

Performability Analysis of Wireless Cellular Networks *

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Abstract

The high expectations of performance and availability for wireless mobile systems has presented great challenges in the modeling and design of fault tolerant wireless systems. The proper modeling methodology to study the degradation of such systems is so-called *performability* modeling. In this paper, we give overview of approaches for the construction and the solution of performability models for wireless cellular networks. First, we start with the Erlang loss model, in which hierarchical and composite Markov chains are constructed to obtain loss formulas for a system with channel failures. Consequently, we develop two level hierarchical models for the wireless cellular system with handoff and channel failures. Then, for a TDMA system consisting of base repeaters and a control channel, we build a hierarchical Markov chain model for automatic protection switching (APS). Finally, we discuss stochastic reward net (SRN) models for performability analysis of wireless systems.

1 Introduction

With the rapid growth of wireless communication services, customers are expecting the same level of availability and performance from wireless communication systems as traditional wire-line networks. In general, wireless systems are characterized by their scarce radio resources which limit not only the service offering but also the quality of service (QoS). Furthermore, service degradation can also be caused by component failures in the wireless system. A failure, depending on its nature, may have different impacts on system performance. The high degree of mobility enjoyed in wireless networks is in turn the cause of inherent unreliability. Compared with wired networks, wireless networks need to deal with disconnects due to handoff, noise and interference, blocked and weak signals and run-down batteries, etc. In addition, the performance and availability of a wireless system is affected by the outage-and-recovery of its supporting functional units. Unplanned as well as planned outages of equipment also contribute to the degradation of the system's availability and performance. From the designer and operator's point of view, it is of great importance to take these factors into account integratively.

Traditional pure performance model that ignores failure and recovery but considers resource contention generally overestimates the system's ability to perform. On the other

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hand, pure availability analysis tends to be too conservative since performance considerations are not taken into account. To obtain realistic composite performance and availability measures, one should consider performance changes that are associated with failure and recovery behavior. The proper modeling methodology to study the degradation behavior of the system is the so-called *performability* modeling [12] [13] [18] [6], which takes into account both availability and performance, thus providing a more complete picture.

Generally, performability evaluation involves two steps: the construction of a suitable model, and the solution of the model. Two common techniques of analyzing performability are (i) a composite continuous-time Markov chain (CTMC) containing both the performance as well as the availability related events, (ii) a two-level hierarchical model where the upper level is a Markov reward model (MRM) that is essentially the availability model with each state of the MRM being assigned a reward rate derived from the lower-level pure performance model. One measure of performability can then be expressed as the expected steady-state reward rate:

$$E[Z] = \sum_j r_j \pi_j$$

where r_j is the reward rate assigned to state j and π_j is the steady-state probability of state j of the upper level MRM. If transient analysis is of interest, then the expected reward rate at time t :

$$E[Z(t)] = \sum_j r_j \pi_j(t)$$

or the expected accumulated reward in the interval $(0, t]$ can be computed:

$$E[Y(t)] = \sum_j r_j \int_0^t \pi_j(\tau) d\tau = \sum_j r_j L_j(t)$$

where $Y(t) = \int_0^t Z(\tau) d\tau$, $\pi_j(t)$ is the transient probability of the upper level MRM being in state j at time t , and $L_j(t)$ is the expected total time spent by the upper level MRM in state j during $(0, t]$.

In this paper, we report recent approaches to performability modeling of wireless communication networks. The paper is organized as follows. In Section 2, we construct composite and hierarchical CTMC models for the combined performance and availability analysis of general wireless systems. In Section 3, the performability of wireless cellular systems with handoff is considered and analyzed. In Section 4, we build hierarchical Markov chain models for APS in TDMA system consisting of base repeaters and a control channel. In Section 5, we demonstrate several stochastic reward net (SRN) models for the performability analysis of wireless systems. Finally, we make our conclusions in Section 6.

2 Erlang loss model [18]

Consider a telephone switching system consisting of n trunks (or channels) with an infinite caller population. If an arriving call finds all n trunks busy, it does not enter the system and is lost instead. The call arrival process is assumed to be Poissonian with rate λ . We assume that call holding times are independent, exponentially distributed random variables with parameter μ and independent of the call arrival process.

