Multiaspect Target Identification with Wave-Based Matched Pursuits and Continuous Hidden Markov Models

Paul Runkle, Lawrence Carin, Luise Couchman, Timothy J. Yoder, and Joseph A. Bucaro

Abstract—Multiaspect target identification is effected by fusing the features extracted from multiple scattered waveforms; these waveforms are characteristic of viewing the target from a sequence of distinct orientations. Classification is performed in the maximum-likelihood sense, which we show, under reasonable assumptions, can be implemented via a hidden Markov model (HMM). We utilize a continuous-HMM paradigm and compare its performance to its discrete counterpart. The feature parsing is performed via wave-based matched pursuits. Algorithm performance is assessed by considering measured acoustic scattering data from five similar submerged elastic targets.

Index Terms—Hidden Markov model, matched pursuits, classification.

1 INTRODUCTION

We are interested in identification of a concealed or distant target, assuming that the putative target has been detected and, therefore, that its nominal location is known. However, the target itself and/or sensor motion. The actual states sampled are hidden since the absolute target-sensor orientation is known. The data under test is noise-free, once the absolute target-sensor orientation is hidden and modeled as a stochastic parameter.

A hidden Markov model (HMM) [1], [2], [3] is used to evaluate the probability $p(y_1, y_2, \ldots, y_N | T_k)$, in particular, it is well-known that wave scattering from most targets is characterized by angular sectors over which the angle-dependent scattered fields are slowly varying. Each such sector is here termed a “state” and the number of states characteristic of a given target is dictated by the target complexity. Assume target $T_k$ is composed of the $M$ states $s = \{s_1, s_2, \ldots, s_M\}$. One can define a state-dependent probability of observing a given feature vector $y_{s_n}$ with $p_{s_n}(y_{s_n} | T_k)$ representing the probability of extracting feature vector $y_{s_n}$ in state $s_{n}$ of target $T_k$. The probability $p(y_1, y_2, \ldots, y_N | T_k)$ can be expressed as:

$$\sum_{q} p_{s}(q | T_k)p_{q}(y_1 | q, T_k)p_{q}(y_2 | q, T_k)\ldots p_{q}(y_N | q, T_k).$$

Here, $q$ represents a sequence of $N$ states, $q = \{q_1, q_2, \ldots, q_N\}$, with $q_k \in S$. The probability $p(y_1, y_2, \ldots, y_N | T_k)$ is evaluated by summing over all possible $M^N$ state sequences $q$, weighted by the associated probabilities $p_{s}(q | T_k)$.

Note that, as modeled, the $N$ scattered waveforms sample $N$ discrete (not necessarily distinct) states characteristic of the target under interrogation. If we assume that the probability of transitioning from one state to the next is dictated only by the current state occupied, then the state sequence $q$ is a Markov chain [4], and the probabilities $p_{s}(q | T_k)$ can be evaluated by a Markov model. More properly, we employ an HMM [1], [2], [3] since the underlying states are hidden and the only observable is the sequence of feature vectors $\{y_1, y_2, \ldots, y_N\}$. In this context, we can exploit the extensive existing HMM literature, wherein HMMs have been developed, for example, for speech-processing [1], [2], [3], target tracking [5], and for handwriting recognition [6], [7], and image processing [8].

As in our previous research [9], we use matched-pursuits feature parsing to calculate the vectors $y_{s_n}$. In that previous research, target classification was performed via discrete HMMs, with such suffering inherent distortion [1], [2] as one quantizes the continuous parameter vectors $y_{s_n}$. Consequently, here we consider continuous HMMs [1], [2], for which the densities $p_{s_n}(y_{s_n} | T_k)$ are represented in terms of continuous mixture distributions. We perform a detailed assessment of the target-identification potential of continuous HMMs via-as-vis discrete HMMs. Further, the results in [9] were presented for synthetic data, while here we consider performance on measured scattering data from different but similar targets. Finally, in the work presented here, we also consider use of the Viterbi algorithm [10], thus allowing estimation of the hidden state sequence $q$, from which the target orientation can be estimated. This issue was not addressed in our previous paper.

