Fast Multipole Method for Scattering from an Arbitrary PEC Target Above or Buried in a Lossy Half Space

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Abstract—The fast multipole method (FMM) was originally developed for perfect electric conductors (PECs) in free space, through exploitation of spectral properties of the free-space Green's function. In the work reported here, the FMM is modified, for scattering from an arbitrary three-dimensional (3-D) PEC target above or buried in a lossy half space. The “near” terms in the FMM are handled via the original method-of-moments (MoM) analysis, wherein the half-space Green's function is evaluated efficiently and rigorously through application of the method of complex images. The “far” FMM interactions, which employ a clustering of expansion and testing functions, utilize an approximation to the Green's function dyadic via real image sources and far-field reflection dyadics. The half-space FMM algorithm is validated through comparison with results computed via a rigorous MoM analysis. Further, a detailed comparison is performed on the memory and computational requirements of the MoM and FMM algorithms for a target in the vicinity of a half-space interface.

Index Terms—Method of moments, scattering.

I. INTRODUCTION

RESEARCHERS have directed significant attention toward development of numerical scattering models, with the method of moments (MoM) representing a prominent example [1]–[7]. For an MoM problem with N unknowns, the memory requirements are of order $O(N^2)$, while the computational complexity depends on whether a direct (LU-decomposition) or iterative (conjugate gradient [2], [8]) matrix solver is applied, the former requiring $O(N^3)$ and the latter $P \cdot O(N^2)$ computations, with P the number of iterations required to achieve convergence. To reduce computational complexity, several researchers have undertaken the development of modified integral-equation solvers [9]–[17], in an effort to achieve better computational efficiency and reduced memory requirements. For example, the adaptive integral method (AIM) [12] exploits properties of the FFT, and researchers have demonstrated memory requirements and computational complexity of $O(N^{1.5})$ and $O(N^{1.5} \log N)$, respectively [12]. Further, there has been significant interest in the fast multipole method (FMM). The simplest two-level (single-stage) FMM [13], [14] has $O(N^{1.5})$ computational complexity and memory

requirements, while a multilevel extension [15]–[17] reduces these numbers to $O(N \log N)$.

While the AIM and FMM algorithms represent promising tools, they have heretofore been applied primarily to free-space scattering [12], [13]–[17], two-dimensional (2-D) analysis in layered media [18], or quasi-planar three-dimensional (3-D) problems in circuit and antenna design [19]. There are many applications for which the free-space model or a quasi-planar approximation are inappropriate. For example, there has been significant interest in radar sensing of buried targets, such as mines [20], unexploded ordnance (UXO) [21], drums [22], etc. In this paper we concentrate on the two-level FMM, with the goal of extending it to the case of a target in the vicinity of a lossy half space.

The FMM clusters the MoM basis elements into M groups [13], [14], and the interactions between distant groups (“far” interactions) are handled by exploiting features of the FMM spectral propagator [13], [14], discussed further below. “Near” interactions are handled in a manner analogous to a conventional MoM analysis [3], [5], [7]. For the calculation of these “near” interactions, we rigorously evaluate the dyadic half-space Green's function [6], [25] via the complex-image technique [26]–[29].

We have developed an approximate but highly accurate method for evaluating the “far” FMM interactions within the context of a half-space problem. In particular, if the target is entirely above or below the half space, the MoM and FMM analyses only require the Green's function for source and observer in the same region. Under this circumstance, each component of the dyadic Green's function can in general be represented in terms of a direct and reflected term [30], [31]. The direct term is handled as in the conventional free-space FMM [13], [14]. Further, the reflected term corresponds to radiation from the expansion function, reflection at the half-space interface, and subsequent propagation to the testing function. If the expansion and testing functions are relatively distant, than this term can be evaluated approximately asymptotically [30], [31]. The reflected term can therefore be represented in terms of an image in real space, with an appropriate amplitude, dictated by the polarization-dependent reflection coefficient.

II. THEORY

A. Integral Equation and MoM Formulation

We utilize an electric-field integral equation (EFIE) formulation [2], [3], [5], which restricts its utility somewhat, due to

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Fig. 1. Geometry for source and observation group in 3-D FMM and generalization to a half-space environment using real images to account for “far” interface interactions.

spurious internal resonances (for closed targets) [2]. However, it has been demonstrated previously [23] that when a closed target is embedded in a lossy medium (e.g., soil), these internal resonances are damped significantly and they therefore present less of a problem computationally. We note, however, that the general FMM formulation can also be applied to a combined field integral equation (CFIE) [2], [15], which is not beset by internal-resonance problems.