We first present an availability model that accounts for failure–repair behavior of trunks; then, we use a performance model to compute performance indices such as blocking probability given the number of non-failed trunks; finally, we combine the two models

together and give performability measures of interest. Assume that the times to trunk failure and repair are exponentially distributed with mean $1/\gamma$ and $1/\tau$, respectively. Also assume that a single repair facility is shared by all the trunks. The availability model is then a homogeneous CTMC with the state diagram shown in Figure 1. Here, the state index denotes the number of non-failed trunks in the system. The steady-state probability for the number of non-failed channels in the system is given by:

$$\pi_i = \frac{1}{i!} (\tau/\gamma)^i \pi_0, \quad i = 1, 2, \dots, n,$$

where the steady-state system unavailability:

$$U = \pi_0 = \left[\sum_{i=0}^n \frac{1}{i!} (\tau/\gamma)^i \right]^{-1}.$$

Consider the performance model with the given number i of non-failed channels. The quantity of interest is the *blocking probability*, that is, the steady-state probability that all trunks are busy, in which case the arriving call is refused service. Note that in this performance model, the assumption is that blocked calls are lost (not re-attempted). The performance model of this telephony system is an $M/M/i$ loss system, and the state diagram is shown in Figure 2. The blocking probability with i channels in the system is given by:

$$P_b(i) = \frac{(\lambda/\mu)^i / i!}{\sum_{j=0}^i (\lambda/\mu)^j / j!}.$$

This equation is known as the *Erlang's B loss formula*. It can be shown to hold even if the call holding time follows arbitrary distribution with mean $1/\mu$ [14].

Now, we construct the composite model for the combined performance and availability analysis and the state diagram is shown in Figure 3. Here, the state (i, j) denotes i non-failed trunks and $j \leq i$ ongoing calls in the system. Note that trunks that are in use as well as those that are free can fail with the corresponding failure rate. This composite model is a homogeneous irreducible CTMC with $(n+1)(n+2)/2$ states, and the steady-state probability can be obtained by solving the linear system of homogeneous equations. Such a solution may be obtained using a software package such as SHARPE [15]. The total call blocking probability is then given by

$$T_b = \sum_{i=0}^n \pi_{i,i}.$$

In a composite model, finding the required measures will be cumbersome and numerically error-prone when the number of trunks is large. To avoid the problems of largeness and stiffness, we can compute the required measure approximately using a hierarchical approach [6, 10, 18, 19]. In this approach, a top-level availability model (Figure 1) is turned into a Markov reward model (MRM), where the reward rates come from a sequence of performance models (Figure 2) and are supplied to the top-level availability model. Attach