The remainder of the text is organized as follows: In Section 2, we provide a detailed explanation of the HMM multiaspect target classifier and its underlying assumptions. Details on estimation of the HMM parameters and their connection to the wave physics are also discussed in Section 2. Example results in which we compare performance of the discrete and continuous HMMs are presented in Section 3, with conclusions discussed in Section 4. The matched-pursuits feature parser is elucidated in the Appendix.

2 CONTINUOUS-HMM TARGET IDENTIFICATION

2.1 Target States and State-Transition Probabilities

Underlying scattering physics dictates that general targets have scattered fields (and, hence, associated feature vectors $y_{s_n}$) that vary strongly with target-sensor orientation. However, one can generally define angular sectors, representative of constrained target-sensor orientations, over which the physics and the scattered fields are slowly varying. We define such angular sectors as states, for reasons that will become clear below. The $N$ observed scattered signals are sample waveforms from $N$ states; the same state may be sampled more than once and some states may not be sampled at all, depending on the target orientation and details of the sensor and/or sensor motion. The actual states sampled are hidden since the target is distant or concealed.

Assume that a given target can be represented by $M$ contiguous states, with (in two dimensions) consecutive angular support of $\gamma_1, \gamma_2, \ldots, \gamma_M$ (see Fig. 1), denoted by states $s = \{s_1, s_2, \ldots, s_M\}$, respectively. Further, let $q_k \in q$ represent the hidden state sampled by the $k$th observed scattered waveform. If the target orientation is assumed uniformly distributed, the initial-state probabilities are defined as...
Moreover, if we assume that the sensor moves in one direction relative to the putative target center (the target has been detected and is now being identified) with angular sampling \(\Delta \varphi\), with \(\Delta \varphi < \varphi_m\), \(\forall \varphi_m\), then the probability of transitioning from state \(m\) to state \(l\), represented by \(a_{ml} \equiv P(q_{l+1} = l\mid q_n = m)\), is given by

\[
\begin{align*}
\text{if } m &= l, \\
\text{if } |m - l| &= 1, \\
\text{if } |m - l| &> 1.
\end{align*}
\]

Like (1), the expressions in (2) are derivable directly from geometrical considerations, with the interested reader referred to [9] for details.

2.2 Maximum-Likelihood Discrimination and Hidden Markov Models

Maximum-likelihood target identification, based on fusing the \(N\) aspect-dependent scattered waveforms, is effected by selecting that target \(T_k\), for which

\[
p(y_1, y_2, \ldots, y_N\mid T_k) \geq p(y_1, y_2, \ldots, y_N\mid T_i) \quad \forall T_i.
\]

We evaluate \(p(y_1, y_2, \ldots, y_N\mid T_k)\) by considering all possible \(M^N\) state sequences, with each weighted by its probability of occurrence, as defined in Section 2.1. In particular,

\[
p(y_1, y_2, \ldots, y_N\mid T_k) = \sum_{m}^{M} \sum_{n}^{M} \cdots \sum_{j}^{M} \sum_{f}^{M} \sum_{i}^{M} a_{mi}a_{nj}a_{lk}a_{fo}
\]

where we sum the indices \(m, n, l, j\) over all \(M\) states for target \(T_j\). In (3), we have utilized the Markov state-transition model from Section 2.1 and we have modeled the statistical properties of \(y_s\), being a function only of the current state occupied, independent of the state sequence and the properties of any other state. These latter two properties are dictated by the manner in which the states \(s_m\) have been defined, each representative of an angular sector over which the wave physics and scattered fields are relatively slowly varying, with each state representing different underlying physics. Finally, while, in general, \(M^N\) possible sequences must be considered in evaluating (3), from (2) it is clear that for our problem only \(M^{N - 1}\) of these have a nonzero probability of occurrence.