The half-space EFIE for a perfectly conducting 3-D target situated entirely in layer $i=1$ (e.g., air) or layer $i=2$ (e.g., soil) of a half-space environment (Fig. 1) can be expressed as [6]

$$\hat{\mathbf{n}} \times \mathbf{E}^{\text{inc}}(\mathbf{x}) = \hat{\mathbf{n}} \times j\omega\mu_i \iint_{S'} \left[\mathbf{I} + \frac{\nabla\nabla}{k_i^2} \right] \cdot \mathbf{G}_{Aii}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}') dS' \quad (1a)$$

or, alternatively, in the mixed potential form of the electric field integral equation (MPIE) [6]

$$\hat{\mathbf{n}} \times \mathbf{E}^{\text{inc}}(\mathbf{x}) = \hat{\mathbf{n}} \times \left[j\omega\mu_i \iint_{S'} \mathbf{K}_{Aii}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}') dS' - \frac{\nabla}{j\omega\epsilon_i} \iint_{S'} K_{\phi e}^{ii}(\mathbf{x}, \mathbf{x}') \nabla' \cdot \mathbf{J}(\mathbf{x}') dS' \right] \quad (1b)$$

where $\mathbf{x}$ is on and $\hat{\mathbf{n}}$ is a unit vector normal to the target surface; $\epsilon_i = \epsilon'_i - j\sigma/\omega$, μ_i and k_i represent (in general complex) permittivity, permeability, and wavenumber of the medium in which the target resides (region i); ω is the angular frequency ($\exp(j\omega t)$ time dependence is assumed and suppressed); and $\mathbf{I}$ represents the unit dyadic. Details on the dyadic half-space Green's functions $\mathbf{G}_{Aii}$ and $\mathbf{K}_{Aii}$, and on the scalar Green's function $K_{\phi e}^{ii}$, have been given by Michalski and Zheng [6], where we use their formulation C. The EFIE in (1) is valid for a general layered medium [6], but here we only consider a half-space, for simplicity (see also explanation in Section II-C).

Like in a conventional MoM solution, the unknown surface current $\mathbf{J}$ is expanded by a set of basis (or expansion) functions $\mathbf{b}_{n'}(\mathbf{x}')$, where we use the well-known RWG vector basis functions introduced in [3], defined on adjacent triangles representing the target's surface. Testing the EFIE (1) with a set of weighting (or testing) functions $\mathbf{w}_n(\mathbf{x})$ tangential to the scatterer surface results in N linear equations $[\mathbf{Z}] \cdot \mathbf{I} = \mathbf{V}$ for the unknown current coefficients $\mathbf{I}_{n'}$, where the $N \times 1$ driving vector $\mathbf{V}$ represents the tangential electric field of the incident wave, tested on the target surface. The elements of the $N \times N$ impedance matrix $[\mathbf{Z}]$ are given by

$$Z_{nn'} = j\omega\mu_i \iint_S \iint_{S'} \mathbf{w}_n(\mathbf{x}) \cdot \left[\mathbf{I} + \frac{\nabla\nabla}{k_i^2} \right] \cdot \mathbf{G}_{Aii}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{b}_{n'}(\mathbf{x}') dS' dS \quad (2a)$$

or

$$Z_{nn'} = j\omega\mu_i \iint_S \mathbf{w}_n(\mathbf{x}) \cdot \iint_{S'} \mathbf{K}_{Aii}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{b}_{n'}(\mathbf{x}') dS' dS - \frac{1}{j\omega\epsilon_i} \iint_S \mathbf{w}_n(\mathbf{x}) \cdot \nabla \iint_{S'} \mathbf{K}_{\phi e}^{ii}(\mathbf{x}, \mathbf{x}') \nabla' \cdot \mathbf{b}_{n'}(\mathbf{x}') dS' dS. \quad (2b)$$

Using the RWG triangular basis [3] and Galerkin testing, the last term in (2b) can be simplified using integration by parts [3].

In the FMM analysis [13], [14], we divide the computation of (expansion function)-(testing function) interactions into “near” and “far” terms. The “far” interactions are best handled via the form in (1a) and (2a), respectively, while for “near” interactions (MoM part of the FMM [13], [14]) (1b) and (2b) are utilized, rigorously accounting for the dyadic half-space Green's function [6]. The half-space Green's function is evaluated using the method of complex images [26]–[29], thereby avoiding direct numerical evaluation of Sommerfeld integrals [25]. Impedance matrix elements representing these “near” interactions are stored in a sparse matrix denoted $[\mathbf{Z}^{\text{near}}]$. The method of complex images for calculating the space-domain Green's function, as well as explicit equations for the impedance matrix elements, including the extraction of self-term singularities, can be found in the literature [26]–[29], [3]–[5].