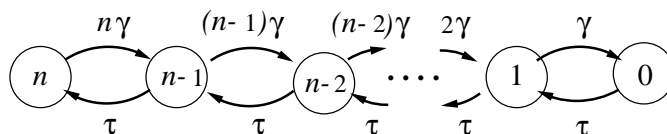


Figure 1: State diagram for the Erlang loss availability model

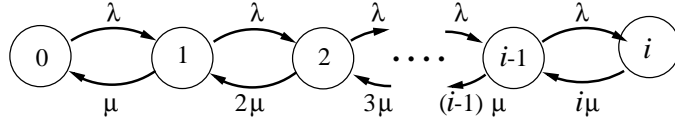


Figure 2: State diagram for the Erlang loss performance model

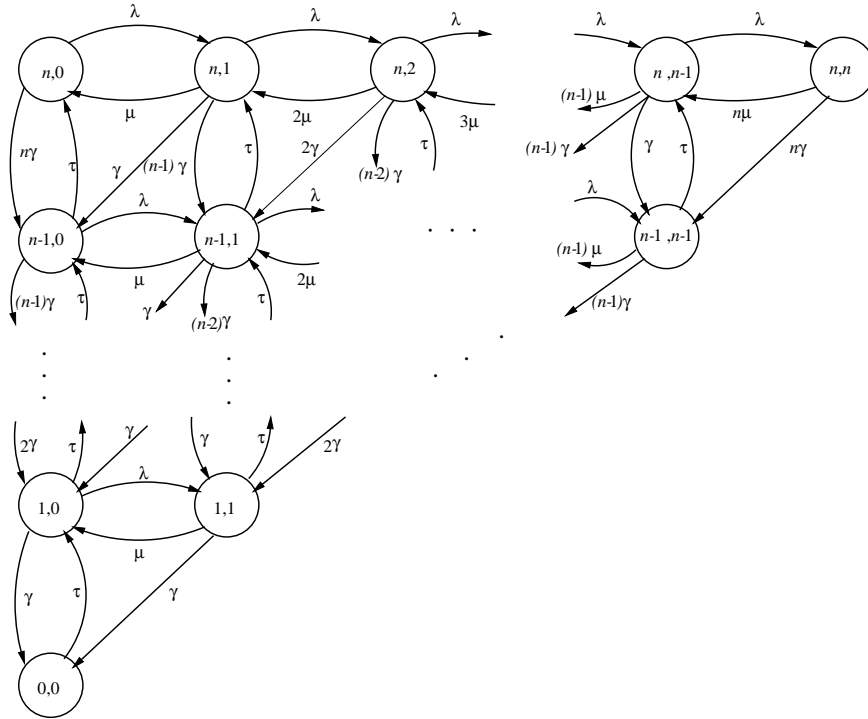


Figure 3: State diagram for the Erlang loss composite model

a reward rate r_i to the state i of the availability model as the blocking probability with i trunks in the system, that is, $r_i = P_b(i)$, $i \geq 1$ and $r_0 = 1$. Then the required total blocking probability can be computed as the expected reward rate in the steady-state and is given by

$$\hat{T}_b = \left[\sum_{i=0}^n r_i \pi_i \right] = \left[\sum_{i=1}^n P_b(i) \pi_i \right] + \pi_0, \quad (1)$$

where π_i is the steady-state probability that i non-failed trunks are there in the system.

Note that the bracketed part on the right-hand-side of equation (1) is the performance loss including that in degraded states while the remaining part is system unavailability U . Thus, this equation shows the total loss probability.

3 Modeling cellular systems with failure and repair [18]

The above Erlang loss formula cannot be used in cellular wireless networks due to the phenomenon of handoff. In this section, we discuss a two level hierarchical performance model for wireless cellular networks with handoff.

The object under study is a typical cellular wireless system. In the system, mobile subscribers (MSs) are provided with telephone service within a geographical area. The

service area is divided into multiple adjacent cells. MSs communicate via radio links to base stations (BSs), one for each cell. When an MS moves across a cell boundary, the channel in the old BS is released and an idle channel is required in the new BS. This phenomenon is called *handoff*. Handoff is an important function of mobility management. To reduce the dropping probability of handoff calls, a fixed number of guard channels is reserved exclusively for the handoff calls [7].

We consider a single cell with limited number of channels, n , in the channel pool. Let the number of guard channels, g ($g < n$), be reserved exclusively for handoff calls. Changing the number of guard channels results in different new call blocking probability and handoff call dropping probability. We notice that channel failures are rather rare compared with the new call or handoff call arrivals and departures. Consequently, we use hierarchical decomposition to obtain an approximate solution: we first present an upper level availability model which accounts for the possible channel failures and repairs. Then, we compute performance indices by constructing a lower level CTMC performance model. Finally, we combine them together and give performability measures of interest.

The upper level model, as shown in Figure 1, describing the failure and repair behavior of the system, is a pure availability model. Let π_i ($i \in [0, n]$) be the steady-state probability of the CTMC being in state i of the upper level model. We know that

$$\pi_i = \frac{1}{i!} (\tau/\gamma)^i \pi_0 \quad (2)$$

where π_0 is the steady state probability of the CTMC in state 0 which equals the steady state unavailability.

$$\pi_0 = \left[\sum_{i=0}^n \frac{1}{i!} (\tau/\gamma)^i \right]^{-1} = U \quad (3)$$

The lower level model, as shown in Figure 4, captures the pure performance aspect of the system [5]. Each state represents the number of talking channels in the system. In Figure 4, $i \in [1, n]$. Let λ_1 be the rate of Poisson arrival stream of new calls and λ_2 be the rate of Poisson stream of handoff arrivals. Let μ_1 be the rate that an ongoing call (new or handoff) completes service and μ_2 be the rate at which the mobile engaged in the call departs the cell. When an idle channel is available in the channel pool and a handoff call arrives, the call is accepted and a channel is assigned to it. Otherwise, the handoff call is dropped. When a new call arrives, it is accepted provided that at least $g + 1$ idle channels are available in the channel pool; otherwise, the new call is blocked.