The form in (3) is characteristic of a hidden Markov model (HMM) [1], [2] and the requisite multiple sums can be evaluated efficiently via the well-known forward-backward algorithm [1], [2]. Alternatively, recently, HMM researchers have employed the Viterbi algorithm [10], which we have applied here. The Viterbi algorithm yields the ML state sequence, defining the new likelihood function

\[
\begin{align*}
p'(y_1, y_2, \ldots, y_N) = \max_{\pi} \left\{ p(y_1\mid q_1, T_k)p(y_2\mid q_2, T_k) \cdots p(y_{N-1}\mid q_{N-1}, T_k)p(y_N\mid q_N, T_k)\right\} &
\end{align*}
\]

In addition to determining the ML state sequence, (4) can be used to refine the initial estimates for the vector \(\pi = [\pi_1, \pi_2, \ldots, \pi_M]\) and the tridiagonal \(M \times M\)-dimensional matrix \(A\), with elements initially defined in (1) and (2), respectively. In particular, recall that (1) and (2) are based on an initial state decomposition, inaccuracies in which will yield errors in \(\pi\) and \(A\), as well as errors in the probabilities \(p(y_i\mid s_m, T_k)\). Here, we use the Viterbi algorithm as a tool to update the state decompositions, as well as the probabilities \(\pi\), \(A\), and \(p(y_i\mid s_m, T_k)\); (1) and (2) and the initial state decomposition can be viewed as starting points for subsequent optimization.

2.3 Continuous HMM Mixture Densities

It remains to describe how the continuous distribution \(p(y_i\mid s_m, T_k)\) is generated; it is defined as the probability of \(y_i\), representing data from state \(s_m\) and target \(T_k\). We consider training data from state \(s_m\) and target \(T_k\), from which we determine the distribution of the associated feature vectors \(y_s\). Using the K-means algorithm [11] in conjunction with the Mahalanobis distance [12] metric, we generate \(L\) discrete vectors \(v_1, v_2, \ldots, v_L\), which coarsely represent the distribution of the training-data generated \(y_s\). The Mahalanobis distance, which is weighted to compensate for heterogeneous features in \(y_s\), is then used to cluster the training data, with \(C_l\) representing the cluster associated with \(v_l\). We represent \(C_l\) with a Gaussian distribution \(g(y_i\mid s_m, T_k)\), with mean \(E(y_i) = v_l\), \(y_s \in C_l\). Moreover, for simplicity, we approximate the components of \(y_s\), as uncorrelated. We note, however, that it has been shown that uncorrelated mixture densities are capable of representing correlated data sufficiently [1], [2]. Finally, we have the mixture distribution:

\[
p(y_i\mid s_m, T_k) = \sum_{l=1}^{L} w_l g(y_i\mid s_m, T_k), \quad \sum_{l=1}^{L} w_l = 1.
\]

The coefficients \(w_l\) are weighted to reflect the number of elements in cluster \(l\) relative to the total number of training data for state \(m\) and target \(T_k\). In particular, if \(N_m\) represents the total number of training vectors available for state \(m\) and \(N_l\) represents the number of these vectors in cluster \(C_l\), then \(w_l = N_l/N_m\).

3 Example Results

3.1 Measurement System and Targets Considered

We consider measured acoustic backscattering data from five submerged elastic targets: 1) a cylindrical air-filled shell; 2) a duplicate shell, stiffened with equally spaced ring stiffeners; 3) a duplicate shell, stiffened with unequally spaced ring stiffeners and augmented by resiliently mounted, elastic internal rods; 4) a slightly larger, nearly periodically ribbed shell; and 5) a duplicate of shell 4, with a large number (~1,000) of attached internal oscillators. Schematics of the targets are shown in Fig. 2. Specifics on the measurement system and further details on the targets measured by the target facility section at the Naval Research Laboratory can be found in [13], [14], [15].