“Far” interactions, applicable to the FMM framework [13], [14], are best described using (2a) and realizing that [6], [30]

$$\begin{aligned} \mathbf{G}_{Aii}(\mathbf{x}, \mathbf{x}') &= \mathbf{I}g_i(\mathbf{x}, \mathbf{x}') + \Delta\mathbf{G}_{Aii}(\mathbf{x}, \mathbf{x}') \\ &= \mathbf{I} \frac{e^{-jk_i|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|} + \Delta\mathbf{G}_{Aii}(\mathbf{x}, \mathbf{x}') \end{aligned} \quad (3)$$

where the new dyadic $\Delta\mathbf{G}_{Aii}$ is defined as the difference between $\mathbf{G}_{Aii}$ and $\mathbf{I}g_i$ (i.e., Δ is not an operator). The dyadic $\mathbf{G}_{Aii}$ is therefore split into a part which accounts for direct interactions (as in the free-space case, but using a homogeneous background medium with in general complex wave number k_i) and a dyadic accounting for interactions with the interface. Consequently, the matrix elements in (2a) for “far” interactions are also split as $Z_{nn'}^{\text{far}} = Z_{nn'}^{\text{far}, \text{hom}} + \Delta Z_{nn'}^{\text{far}}$, with $Z_{nn'} = Z_{nn'}^{\text{near}} + Z_{nn'}^{\text{far}}$. As discussed above, $Z_{nn'}^{\text{far}}$ is computed using the form

in (2a), with $Z_{nn'}^{far,hom}$ employing the homogeneous-medium Green's function g_i and $\Delta Z_{nn'}^{far}$ employing ΔG_{Aii} [see (3)].

B. Free-Space Fast Multipole Method

As is well known from [13], [14], in the 3-D free-space FMM the scalar Green's function $g_i(\mathbf{x}, \mathbf{x}')$ is expanded using the addition theorem [32] resulting in the plane wave decomposition

$$\begin{aligned} g_i(\mathbf{x}, \mathbf{x}') &= \frac{-jk_i}{(4\pi)^2} \iint_{4\pi} e^{-jk_i \hat{\mathbf{k}} \cdot (\mathbf{x} - \mathbf{x}_m)} T_L \\ &\quad \cdot (k_i X_{m'm}, \hat{\mathbf{k}} \cdot \hat{\mathbf{X}}_{m'm}) e^{+jk_i \hat{\mathbf{k}} \cdot (\mathbf{x}' - \mathbf{x}_{m'})} d^2 \hat{\mathbf{k}} \quad (4a) \\ T_L(k_i X_{m'm}, \hat{\mathbf{k}} \cdot \hat{\mathbf{X}}_{m'm}) &= \sum_{l=0}^L (-j)^l (2l+1) h_l^{(2)}(k_i X_{m'm}) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{X}}_{m'm}) \quad (4b) \end{aligned}$$

where the distance vector $\mathbf{R} = |\mathbf{x} - \mathbf{x}'|$ has been subdivided into a vector from the source point $\mathbf{x}'$ to the center $\mathbf{x}_{m'}$ of a corresponding source group, a vector from an observation group center $\mathbf{x}_m$ to the observation point $\mathbf{x}$, and a vector $\mathbf{X}_{m'm}$ connecting the group centers (Fig. 1), i.e., $\mathbf{R} = \mathbf{x} - \mathbf{x}' = (\mathbf{x} - \mathbf{x}_m) + (\mathbf{x}_m - \mathbf{x}_{m'} - (\mathbf{x}' - \mathbf{x}_m)) = (\mathbf{x} - \mathbf{x}_m) + \mathbf{X}_{m'm} - (\mathbf{x}' - \mathbf{x}_{m'})$. Therefore, the target surface is first partitioned into groups $m = 1, \dots, M$, each of which has an average number of $A_m \approx N/M$ basis/weighting functions. Inside group m the elements are labeled as $\alpha = 1, \dots, A_m$ [13], [14]. The group information $n(m)$, is stored in matrix form. As was shown in [13] and [14], for the two-level FMM described here, the optimal number of groups (to minimize CPU and RAM) is given by $M \propto \sqrt{N}$. Approximate equations for the number of terms L needed in (4b) are given in [13], [14], [17] for the case of a lossless environment with $k_i = k_0$ (vacuum). For a lossy background medium (e.g., soil) more terms are required, as discussed further in Section II-C. Therefore, in our implementation L is determined adaptively, depending on the FMM grouping scheme and the (complex) wavenumber. For the accurate numerical integration over the solid angle 4π in (4a), a quadrature rule with $K = 2L^2$ points is applied [13], [14]; although more efficient quadrature rules are available [33].

C. Extension to Half-Space Fast Multipole Method

In addition to the evaluation of the dyadic Green's function in the calculation of $[\mathbf{Z}^{near}]$ (here using the method of complex images [26]–[29]), it is essential to include the effects of the “far” interface interactions represented by $[\Delta \mathbf{Z}^{far}]$ [see (3)]. Using complex images, each term ΔG_{Aii}^{xx} , ΔG_{Aii}^{zz} and Q_{Aii} in the space-domain half-space dyadic Green's function [6]

$$\begin{aligned} \Delta G_{Aii}(\mathbf{x}, \mathbf{x}') &= (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) \Delta G_{Aii}^{xx} + \left(\hat{\mathbf{z}}\hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{z}}\hat{\mathbf{y}} \frac{\partial}{\partial y} \right) Q_{Aii} \\ &\quad + \hat{\mathbf{z}}\hat{\mathbf{z}} \Delta G_{Aii}^{zz} \quad (5) \end{aligned}$$

can be expressed in terms of a sum of free-space Green's functions [26]–[29] with image sources located in complex space.