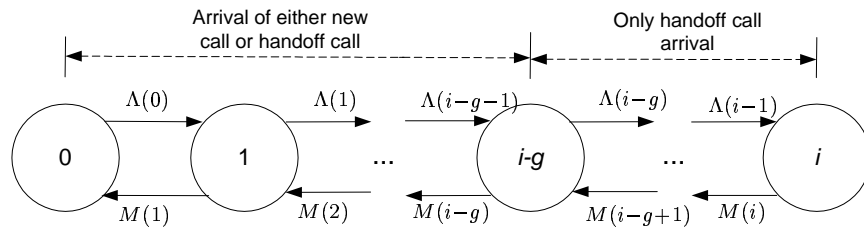


Figure 4: CTMC Performance Model of Wireless Handoff

The state-dependent arrival and departure rates in the birth-death process of Figure 4 are given by

$$\Lambda(j) = \begin{cases} \lambda_1 + \lambda_2, & \text{if } j < i - g \\ \lambda_2, & \text{if } i - g \leq j < i \end{cases} \quad (4)$$

and $M(j) = j(\mu_1 + \mu_2)$, $j = 1, 2, \dots, i$.

Let the steady-state probability of the CTMC being in state j be denoted by $\pi_j^{(l)}$.

Let $\lambda = \lambda_1 + \lambda_2$, $\mu = \mu_1 + \mu_2$, and $A = \frac{\lambda}{\mu}$, $A_1 = \frac{\lambda_2}{\mu_1 + \mu_2}$.

After finding $\pi_j^{(l)}$ [5], we can write the expression for the dropping probability for handoff calls $P_d^{(l)}(i)$ in the lower model.

$$P_d^{(l)}(i) = \pi_i^{(l)} = \frac{\frac{A^{i-g}}{i!} A_1^g}{\sum_{j=0}^{i-g-1} \frac{A^j}{j!} + \sum_{j=i-g}^i \frac{A^{i-g}}{j!} A_1^{j-(i-g)}}. \quad (5)$$

Similarly, the expression for the blocking probability of new calls $P_b^{(l)}(i)$ in the lower level model:

$$P_b^{(l)}(i) = \sum_{j=i-g}^i \pi_j^{(l)} = \frac{\sum_{j=i-g}^i \frac{A^{i-g}}{j!} A_1^{j-(i-g)}}{\sum_{j=0}^{i-g-1} \frac{A^j}{j!} + \sum_{j=i-g}^i \frac{A^{i-g}}{j!} A_1^{j-(i-g)}}. \quad (6)$$

To get the numerical measures for the whole system, the lower level performance model is solved and its results are passed as reward rates to the upper level availability model.

We denote respectively $P_d^{(t)}$ and $P_b^{(t)}$ as the total dropping and total blocking probability obtained from the performability model. The approximate dropping probability is obtained as

$$P_d^{(t)} = U + \sum_{i=1}^n (\pi_i \cdot P_d^{(l)}(i)), \quad (7)$$

The approximate blocking probability is obtained as

$$P_b^{(t)} = \sum_{i=0}^g \pi_i + \sum_{i=g+1}^n (\pi_i \cdot P_b^{(l)}(i)) \quad (8)$$

4 Hierarchical model for APS in a TDMA system [2]

4.1 Wireless cellular systems with failures

A TDMA system with hard handoff in which a cell has multiple base repeaters, say N_b , is considered in this model. Each base repeater provides a number of channels, say M , for mobile terminals to communicate with the system. Therefore a total of $N_b M$ channels are available when the whole system is working properly. Normally, one of the channels is dedicated to transmitting control channel messages. Such a channel is called a control channel. The total number of available talking channels is $N_b M - 1$. We also assume that the control channel is selected randomly out of $N_b M$ channels. Failure of the control channel will cause the whole system to fail. To avoid this undesirable situation, an automatic protection switching (APS) scheme is suggested in [20] so that the system automatically selects a channel from the rest of the available channels to substitute the failed control channel. If all channels are in use (talking), then one of them is forcefully terminated and is used as the control channel.