Each of the targets was ensnosed by an acoustic waveform with bandwidth from approximately 11-40 kHz, corresponding to relative target dimensions of \(2.9 \leq \kappa A \leq 10.4\), where \(\kappa\) is the wavenumber and \(\alpha\) is the average radius of the targets (see
To realize waveforms that approximate a minimum time-bandwidth product, we have deconvolved the spectrum of the incident waveform and have synthesized scattered waveforms assuming the incident-pulse shape in Fig. 3 (representative of a Blackman pulse [16]). This step is not necessary, but we find it convenient to utilize an incident pulse with minimum temporal support. In Fig. 4, we plot the frequency-dependent backscattered signature of each target, as a function of the incidence angle, assuming the incident spectrum in Fig. 3. As expected from the similarity of the target geometries, the angle and frequency dependence of the backscattered fields are similar for all targets.

3.2 Example Mixture Distributions

Before presenting classification results, we examine example mixture distributions computed as discussed in Section 2.3. In Fig. 5, we present distributions for Target 4, states 1, 3, and 5. For all five targets, we considered a five-state decomposition, motivated by similar underlying physics. To describe the five
states and the associated constitutive physics, consider Target 1 (Fig. 2), the simplest of the targets. State 1, $0^\circ -18^\circ$, corresponds to specular diffraction from the ends of the shell; state 2, $19^\circ -60^\circ$, is absent elastic scattering and is characterized primarily by compressional diffraction; in state 3, $61^\circ -71^\circ$, the compressional wave travels straight down the shell (phase matched); state 4, $72^\circ -82^\circ$, is dominated by helical membrane waves; and state five, $82^\circ -90^\circ$, is principally characterized by scattering from the target broadside. We reiterate that this angular decomposition is used to delineate the initial state decomposition (cf. (1) and (2)), with these refined via Viterbi reestimation [10], as discussed in Section 2.2.

In Fig. 5, the contours represent the continuous probability distribution described in (5), the “X” symbols denote the discrete $v_i$ computed via K-means [11], and the points identify the training data, after being parsed via wave-based matched pursuits. We have utilized three matched-pursuits iterations and, consequently, the parameter vectors $y_n$ are eight-dimensional (see Appendix). Further, we have used $L = 4$ discrete $v_i$, thus representing the mixture distributions as a sum of four eight-dimensional Gaussian distributions. At least in the two-dimensional subspace shown in Fig. 5, the different states clearly have distinctive characteristics, which will aid multiaspect identification.

### 3.3 Comparison of Discrete and Continuous HMM Performance

As an alternative to the continuous mixture density defined in (5), one can use a discrete probability function to describe the state-dependent distribution of the feature vectors $y_n$. This is done by defining $J$ discrete elements $c_j$ in the parameter space defined by $y_n$, with these elements constituting a “codebook” $C$ [11], [17], [18]. Each $y_n$ is mapped to its nearest neighbor in $C$ (using the Mahalonobis distance [12]). In this manner, the continuous feature vectors $y_n$ are effectively discretized into $J$ regions, with $b_{jm}$ representing the probability of realizing codebook element $c_j$ in state $m$. The probabilities $b_{jm}$ define a $J \times M$-dimensional matrix $B$, which is “learned” during a training phase similar to that discussed in Section 2.

A discrete HMM has the advantage of a representation completely defined by the vector $\pi$ and the matrices $A$ and $B$. However, the quantization $y_n \rightarrow c_j$ yields a “distortion” [1], [2], defined as the distance between the continuous feature vector $y_n$ and the codebook element $c_j$. The distortion can be reduced by increasing the size of the codebook. However, eventually the codebook becomes sufficiently large to approximate a continuous distribution, suggesting use of the latter (i.e., a continuous HMM).