The plane wave expansion (4) used in the 3-D free-space FMM remains valid for this more general case of a complex

Fig. 2. Number of terms needed in (4) as a function of $|kd|$ for a relative error smaller than 10^{-3} , depending on the ratio k''/k' for a complex wavenumber $k = k' - jk''$. The empirical formula $L = kd + \gamma \ln(\pi + kd)$ is also plotted, with kd replaced by $|kd|$, is also plotted for comparison. Here only the direct interaction (i.e., free-space, but lossy background) is considered.

wavenumber k_i and complex source points, although the convergence is slower. This is especially true for the complex distance vectors. While complex wavenumbers $k = k_i$ can be handled by slightly increasing the number L of terms in (4) (at least for moderate losses, see Fig. 2), we have not yet found a solution to avoid the generally large upper limit L needed for general complex source points. To further illustrate this, we consider the addition theorem [13], [14], [32]

$$\begin{aligned} \frac{e^{-jkR}}{R} &= \frac{e^{-jk\sqrt{(\mathbf{X}+\mathbf{d}) \cdot (\mathbf{X}+\mathbf{d})}}}{\sqrt{(\mathbf{X}+\mathbf{d}) \cdot (\mathbf{X}+\mathbf{d})}} \\ &= -jk \sum_{l=0}^{L=\infty} (-1)^l (2l+1) j_l(kd) h_l^{(2)}(kX) P_l(\hat{\mathbf{d}} \cdot \hat{\mathbf{X}}) \quad (6) \end{aligned}$$

on which the plane wave expansion (4) is based. Theoretically, the expansion in (4) converges as long as $|d| < |X|$ is satisfied, wherein the distance R , wavenumber k , $X = \sqrt{\mathbf{X} \cdot \mathbf{X}}$, and $d = \sqrt{\mathbf{d} \cdot \mathbf{d}}$ can be arbitrarily complex [32]. As discussed in [13], for real arguments (i.e., real wavenumber k , real source point $\mathbf{x}'$, real group center $\mathbf{x}_{m'}$, real vectors $\mathbf{d}$ and $\mathbf{X}$), the spherical Bessel function $j_l(kd)$ and spherical Hankel function of second kind $h_l^{(2)}(kX)$ are approximately of constant magnitude for $l < kd$ and $l < kX$, respectively. The l th-order Legendre polynomial is strictly bounded according to $|P_l(\hat{\mathbf{d}} \cdot \hat{\mathbf{X}})| \leq 1$. For $l > kd$ and $l > kX$, respectively, $j_l(kd)$ decays quickly and $h_l^{(2)}(kX)$ grows quickly. Consequently, L must be chosen large enough such that convergence is achieved, but it must not greatly exceed kX , or the sum becomes numerically unstable. For the free-space FMM with real arguments (e.g., $k = k_0$) it has been shown in [13], [14], [17] that, assuming a maximum cluster diameter d , approximately $L = kd + \gamma \ln(\pi + kd)$ terms in the series expansion are sufficient; with $\gamma \approx 1$ for a relative error less than $10^{-1} = 10\%$, $\gamma \approx 3$ for 10^{-3} and $\gamma \approx 5$ for 10^{-6} . If two clusters are situated such that $kX < L$, the clusters are too close for the representation in (4a) and (4b) to be valid [13] and therefore the interactions between those clusters

Fig. 3. Number of terms needed in (4) as a function of $|kd| = k_0 d$ for a relative error smaller than 10^{-3} for a scatterer above soil with relative permittivity $\epsilon_r = 6 - j1$. The source and observation group centers are both located at $z_{m'} = z_m = 0.4d$ and the separation was set to $k_0 X_{m'm} = L$. In addition to the number of terms required for the direct interaction, the results are given for a real image when considering all complex images.

have to be treated in the sparse part $[Z^{\text{near}}]$ of the original MoM impedance matrix (see Section II-A).

For image sources in complex space at least one of the vectors $\mathbf{X}$ or $\mathbf{d}$ is complex (depending on the specific choice of the image group center $\mathbf{x}_{m'}$) and the condition $L = kd + \gamma \ln(\pi + kd)$ is no longer valid, which is also the case for real image sources but a lossy background [i.e., complex wavenumber $k = k' - jk''$]. Fig. 2 illustrates the increase in L for a complex wavenumber, depending on the relative losses of the surrounding medium (represented by the ratio k''/k'), where here we truncate the series expansion for a relative error less than 10^{-3} , and the distance between the group centers is set to $|kX| = L$. For moderate losses, the required number of terms in the addition theorem only increases slightly. In contrast, the (in general) significant increase of L for complex images is illustrated in Fig. 3, where we plot the required number of terms L for a target above soil of relative permittivity $\epsilon_r = 6 - j1$. The source and observation group centers are both located at $z_{m'} = z_m = 0.4d$, separated by a distance which satisfies $|k_1 X_{m'm}| = k_0 X_{m'm} = L$. Remembering that $K = 2L^2$ terms are utilized in the numerical evaluation of (4a), the computational and memory requirements increase dramatically when using complex images.