A cell as a whole is subject to failures which make all channels in it inaccessible, causing a full outage. In practice, this type of failure may occur when the communication links between base station controller and base repeaters do not function properly, or a critical function unit (such as base station controller) fails. In this model, we will refer to this

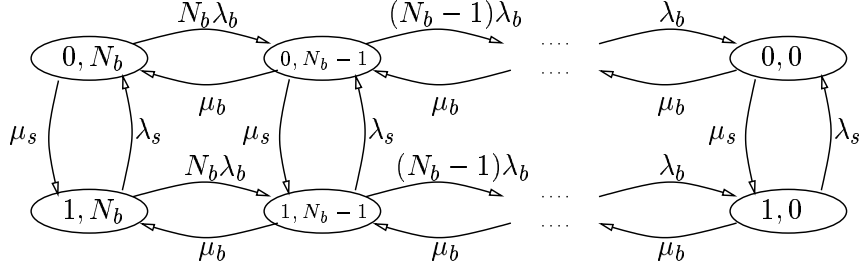


Figure 5: Markov chain of availability model

type of failure as the platform failure. Each base repeater is also subject to failure which disables the channels that it provides. In a system without APS, if a failed base repeater happens to be the one hosting the control channel, it results in a full outage, same as the situation caused by a platform failure.

We use the traditional two-level performability model: we first present an availability model which accounts for the failure and repair of base repeaters; second, we use a performance model to compute performance indices given the number of non-failed base repeaters; finally, we combine them together and give corresponding loss formulas.

4.2 The availability model

All failure events are assumed to be mutually independent. Times to platform failure and repair are assumed to be exponentially distributed with mean $1/\lambda_s$ and $1/\mu_s$, respectively. Also assume that times to base repeater failure and repair are exponentially distributed with mean $1/\lambda_b$ and $1/\mu_b$ respectively, and that a single repair facility is shared by all the base repeaters.

Let $s \in S = \{0, 1\}$ denote a binary value indicating whether or not the system is down due to a platform failure (0: system down due to a platform failure; 1: no platform failure has occurred). Also let $k \in B = \{0, 1, \dots, N_b\}$ denote the number of non-failed base repeaters. The 2-tuple (s, k) , $s \in S, k \in B$ defines a state in which the system is undergoing a (no) platform failure if $s = 0$ (if $s = 1$) and k base repeaters are up. The underlying stochastic process is a homogeneous CTMC with state space $S \times B$. Let $P(s, k; N_b)$ be the corresponding steady state probability. The state diagram of this irreducible CTMC is depicted in Figure 5.

Solving the CTMC, we have

$$P(s, k; N_b) = \begin{cases} \frac{\lambda_s}{\lambda_s + \mu_s} \frac{1}{k!} \left(\frac{\mu_b}{\lambda_b}\right)^k \left[1 + \sum_{j=1}^{N_b} \frac{1}{j!} \left(\frac{\mu_b}{\lambda_b}\right)^j\right]^{-1}, & \text{if } s = 0, \\ \frac{\mu_s}{\lambda_s + \mu_s} \frac{1}{k!} \left(\frac{\mu_b}{\lambda_b}\right)^k \left[1 + \sum_{j=1}^{N_b} \frac{1}{j!} \left(\frac{\mu_b}{\lambda_b}\right)^j\right]^{-1}, & \text{if } s = 1. \end{cases} \quad (9)$$

The system unavailability corresponds to all the states in which either the system has a platform failure that brings the whole system down, or in a system without APS, a base repeater hosting the control channel fails, or the system even without platform failure has no working base repeater left. For a system without APS, the probability that one of the $(N_b - k)$ failed base repeaters happens to host the control channel is $(N_b - k)/N_b$. Denote $U(N_b)$ as the steady state system unavailability. For both systems with and without APS,

we thus write unavailability as

$$U(N_b) = \begin{cases} \sum_{k=0}^{N_b} P(0, k; N_b) + \sum_{k=0}^{N_b} P(1, k; N_b) \frac{N_b - k}{N_b}, & \text{w/o APS} \\ \sum_{k=0}^{N_b} P(0, k; N_b) + P(1, 0; N_b), & \text{w/ APS.} \end{cases} \quad (10)$$

4.3 Performability

For each of the states of the availability model of Figure 6, Equation(5) and (6) in Section 3 provide performance indices given the number of non-failed channels.

