.

The time domain singular vector is then given by:

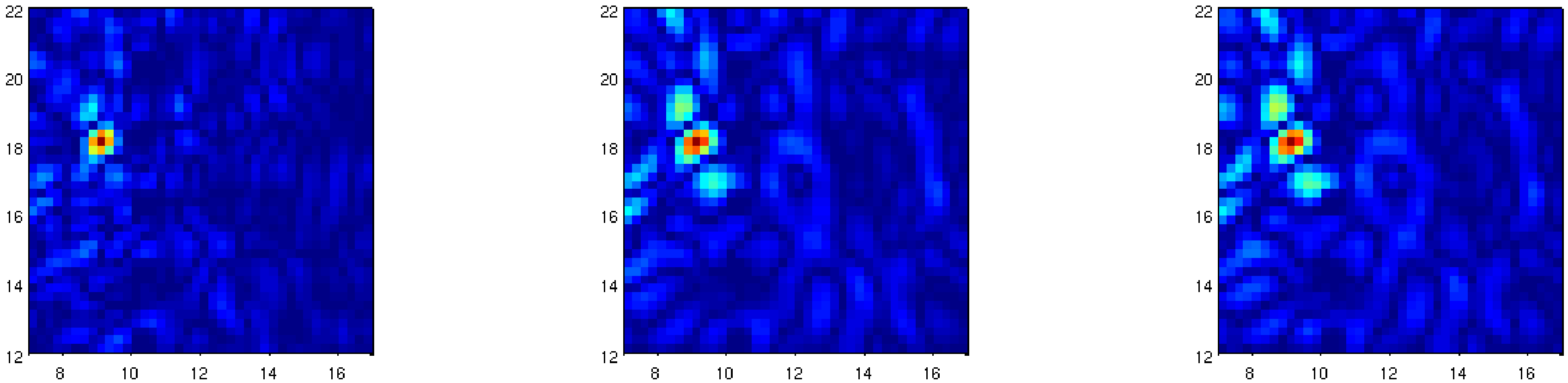
$$U_k^{(p)}(t) = \frac{1}{2\pi}\int e^{-i\omega t}\sigma(\omega)\widehat{U}_k^{(p)}(\omega)d\omega$$

It is possible to apply any of the above imaging algorithms to the Response Matrix projected on each singular vectors.

Optimal entropy stoping - Projection on the first singular vector

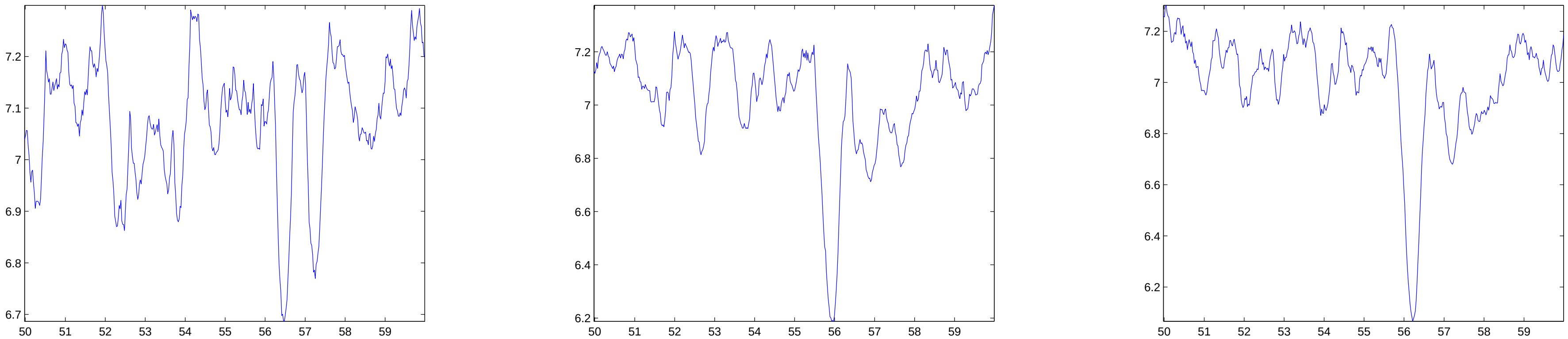


Entropy versus time – illumination #2, #6 and #10 (second row of sensors)