Therefore, we searched for an alternative (though approximate) formulation. Considering the fact that the dyadic half-space Green's function is rigorously accounted for in the “near” interaction matrix $[Z^{\text{near}}]$, using a far-field reflection coefficient approximation [30], [31] to include “far” interface interactions seems appropriate (as demonstrated in Section III). Under this assumption it is sufficient to utilize a single real image to approximately account for the reflection at the interface [30], [31]. There is only one set of real image clusters, as opposed to dyadic-component-dependent multiple complex-image locations. This property reduces the overall CPU and memory requirement. In fact, the storage and CPU are approximately twice that of a free-space FMM (see Section III), due to the extra set of image clusters. The location of the (approximate) real image

at $(x', y', -z')$ (assuming the interface at $z = 0$) and the corresponding real image source group are easily determined (Fig. 1). Generalizing the free-space FMM algorithm is now straightforward. In the preprocessing stage [13], [14] we only have to include additional calculations of the translation operators between all image group and observation group centers and the spectral Fourier transforms of the image expansion functions.

To account for the polarization-dependent far-field reflection of a wave traveling from the source point $\mathbf{x}'$ to the observation point $\mathbf{x}$, we introduce the asymptotic expression for the reflection dyadic [30], [31]

$$\mathbf{r}^v(\mathbf{x}, \mathbf{x}') = \hat{\mathbf{h}} \hat{\mathbf{h}} R_{TE}^v(\vartheta) + [\mathbf{I} - \hat{\mathbf{h}} \hat{\mathbf{h}}] R_{TM}^v(\vartheta)$$

with

$$\hat{\mathbf{h}} = \frac{\hat{\mathbf{z}} \times (\mathbf{x} - \mathbf{x}')}{|\hat{\mathbf{z}} \times (\mathbf{x} - \mathbf{x}')|} \quad (7)$$

where ϑ represents the angle between the ray-optical reflection path and the unit vector $\hat{\mathbf{z}}$ normal to the interface (Fig. 1). The material properties and the angle ϑ determine the Fresnel reflection coefficients $R_{TE, TM}^v = R_{TE, TM}^{12}$ for a target situated in layer $i=1$ (e.g., air) or $R_{TE, TM}^v = R_{TE, TM}^{21}$ for a target in layer $i=2$ (e.g., soil), respectively. The polarization is denoted as TE_z (electric field perpendicular to the plane of incidence, i.e., parallel to the unit vector $\hat{\mathbf{h}}$) and TM_z (magnetic field perpendicular to the plane of incidence), respectively.

III. RESULTS

In all examples below, the scattered fields are computed via the FMM algorithm developed here, as well as with a rigorous MoM solution [7], wherein the dyadic Green's function is characterized rigorously via the method of complex images [26]–[29]. The accuracy, computational complexity, and memory requirements of the FMM are calibrated through comparison with the MoM algorithm. The FMM model discussed here is easily modified to the free-space case [13], [14]; we therefore consider the first target (triangular) in free space as well, such that the accuracy of the free-space FMM (vis-a-vis the MoM) can be assessed relative to the accuracy of the half-space FMM. This is particularly important because the free-space FMM makes no approximation to the Green's function beyond those in (4), while the half-space FMM code developed here makes an additional approximation (real images) to the half-space Green's function (for the “far” FMM interactions).

Concerning the FMM, recall that $M \propto \sqrt{N}$ clusters are optimal [13], [14], where N is the number of unknowns. For simplicity, we have used $M = 2^q$ FMM clusters, where q is an integer. While this approach is slightly suboptimal for the two-level FMM applied here, a similar clustering is employed in the multilevel FMM [15]–[17]. As mentioned in Section II-B, the number of terms L in (4b) is determined adaptively, where, for the problems considered here, L is typically in the range 6–15 (when truncating the series for a relative error of 10% or less, as suggested in [14], [17]). With regard to the MoM, the number of complex-image terms is also determined adaptively, and for the problems considered here 5–15 complex images are

Fig. 4. Radar cross section of a 90°-trihedral of edge length 100 cm (hypotenuse = 141 cm) with one plate parallel to the xy -plane (see inset). The incident plane wave direction is given by $\phi_i = 20^\circ$ (relative to trihedral symmetry plane) and $\theta_i = 60^\circ$ (30° from grazing). The figure shows the bistatic RCS (here we only plot VV and HH) for $\theta_s = 50^\circ$ and a frequency of 1 GHz (number of unknowns $N = 5202$). (a) Trihedral 30 cm above lossy Yuma soil with 10% water content [20], [22]. (b) Trihedral in free-space.

typical. We reiterate that the “near” FMM terms employ a complex-image-computed Green’s function. Finally, in the MoM solution, we have used a direct (LU-decomposition) solver of order $O(N^3)$ complexity, which could be reduced to $P \cdot O(N^2)$ if a conjugate-gradient solver [2], [8] was applied.