We notice that calls can be blocked (or dropped) due to system *being down* or *being full*. The former type of loss is captured by the pure availability model while the latter type of loss is captured by the pure performance model. We now wish to combine the two types of losses. The primary vehicle for doing this is to determine pure performance losses for each of the availability model states and attach these loss probabilities as reward rates (or weights) to these states. Such a Markov reward model has been called a performability model. We list reward rates for the states of the availability model in Table 1 for systems

State (s, k)	Reward rate	
	New call blocking	Handoff call dropping
$(0, k)$, for $k = 0, \dots, N_b$	1	1
$(1, 0)$	1	1
$(1, k)$, for $k = 1, \dots, N_b$	1 , if $kM - 1 \leq g$ $\frac{N_b - k}{N_b} +$ $P_b^{(l)}(kM - 1) \frac{k}{N_b}$, o.w.	$\frac{N_b - k}{N_b} +$ $P_d^{(l)}(kM - 1) \frac{k}{N_b}$

Table 1: Reward rates for systems without APS

without APS and Table 2 for system with APS. Let us first consider states of system being down.

Clearly, for both systems without and with APS, a cell is not able to accept any new calls or handoff calls if it has platform failure which corresponds to the states $(0, k)$ for $k = 0, \dots, N_b$, or all base repeaters are down which corresponds to the state $(1, 0)$. Therefore, reward rates of both overall new call blocking and handoff call dropping are 1's.

In addition, for a system without APS, control channel may go down in states $(1, k)$ for $k = 1, \dots, N_b$ with probability $(N_b - k)/N_b$ and cause new call blocking and handoff call dropping. This corresponds to the rates with $(N_b - k)/N_b$ in the last row of Table 1. All cases mentioned above contribute to system unavailability, $U(N_b)$. Hence, system unavailability, $U(N_b)$, also consists of one of the parts of the overall new call blocking probability and handoff call dropping probability.

We now consider states in which the system is not undergoing a full outage caused by failures of platform, control channel (if system w/o APS) or all base repeaters being down.

The corresponding states are $(1, k)$ for $k = 1, \dots, N_b$. The total number of available channels in state $(1, k)$ is $kM - 1$. Thus, new call blocking probability and handoff call

State (s, k)	Reward rate	
	New call blocking	Handoff call dropping
$(0, k)$, for $k = 0, \dots, N_b$	1	1
$(1, 0)$	1	1
$(1, k)$, for $k = 1, \dots, N_b$	1, if $kM - 1 \leq g$ $P_b^{(l)}(kM - 1)$, o.w.	$P_d^{(l)}(kM - 1)$

Table 2: Reward rates for systems with APS

dropping probability in these states are $P_b^{(l)}(kM - 1)$ and $P_d^{(l)}(kM - 1)$, respectively. Thus, these probabilities are used as reward rates to these states for overall new call blocking and handoff call dropping.

For a system without APS, we note that the probability of not having the control channel down in state $(1, k)$ for $k > 0$ is k/N_b . Therefore, the reward rates, $P_b^{(l)}(kM - 1)$ and $P_d^{(l)}(kM - 1)$, are also weighted by k/N_b (shown in the last row of Table 1).

Also, in case that the number of idle channels is less than the number of guard channels, *i.e.*, $kM - 1 < g$ for states $(1, k)$, $k = 1, \dots, N_b$, a cell is not able to set up any new calls because all available channels are reserved for handoff calls. Hence, the reward rates for new call blocking assigned to the corresponding states are 1's.