Backpropagated field at minimum of entropy

Optimal entropy stoping - Projection on the second singular vector



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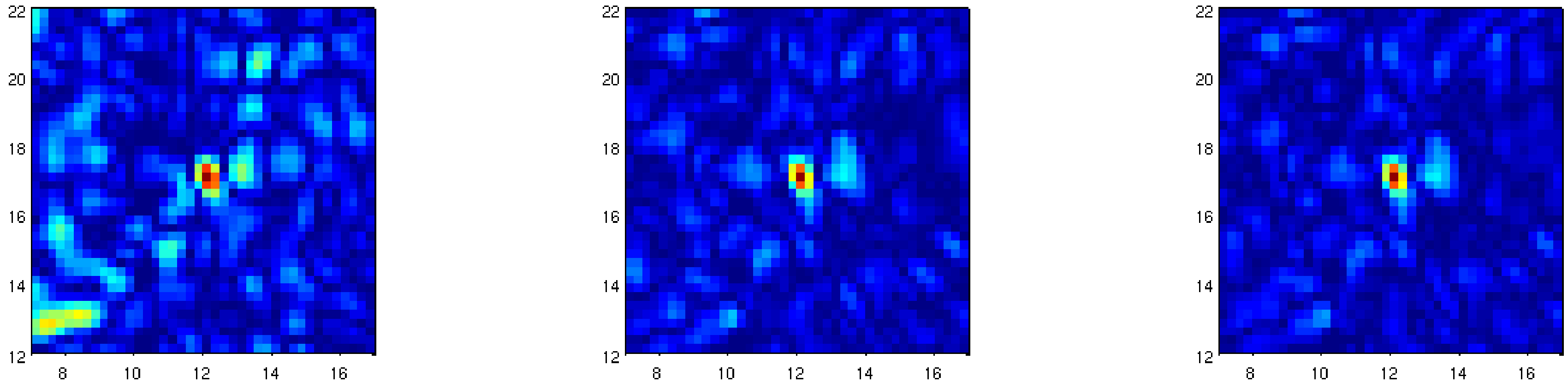
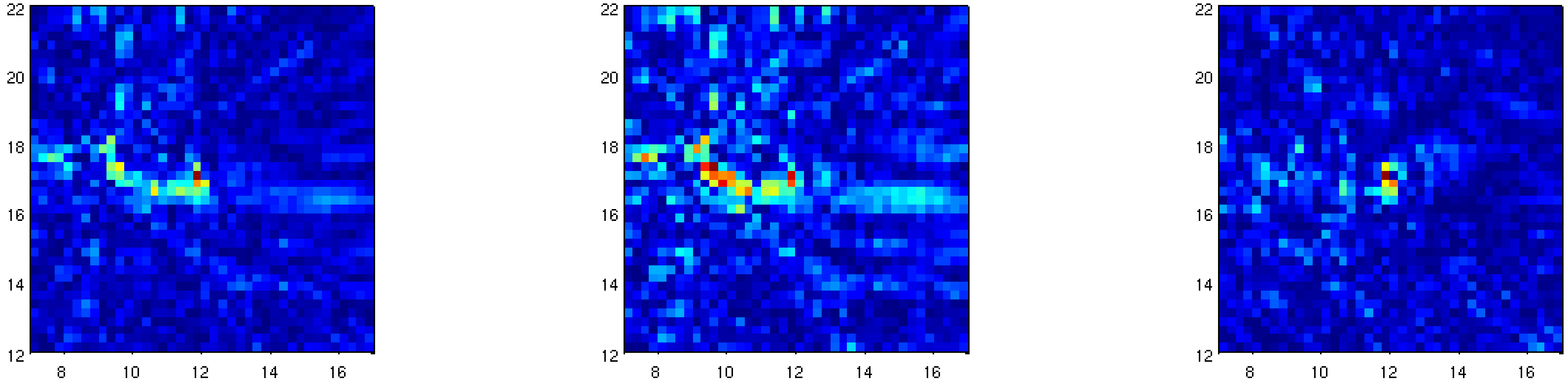


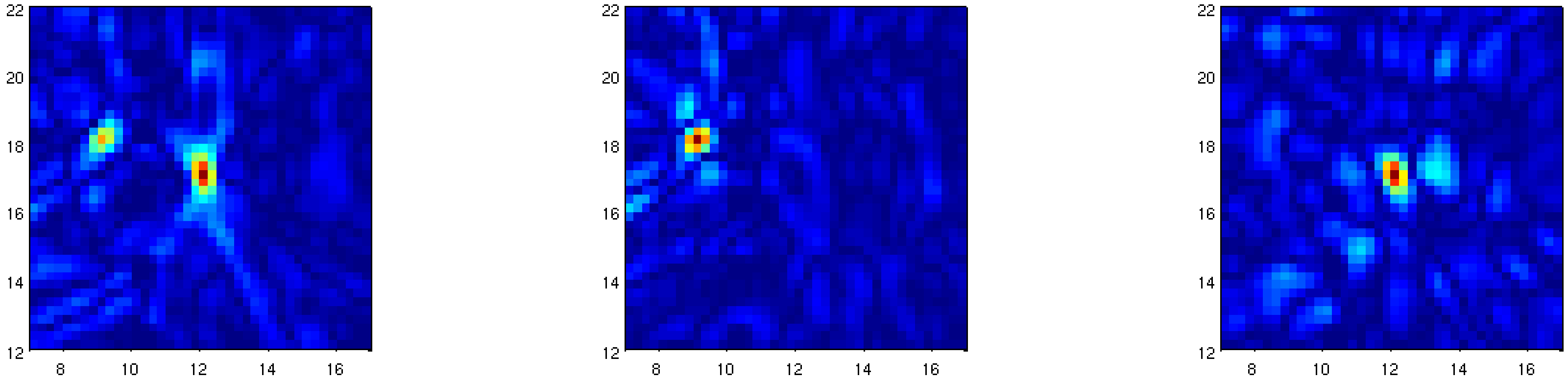
Backpropagated field at minimum of entropy

We can improve the Signal to Noise Ratio by cumulating the images.

KM



TR



Raw traces *1st sing. vec.* *2nd sing. vec.*

Conclusion

Kirchhoff Migration is not reliable in a complex background.

Echo-Mode Time reversal imaging with optimal compensation using Shannon entropy or TV norm proves to work very well:

- It is stable
- Gives a reliable image of each defect

Work in progress:

- Investigate how other algorithms (Migration, MUSIC,...) perform when (part of) the actual background is known (broadband *vs.* narrowband),
- Use a coherent interferometry algorithm — he traces have a lot of delay spread, which is a case when intereferometry is expected to perform well
- Address the question of optimal illumination
- Address the question of optimal allocation of sensors