We perform a comparison between the discrete and continuous HMM. With regard to the former, we utilize a 64-element codebook, and the vector $\pi$ and matrices $A$ and $B$ are optimized via the Viterbi algorithm (Section 2.2 and [10]). The codebook was generated via the K-means algorithm [11], similar to the generation of the $v_i$ used in the continuous HMM (Section 2.3). It should be noted here that the complexity of the continuous and discrete HMMs are similar. For the discrete HMM, 64 codebook elements were used, while, for the continuous HMM, $L = 4$ mixture densities were utilized and 16 parameters are estimated for each (eight means and variances). For the continuous HMMs, we must also estimate the weights $w_j$ in (5). In Fig. 6, we show a comparison of discrete and continuous HMM performance, as a function of number of observations $N$, assuming 5° angular sampling. These results are for noisy data, with the noise having a power spectral density (PSD) identical to the spectrum of the incident pulse (Fig. 3) in an attempt to simulate additive clutter. For both the discrete and continuous HMMs, training was performed by using all 360 backscattered waveforms from each target (1° sampling was used in the measurements) and 15 noise realizations were considered for each scattered waveform. Since there are 181 unique observation
sequences, we trained on $15 \times 181$ sequences. The testing was performed using all possible sequences of the 360 backscattered waveforms (for the $N$ measurements characteristic of a given sequence) and four noise realizations. Since the noise realizations used for testing and training were different, the HMMs were tested and trained on distinct data.

In Fig. 6, $N = 10$ samples corresponds to a total of $45^\circ$ of data ($5^\circ$ sampling), for which we see an average misclassification probability of less than 0.1, for both the discrete and continuous HMMs. As the number of samples $N$ decreases, corresponding to reduced angular sampling, the rate of misclassification increases, as expected. Moreover, it is clear from these results that the continuous HMM consistently outperforms its discrete counterpart. The relatively inferior discrete-HMM performance is attributed to the distortion induced in the mapping of the continuous $y_n$ to the discrete $c_i$.

Fig. 5. Continuous mixture distributions for target 4 (see Fig. 2). The contours represent the continuous probability distribution described in (5), the "X" symbols denote the discrete $v_n$; computed via K-means [11], and the points identify the training data after being parsed via wave-based matching pursuits. These results represent a two-dimensional projection on an eight-dimensional space, with results shown for the $\left(\omega_1, \tau_2 - \tau_1\right)$ subspace. (a) State 1, (b) state 3, (c) state 5.

Fig. 6. Comparison of average discrete and continuous HMM performance as a function of number of observations $N$, assuming $5^\circ$ angular sampling. Results are shown for additive colored noise and the noise power spectral density (PSD) is identical to the spectrum of the incident pulse (Fig. 3). An average SC-SNR of 20 dB is considered.
With regard to defining the signal-to-noise ratio (SNR), algorithm performance is determined in large measure by the ability of the matched-pursuits algorithm (see Appendix) to reliably extract features from noisy data. For a matched filter applied to the sampled signal \( f_f \), the SNR is defined as \( <f_f/f> / \sigma^2 \), where \( \sigma^2 \) is the noise variance [12]. The matched-pursuits algorithm performs matched-filter detection on features (denoted \( e \) in the Appendix) in the waveform \( f_f \). If \( e_i \) represents the normalized feature element selected on the \( i \)th matched-pursuits iteration, for noise-free data, the matched-pursuits figure of merit is \( <e_f e_i> / \sigma^2 \), defined as the signal component SNR (SC-SNR) [19]. We have found that an SC-SNR of in excess of approximately 5 dB is required for reliable matched-pursuits performance [19].

The results in Fig. 6 are for an average SC-SNR of 20 dB, for three matched-pursuits iterations, averaged for all 360 scattered waveforms for a particular target.

The signatures from the targets in Fig. 2 are highly aspect dependent, motivating the five-state HMM decomposition discussed above. For example, the energy in waveforms scattered from state 5 (characteristic of scattering for incidence angles at or near the target broadside) are of much larger amplitude than waveforms scattered in state 1 (characterized by relatively weak diffraction from the ends of the target). Therefore, for a fixed noise (clutter) variance, the SC-SNR and SNR are also strongly state dependent. To illustrate this, and to assess the noise level characteristic of the results in Fig. 6, in Table 1 we plot the average SC-SNR in each of the five states, for each target. The data in Table 1 correspond to the 20 dB average SC-SNR scenario described in Fig. 6.