In all the computations, we have assumed linearly polarized plane-wave incidence. The scattered fields are computed in the far zone, via an asymptotic approximation to the half-space dyadic Green’s function [30], [34].

A. Trihedral Fiducial Target

We first consider the trihedral target inset in Fig. 4(a), situated above a lossy half space. The soil is characterized as Yuma soil, with 10% water content [20], [22]. As depicted in Fig. 4(a), the trihedral is composed of three 45°-45°-90° triangles, each with a 1.41 m hypotenuse. The trihedral is placed 30 cm above the soil, with one of its triangles parallel to the air-ground interface. Such that the code is tested thoroughly, we first present bistatic results for incidence angles $\theta_i = 60^\circ$ and $\phi_i = 20^\circ$, and the

scattered fields are observed for $\theta_s = 50^\circ$ and $-180^\circ \leq \phi_s \leq 180^\circ$. The results in Fig. 4(a) present the bistatic scattered fields for VV and HH polarization, for both the FMM and MoM solutions, at a frequency of 1 GHz. At this frequency the hypotenuse is $4.7\lambda_0$, where λ_0 is the free-space wavelength. We see that the agreement between the FMM and MoM results is excellent. Similar agreement was found for the cross-polarized (VH and HV) fields, but these are not shown here, such that Fig. 4(a) is easy to read.

While the agreement between the MoM and FMM results is quite good, there are slight discrepancies in some places. While the MoM has few approximations, the FMM algorithm has three additional sources of errors. First, considering (4), the integration over the unit sphere in (4a) is approximated via a $K = 2L^2$ point quadrature rule [13]. Further, the sum in (4b) must be truncated appropriately, as discussed in Section II-C. These same approximations are present in the free-space FMM algorithm [13], [14]. The half-space problem, as modeled here, introduces a third source of error. In particular, we have approximated the half-space Green’s function (for the “far” FMM terms) via real images, with appropriate polarization-dependent reflection coefficients. To examine more closely the source of errors in the FMM results of Fig. 4(a), we consider the same trihedral, situated in free space. If the agreement between the MoM and FMM algorithms is better for the free-space case than for the half-space results in Fig. 4(a), then we conclude that the half-space Green’s function approximation applied here is inadequate.

In Fig. 4(b) we consider the same case as in Fig. 4(a) but the soil half space is absent. We see by comparing Fig. 4(a) and (b) that the free-space and half-space FMM results demonstrate the same level of accuracy, relative to the MoM computations. This yields further confidence in the accuracy of the real-image approximation to the half-space Green’s function, which has been examined previously in the context of the MoM [31]. With regard to the FMM results in Fig. 4(a), we also examined the solution when the reflection term was eliminated (i.e., neglecting reflections in “far” interactions), to see if the real image points were actually important. We found that the accuracy of the half-space FMM results deteriorated dramatically if the real image term was ignored, underscoring the importance and accuracy of this term.

The bistatic results in Fig. 4 provide a thorough test of the half-space FMM’s accuracy, but the monostatic RCS is of more interest for practical radar systems. In Fig. 5 we consider the MoM- and FMM-computed backscattered RCS from the same trihedral as considered above, for the half-space and free-space cases, respectively. We consider a frequency of 600 MHz, with angles of incidence and scatter $\theta = 60^\circ$ and $-180^\circ \leq \phi \leq 180^\circ$. We see that for these backscatter results, the agreement between the FMM and MoM results is almost perfect, for both the free-space and half-space cases.

In comparing the results in Figs. 4 and 5, we note that there is often a noticeable difference between the signature of the trihedral in free space and above soil. This is an important issue for radar calibration via such fiducial targets, since most previous results for trihedral scattering have been for the free-space case [7], [35], [36]. The importance of the FMM becomes even more evident as the size of the trihedral increases. For example,

Fig. 5. Backscattering (monostatic RCS) for the trihedral in Fig. 4 at a frequency of 600 MHz (number of unknowns $N = 1770$), for varying $\phi = \phi_i = \phi_s$ and $\theta = \theta_i = \theta_s = 60^\circ$. (a) Trihedral 30 cm above lossy Yuma soil with 10% water content [20], [22]. (b) Trihedral in free-space.

ultra-wideband synthetic aperture radar (SAR) systems operate over frequencies from 50 to 1200 MHz [20], [22], for which very large trihedrals are required, to account for the lowest frequencies. The efficacy of applying the FMM to this problem has been examined in detail, in a separate paper [37].