Now let $G = \lfloor (g + 1)/M \rfloor$. Summarizing Table 1 and Table 2, the total call blocking probability can be written as the expected steady state reward rate,

$$\begin{aligned}
P_b^{(t)}(N_b) &= U(N_b) \\
&+ \begin{cases} \mathbf{1}(G > 0) \sum_{k=1}^G P(1, k; N_b) \left(\frac{k}{N_b} \right) \\ + \sum_{k=G+1}^{N_b} P(1, k; N_b) P_b^{(l)}(kM - 1) \left(\frac{k}{N_b} \right), & \text{w/o APS} \\ \mathbf{1}(G > 0) \sum_{k=1}^G P(1, k; N_b) \\ + \sum_{k=G+1}^{N_b} P(1, k; N_b) P_b^{(l)}(kM - 1), & \text{w/ APS} \end{cases} \quad (11)
\end{aligned}$$

where $\mathbf{1}(e)$ is the indicator function: $\mathbf{1}(e) = 1$ if expression e is true; $\mathbf{1}(e) = 0$, otherwise. Similarly the total handoff call dropping probability can be given as

$$\begin{aligned}
P_d^{(t)}(N_b) &= U(N_b) \\
&+ \begin{cases} \sum_{k=1}^{N_b} P(1, k; N_b) P_d^{(l)}(kM - 1) \frac{k}{N_b}, & \text{w/o APS} \\ \sum_{k=1}^{N_b} P(1, k; N_b) P_d^{(l)}(kM - 1), & \text{w/ APS.} \end{cases} \quad (12)
\end{aligned}$$

5 SRN models for wireless cellular systems with failures

In order to automate the generation and solution of large CTMCs or MRMs, a higher-level language is often desired, stochastic Petri net (SPN) and its derivatives are commonly used for this purpose.

5.1 Introduction to SRN

Stochastic reward net (SRN) is an extension of Petri net (PN), which is a high level description language for formally specifying complex systems. A PN is a bipartite directed graph with two types of nodes: places and transitions. Each place may contain an arbitrary (natural) number of tokens. For a graphical presentation, places are depicted as circles, transitions are represented by bars and tokens are represented by dots or integers in the places. Each transition may have zero or more input arcs, coming from its input places; and zero or more output arcs, going to its output places. A transition is enabled if all of its input places have at least as many tokens as the multiplicity of the corresponding input arc. When enabled, a transition can fire and will remove from each input place and add to each output place the number of tokens corresponding to the multiplicities of the input/output arcs. A marking depicts the state of a PN which is characterized by the assignment of tokens to all its places. With respect to a given initial markings, the reachability set is defined as the set of all markings, that are reachable through any possible firing sequences of transitions starting from the initial marking. To get the performance and reliability/availability measures of a system, appropriate reward rates are assigned to its SRN. As SRN is automatically transformed into a Markov reward model, steady state and/or transient analysis of the Markov reward model produces the required measures of the original SRN. In this paper, we use the tool SPNP [3] to specify and solve the SRN models.

5.2 Basic Hierarchical performability model

Here, we consider the same wireless cellular system as the system in Section 3. We build a two level SRN model for performability analysis. The upper level model, as shown in

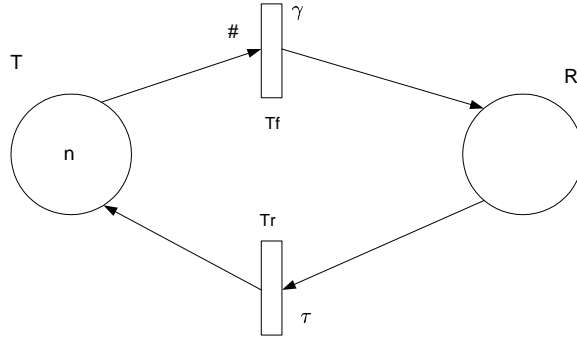


Figure 6: Upper level pure availability model

Figure 6, describes the failure and repair behavior of the system. The number of tokens in place T represents the number of channels that are currently non-failed in the cell. The number of tokens in place R represents the number of channels that have failed. Transition Tf with rate γ represents the failure of a channel while transition Tr with rate τ represents the repair of a channel.