The results in Fig. 6 present the average misclassification probability across all possible target orientations, for the discrete and continuous HMMs, a misclassification defined as identifying the data as characteristic of target \( T_i \) when it was actually scattered from target \( T_j \). Additional information is presented in the form of a confusion matrix, as presented in Tables 2 and 3, for the discrete and continuous HMMs, respectively. Using the target designations described in Section 3.1, the confusion matrix considers data from target \( T_k \) and quantifies the probability that it is classified as target \( T_i \), for \( i \) and \( k \) from one to five (for the five targets considered here). Several observations can be made from these tables, which considered the same 20 dB average SC-SNR data presented in Fig. 6, for the case of \( N = 5 \) scattered waveforms (20° of data, with 5° sampling). First, as in Fig. 6, these tables demonstrate that the continuous HMM consistently outperforms its discrete counterpart. Moreover, for both the continuous and discrete HMM, data from target 4 had the highest probability of being properly classified (96.27 percent and 99.86 percent, for the discrete and continuous HMM, respectively). This is attributed to the Bloch wave [13] excited on the ribbed shell, which is characterized by an angle-dependent frequency that is clearly visible and distinct in Fig. 4. These results indicate that the wavefront-resonance matched-pursuits dictionary (see Appendix) and HMM do an adequate job of exploiting this feature.

### 3.4 Continuous-HMM Performance for Noisy Data

Having established the superiority of the continuous HMM for this data set, we examine its performance in greater detail. In Fig. 7, we present the probability of misclassification for each of the five targets, as a function of the average SC-SNR. Results are presented for an average SC-SNR of 10, 15, and 20 dB, and presented as a function of the number of scattered waveforms \( N \), using 5° angular sampling. Consistent with the results in Tables 2 and 3, data scattered from target 4 was consistently misclassified least frequently. As expected, as the average SC-SNR is diminished, the probability of misclassification increases.
3.5 State-Sequence Estimation

If, in addition to identifying the target, one can estimate the underlying state sequence \( q \), the target orientation can be estimated. The Viterbi algorithm \([10]\) yields the maximum-likelihood estimation for the state sequence \( q \), as symbolized in (4), and this sequence can be used to estimate the target orientation. In Fig. 8, we plot the number of states that were incorrectly identified, for each of the five targets, for cases in which a correct target identification was realized (not all states need be correctly identified for the target to be identified correctly). The results in Fig. 8 correspond to sequences of length \( N = 10 \), with an average SC-SNR of 15 dB. We see that, for most of the five targets, the number of errors in state classification is relatively small. Interestingly, from Fig. 8, the target with the least complexity (the empty shell, target 1) was most likely to cause state-identification errors while still correctly classifying the target. This implies that there is less state-dependent diversity in the underlying physics (scattered fields) for this target than for the other four, which we attributed to the fact that targets 2-5 were characterized by more complicated internal structure.

Fig. 7. Average probability of misclassification as a function of the average SC-SNR. Results are presented as a function of the number of scattered waveforms \( N \), using 5° angular sampling.

Fig. 8. The number of states that were incorrectly identified for an \( N = 10 \) length sequence for cases in which a correct target identification was realized. An average SC-SNR of 15 dB is considered. For the 20 dB case (see Fig. 7), all targets had an indistinguishably small number of state misclassifications. (a) Target 1, (b) target 2, (c) target 3, (d) target 4, and (e) target 5.
4 Conclusions
A hidden Markov model (HMM) has been investigated for multiaspect target identification. While the basic classification paradigm is applicable to a general feature parser, here we have employed a wave-based matched-pursuits algorithm. Using measured data from five submerged elastic targets we demonstrated the marked benefit of employing a continuous HMM, vis-à-vis a discrete HMM. Further, through use of the Viterbi algorithm [10], we approximately extracted the hidden state sequence \( q \) from which we can approximate the target orientation.