Before leaving the trihedral target, we examine the memory requirements and computational speed of the FMM and MoM algorithms, for the results in Fig. 4. In Fig. 6 we compare the FMM and MoM computation speed and memory requirements, for the trihedral in free space as well as over the soil, as a function of the number of unknowns (approximately equal to 1.5 times the number of triangles [3]). The smallest number of unknowns $N = 220$ corresponds to a frequency of 200 MHz, while the largest $N = 9522$ corresponds to a frequency of 1.4 GHz. We see that, for the MoM solution, the computation time for the free-space and half-space problems coalesce after N becomes sufficiently large, representing the point at which the $O(N^3)$ LU-decomposition overwhelms the $O(N^2)$ matrix fill time. These results were computed on a HP9000/C240 workstation. The results in Fig. 6 also demonstrate the significant memory savings gained by the FMM, at $N > 2500$ for the

Fig. 6. Required RAM and total CPU time (on HP9000/C240) for bistatic RCS calculations in Fig. 4 (720 scattering angles), for frequencies ranging from 200 to 1400 MHz (corresponding to $N = 220$ –9522 unknowns) using MoM (with LUD) and FMM. The CPU time for the MoM solution at $f > 1000$ MHz ($N > 5202$) has been extrapolated.

half-space problem. Finally, we note that the memory requirements and computation time of the half-space FMM, as formulated here, is about twice that of the free-space code. This is attributed to the extra set of image sources, at real spatial positions, which are absent for the free-space problem.

B. Buried 55-Gallon Drum

We consider a 55-gallon drum (90-cm length, 60-cm diameter) buried with its axis parallel to the air–ground interface. Further, the axis of the target is at a depth of 100 cm. For this example we consider Puerto Rico soil, as reported in [38], with 10% water content by weight. The wavelength in this soil is reduced in size by more than a factor of two (relative to free space), due to the soil’s relatively high dielectric constant between about $\epsilon'_r = 5$ and $\epsilon'_r = 6.5$ [38]. We consider normal incidence relative to the air–ground interface, addressing separately the case of perpendicular (V) and parallel (H) polarization relative to the target axis. In Table I we present the copolarized VV and HH backscattered RCS of the target, as a function of frequency, from 100–1000 MHz. The MoM results are only computed up to 500 MHz, above which the MoM computation time (Fig. 7) becomes prohibitive. However, for frequencies below 600 MHz, there is excellent agreement between the rigorous MoM solution and the FMM results.

In Fig. 7 we plot the computation time and memory requirements of the MoM and FMM algorithms, for the results in Table I. All MoM computation times for frequencies over 500 MHz are estimated. These results, which were computed on a SGI Origin 2000 supercomputer (only one processor used), demonstrate the significant gains afforded by the FMM, in both computation time and memory. Upon close inspection of the FMM results in Fig. 7, one notices slight undulations in the data. This, as discussed above, is due to the fact that we utilize $M = 2^q$ FMM clusters, where q is an integer. As N is varied, $M = 2^q$ approaches and recedes the optimal $M \propto \sqrt{N}$, causing a slight undulation in the results (as the algorithm, with $M = 2^q$ clusters, approaches and recedes optimality).

Fig. 7. Required RAM and CPU time (on SGI R10000/195 MHz) for monostatic RCS calculations of 55-gallon drum in Table I using MoM (with LUD) and FMM, i.e., $N = 576$ –46 623 unknowns corresponding to $f = 100$ –1000 MHz. The CPU time for the MoM solution at $f > 500$ MHz ($N > 11841$) has been extrapolated.

TABLE I

BACKSCATTERING (RCS IN [dBsm]) OF A 55-GALLON DRUM (LENGTH = 90 cm, DIAMETER = 60 cm) BURIED IN PUERTO RICO CLAY (10% WATER BY WEIGHT) [38] AS A FUNCTION OF FREQUENCY FOR A NORMALLY INCIDENT PLANE WAVE ($\theta = 0^\circ$). THE DRUM IS SITUATED WITH ITS SYMMETRY AXIS 100 cm BELOW THE HALF-SPACE BOUNDARY, PARALLEL TO THE INTERFACE. THE INCIDENT ELECTRIC FIELD IS PERPENDICULAR TO THE CYLINDER AXIS FOR VV AND PARALLEL FOR HH POLARIZATION

f [MHz]	N (unknowns)	VV (MoM)	HH (MoM)	VV (FMM)	HH (FMM)
100	576	-15.916	-13.600	-15.917	-13.598
200	2070	-13.238	-12.658	-13.247	-12.682
300	4506	-12.643	-12.468	-12.649	-12.469
400	7812	-13.276	-13.156	-13.321	-13.098
500	11841	-14.738	-14.549	-14.754	-14.526
600	16536	—	—	-16.495	-16.314
700	22200	—	—	-18.248	-18.149
800	27687	—	—	-20.119	-20.072
900	35568	—	—	-22.137	-22.102
1000	46623	—	—	-24.181	-24.127