The lower level SRN model, as shown in Figure 7, captures the pure performance aspect of the system. In Figure 7, $i \in [1, n]$. The number of tokens in place P_{talk} represents the number of channels that are occupied by either a new call or a handoff call. The firing of transition $T_{\text{new_call}}$ represents the arrival of new calls and the firing of transition $T_{\text{handoff_in}}$ represents the arrival of a handoff call from neighboring cells. A handoff call will be dropped only when all channels are occupied (i.e., $\#P_{\text{talk}} = i$). This is realized by an inhibitor arc from place P_{talk} to $T_{\text{handoff_in}}$ with multiplicity i . A new call, however, will

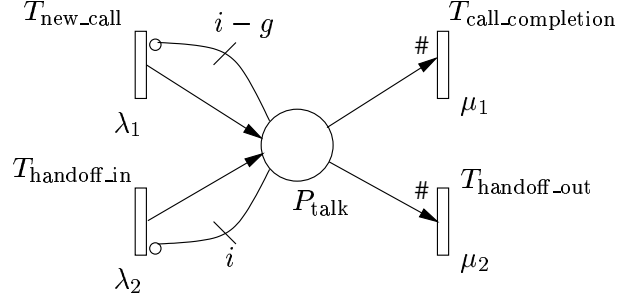


Figure 7: The SRN of lower level wireless loss model

be blocked if there are no more than g idle channels. This is simply reflected in the SRN by the inhibitor arc from place P_{talk} to transition $T_{\text{new_call}}$ with multiplicity $i - g$. The firings of transition $T_{\text{call_completion}}$ and $T_{\text{handoff_out}}$ represent the completion of a call and the departure of an outgoing handoff call, respectively. The rates of transitions $T_{\text{call_completion}}$ and $T_{\text{handoff_out}}$ are marking-dependent, as indicated by the two “#” symbols next to the transition.

Two steady-state measures are of interest from the SRN model of Figure 7, namely, the new-call blocking probability, $P_b^{(l)}(i)$, and the handoff-call dropping probability, $P_d^{(l)}(i)$. We obtain these two measures by computing the expected steady-state reward rate for the SRN model with the proper assignment of reward rates to the markings.

$$P_b^{(l)}(i) = \sum_{j \in \Omega} (r_b)_j \pi_j^{(l)} \quad (13)$$

$$P_d^{(l)}(i) = \sum_{j \in \Omega} (r_d)_j \pi_j^{(l)} \quad (14)$$

The reward rates to the marking j for the new-call blocking probabilities are

$$(r_b)_j = \begin{cases} 1, & \#P_{\text{talk}} \geq i - g \\ 0, & \#P_{\text{talk}} < i - g \end{cases}$$

and that for the handoff dropping probabilities are

$$(r_d)_j = \begin{cases} 1, & \#P_{\text{talk}} = i \\ 0, & \#P_{\text{talk}} < i. \end{cases}$$

Thus, we denote respectively $P_d^{(t)}$ and $P_b^{(t)}$ as the dropping and blocking probability in the performability model. The approximate dropping probability is obtained through

$$P_d^{(t)} = \pi_0 + \sum_{i=1}^n (\pi_i \cdot P_d^{(l)}(i)), \quad (15)$$

where π_i ($i \in [0, n]$) is the steady state probability of marking i for the upper level model.

The approximate blocking probability is obtained through

$$P_b^{(t)} = \sum_{i=0}^g \pi_i + \sum_{i=g+1}^n (\pi_i \cdot P_b^{(l)}(i)) \quad (16)$$

6 Conclusion

During the last decade we have witnessed a tremendous growth within the wireless communication industry. Customers want speed and improved performance, but only if it comes with reliable services. This requires fundamental rethinking of the traditional pure performance model that ignores failure, repair or recovery but mainly concentrates on resource contention. To reflect a real-world system in realistic way, availability, capacity and performance issues of a network should be considered in an integrated way.

In this paper, we have presented the CTMC, MRM and SRN models for performability study of a variety of wireless systems. By solving the two-level models, we can compute performability measures, such as call blocking probability and handoff call dropping probability, for wireless systems and wireless cellular systems with handoff, base repeaters, and control channels. Compared with composite models, the more robust and less time-consuming hierarchical models are proven to be able to provide high accuracy. It is expected that the models presented in this paper will be useful in wireless network design and operation.

Further study might include performability study in multi-media wireless system in the presence of multiple control channels and corresponding fault-tolerant protection schemes, and performability study in differentiate QoS services, IP wireless mobile system, and survivability of cellular systems.

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