There are several propitious directions for future research. For example, the HMM construct is applicable to a general feature parser. It is worthwhile to consider alternatives to matched-pursuits, to quantify the link between HMM classification performance and the associated feature vectors employed. Further, all results presented here were for processing multiple time-domain waveforms, corresponding to viewing the target from a sequence of distinct orientations. A similar approach could also be applied to a sequence of (two-dimensional) images, with these similarly corresponding to multiple views of a putative target.

Appendix

Wave-Based Matched Pursuits

Matched pursuits is an algorithm developed by Mallat and Zhang [20] for decomposition of a sampled waveform \( f = \{f_1, f_2, \ldots, f_K\} \) in terms of a prescribed set of normalized vectors \( e_i \in D \), where \( D \) is termed a “dictionary.” The dictionary can take an arbitrary form and, therefore, can be tailored to the features that underlie the waveform(s) under study. We perform inner products, \( \langle f, e_i \rangle = f^T e_i \), between \( f \) and the sampled dictionary elements \( e_i \), and select that dictionary element \( e_i \) for which \( \langle f, e_i \rangle \) is defined, and the process is repeated on \( f_i \). After \( I \) matched-pursuits iterations, we effect the decomposition

\[
 f = \sum_{i=1}^{L} R_{-1} e_i + R_{1},
\]

where \( R_0 \equiv f \). The goal is to define a dictionary for which the principal features characteristic of \( f \) can be represented with the smallest number of matched-pursuits iterations \( I \), with the remainder \( R_0 \) containing noise or superfluous content.

As discussed in [20], the dictionary search at each matched-pursuits iteration is performed by first utilizing a discrete, finite approximation to the dictionary \( D \), followed by a gradient search in the continuous parameter space. For the wavefronts and resonances of interest for wave scattering, we use the discretized dictionary

\[
 e(k; l, p | q) = \cos(2\pi(k - q2^{-p})(q2^{-p} + \alpha) \exp(-(k - q2^{-p})^2 | q2^{-p} - U(k - q2^{-p})),
\]

where \( k \) is the temporal sampling parameter, \( N_i \) is the nominal length of the incident pulse, \( R \) represents the number of samples in \( f \) and \( e_i \), and \( U(x) = 0 \) for \( x < 0 \) and \( U(x) = 1 \) for \( x > 0 \). The parameter reflects the expected longest duration of the resonant (extended duration) features and we typically choose \( \xi = 4 \). In the initial coarse dictionary, parameters \( p, l, \) and \( q \) are integers, representing, respectively, the element duration, angular frequency, and temporal shift. By making \( p \) large or small, \( (A2) \) can represent long or short duration features, characteristic of resonances and wavefronts, respectively. The search over the coarse dictionary and subsequent gradient search are performed in a manner similar to that in [20].

After applying matched pursuits on the \( n \)th scattered waveform, we generate a \((3I - 1)\)-dimensional parameter vectors \( y_n \).

\[
 Each \ y_n \ is of the form \ [\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_i, \varphi_1, \varphi_2, \ldots, \varphi_j, \psi_n, \varphi_1, \varphi_2, \ldots, \varphi_j] \ where \ \alpha = 2^p, \ \varphi = 2^{(-p)} \ and \ \psi = 2n2^{-p}. \ Parameters \ \alpha_i, \ \alpha_j, \ and \ \psi \ are extracted on the \( n \)th matched-pursuits iteration. Not that \( y_n \) only contains relative shifts \( \varphi \), which are independent of the target-sensor distance and, therefore, only characteristic of the target itself. The requisite number of matched-pursuits iterations, \( I \), is dependent on the scattering complexity and on dictionary compactness; for the data considered in Section 3, we found three matched-pursuits iterations sufficient.

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