C. Buried Model Unexploded Ordnance (UXO)

The final set of results correspond to an MK 146 MOD 2 U.S. Navy projectile, with dimensions and shape shown in the inset in Fig. 8, buried in Yuma soil with 5% water content [20], [22], the target symmetry axis in the yz -plane. The plane wave is incident at $\phi_i = -90^\circ$ (in the yz -plane) and $\theta_i = 60^\circ$, and the bistatic scattered fields are observed at angles $\theta_s = 60^\circ$ and $-180^\circ \leq \phi_s \leq 180^\circ$, with a 500-MHz operating frequency. The MoM versus FMM computation and memory requirements are similar to those found for the buried drum (Fig. 7) and the trihedral (Fig. 6) and therefore are not shown here, for brevity.

Considering the results in Fig. 8, we see that the FMM and MoM algorithms agree almost exactly. The depth and orientation of the target in Fig. 8 is realistic (given the target size), but this is a very favorable case for the FMM. In particular, recall that the additional approximation we have introduced [beyond the usual FMM approximations in (4)] is the use of a single real image to represent the reflection at the air–ground interface. This term is needed to account for multiple interactions of

Fig. 8. Bistatic radar cross section of a model UXO (cylinder with hemispherical endcap) of length 153 cm and diameter 40.6 cm (see inset). The model UXO is buried in Yuma soil of 5% water content [20], [22] with the target symmetry axis in the yz -plane, at an angle of 30° relative to the z -axis and the nose down at a depth of $z = -217.5$ cm. The RCS is plotted for varying ϕ_s at an angle $\theta_s = 60^\circ$ (30° from grazing) and a frequency of 500 MHz (corresponding to $fwrN = 7167$), assuming a plane wave incident at $\phi_i = -90^\circ$ (in yz -plane) and $\theta_i = 60^\circ$.

Fig. 9. Same as Fig. 8 but using an angle of 75° between target axis and z -direction (15° tilt relative to horizontal xy -plane), the nose down at a depth of $z = -89.3$ cm (the closest distance between target and interface is 30 cm), and a plane wave incident from $\phi_i = 0^\circ$ (in xz -plane) and $\theta_i = 60^\circ$ (compare to inset in Fig. 8). The bistatic RCS is plotted for varying ϕ_s at an angle $\theta_s = 50^\circ$ (40° from grazing).

electromagnetic energy between the target and air–ground interface. For the lossy soil, and relatively deep target, such interactions are less important. Consequently, as a better test of the half-space FMM developed here, we consider the same ordnance considered in Fig. 8, with the angle between the target axis and z -axis (see Fig. 8) set to 75° . Moreover, the closest distance between the target and the interface is 30 cm, rather than 75 cm in Fig. 8. While such a shallow depth is unlikely for a target of this size, the target–interface interactions are considerably more important for this case [although not as important as for a target above the interface, as in Figs. 4(a) and 5(a)]. Results are shown in Fig. 9, where we consider the same frequency, but modified incidence ($\theta_i = 60^\circ$ and $\phi_i = 0^\circ$, i.e., in the xz -plane) and scattering angles ($\theta_s = 50^\circ$). We note that the bistatic RCS of this shallow target is considerably larger than that of the deeper

target (Fig. 8), and the agreement between the MoM and FMM results is again excellent.

IV. CONCLUSION

The FMM has been extended to the case of 3-D perfectly conducting targets situated above or within a lossy half space. The “near” FMM terms are handled as in a MoM analysis, wherein the half-space dyadic Green’s function is characterized rigorously via the method of complex images [26]–[29]. For the “far” FMM terms, the reflected terms in the dyadic half-space Green’s function are approximated via real images with polarization-dependent amplitudes [30]; this technique having proven useful in previous MoM research [31]. The location of the real image is the same for each component of the dyadic. Thus, in addition to being accurate, this approximation yields significantly reduced computational and memory requirements.

Results were presented for targets above and below a half space, with good agreement *vis-a-vis* a reference MoM solution. Nevertheless, the subsurface problem is worthy of further examination. In particular, the real images are required for accurate treatment of the target-interface interaction. However, for deep targets and for relatively lossy soils, the images may be unnecessary, yielding a half-space FMM that replicates the free-space version [13], [14] (although the incident field and the scattered far-field calculations are slightly more complicated). This is likely to be a strong function of target type, depth, and orientation, as well as of the detailed soil characteristics. Moreover, recall that the “far” FMM interactions are handled via the spectral propagator in (4b). If the target is buried in a lossy half space (i.e., complex k_L), T_L is small after sufficiently large intercluster distance $X_{m'm}$. Consequently, for the buried-target case, a further algorithm acceleration and reduction in memory can be achieved by ignoring intercluster interactions for sufficiently large $X_{m'm}$.

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