Availability and Performance Evaluation for Automatic Protection Switching in TDMA Wireless System *

Hairong Sun, Yonghuan Cao and Kishor S. Trivedi
{hairong, ycao, kst@ee.duke.edu}
CACC, Department of ECE
Duke University, Durham, NC 27708

James J. Han
Corporate Software Technology Center
Motorola
Schaumburg, IL 60196-1065

Abstract

In this paper, we compare the availability and performance of a wireless TDMA system with and without automatic protection switching. Stochastic reward net models are constructed and solved by **SPNP** *(Stochastic Petri Net Package). Hierarchical decomposition is adopted to simplify the analysis. The optimization of the number of guard channels reserved for the handoff calls is studied. Numerical results prove the accuracy of the decomposition and show the significant improvement of the availability and the performance indexed as the new call blocking probability and handoff call dropping probability, brought by automatic protection switching.*

1 Introduction

The wireless communication networks are more subject to the channel impairment than wireline communication networks. The unreliability of communication stems from disconnects at the physical layer due to the handoffs of the mobile terminals, noise and interference, run-down batteries, blocking by buildings, and weak signals etc. In this paper, we consider a TDMA (Time Division Multiple Access) wireless system [11], where the base transceiver system of each cell has N base repeaters (also called base radio, BR in short), one controller and a local area network connecting these subsystems (see Fig. 1). Each base repeater provides M time-division-multiplexed channels. The cell reserves **one** channel for signaling transfer, which resides in one of N base repeaters. Therefore, there are $MN-1$ traffic channels and 1 control channel in this system. This architecture is general and can be used to describe the existing TDMA wireless systems, such as GSM (Global System for Mobile Communications) and USDC (United States Digital Cellular) [10].

*This work was supported in part by the NSF-Korea International Program under Grant No. NSF-INT-97-27221 and by a followship from Motorola Inc. to Duke University.

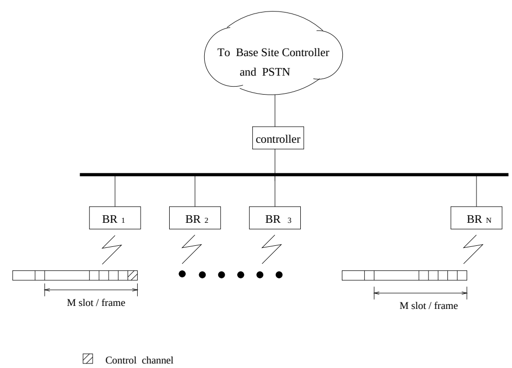


Figure 1. A Base Transceiver System for TDMA

There are several kinds of failures that may occur in the system:

- the controller or the local area network connecting the base repeaters and controller going down causing the system as a whole to go down. This event is called *sys_down* in this paper.
- the base repeater where the control channel resides going down causing the system as a whole to go down. This event is called *ctrl_down* in this paper.
- any other base repeater where the control channel does not reside going down does not cause the system as a whole to go down, but system is degraded (partially down). This event is called *br_down* in this paper.

The first two cases will cause the whole system to shut down. The last case will stop all the ongoing communications on the failed base repeater and reduce the capacity of the system. In such a TDMA system, the system availability is certainly a measure of importance. Two other performance metrics are of great interest as well, blocking probability of new calls and dropping probability of handoff call which conflict with each other due to the sharing of the common but limited channel resource between these two kinds of calls.

In order to increase the availability of the TDMA wireless system, a proposal is made in [4] to introduce Automatic Protection Switching (APS), which automatically switches the ongoing talks on the failed channels to the idle channels. This mechanism is extended in [5] to the system with multiple base repeaters, which allows the system to automatically switch all the ongoing calls on the failed base repeater to the working base repeaters. However, only traffic channels are considered in [4] and [5]. Evidently, it will complicate the analysis by also taking the control channel into account.

This paper studies the availability and performance of the TDMA wireless system with and without APS considering both traffic and control channels, not just focusing on performance of handoff and channel assignment which has been investigated extensively in the literature [6, 7, 8]. Stochastic reward net (SRN) models are constructed and solved by using **SPNP** (Stochastic Petri Net Package)[3]. Hierarchical models using decomposition techniques [2] are also provided to simplify the evaluation of availability and performance, which save significant computational effort. Another contribution of this paper is the optimization of the number of guard channels reserved for the handoff calls.

The organization of the paper is as follows. In Section 2, the SRN model and hierarchical model for TDMA wireless system *without* APS are presented. In Section 3, the SRN model and hierarchical model for TDMA wireless system *with* APS are given. Numerical results are discussed in Section 4. Conclusions are drawn in Section 5.

2 TDMA Wireless System *without* Automatic Protection Switching

2.1 SRN Model of System without APS

In order to quantify the impact of *br_down* on the system, we should know the number of ongoing talking channels on the failed base repeater when it fails. Suppose the channels are allocated randomly to the users. Denote the number of talking channels in the whole system as k, and the number of idle channels in the whole system as j ($j+k = MN-1$), when the failure *br_down* occurs. Then the probability, a_i, of i talking channels residing in the failed base repeater, is given by

- if $k < M$,

$$a_i = \begin{cases} 0, & i > k \\ \dfrac{\binom{M(N-1)-1}{k-i}\binom{M}{i}}{\binom{MN-1}{k}}, & i \le k \end{cases} \tag{1}$$

- if $M \le k < M(N-1)-1$,

$$a_i = \frac{\binom{M(N-1)-1}{k-i}\binom{M}{i}}{\binom{MN-1}{k}} \tag{2}$$

- if $k \ge M(N-1)-1$,

$$a_i = \begin{cases} 0, & i < M-j \\ \dfrac{\binom{M(N-1)-1}{j-M+1}\binom{M}{M-i}}{\binom{MN-1}{j}}, & i \ge M-j \end{cases} \tag{3}$$

The SRN model for a TDMA system without APS is depicted in Fig. 2. In words, we describe the model as follows:

Whenever there is one token in Place P_{on}, the system is working perfectly, without any outage or partial outage. Transition T_{sys-d} represents the occurrence of *sys_down*. When Transition T_{sys-d} fires, it removes the token from Place P_{on} and deposits one token in Place P_{sys_down}. Transition T_{sys-r} represents the corresponding repair. Upon its firing, Transition T_{sys-r} returns the token in Place P_{sys_down} to Place P_{on}.

The occurrence of *ctrl_down* event is represented by Transition T_{ctrl-d}. Upon its firing, Transition T_{ctrl-d} removes the token in Place P_{on} and deposits one token in Place P_{ctrl_down}. Transition T_{ctrl-r} represents the corresponding repair. Upon its firing, Transition T_{ctrl-r} returns the token from Place P_{ctrl_down} to P_{on}. The pair of transitions, T_{br-d} and T_{br-r}, with Place P_{br_down}, representing *br_down* failure and the following repair, operate in the same manner. We point out that, in this paper, *sys_down*, *ctrl_down* and *br_down* are assumed to be exclusive events. Thus, for example, when one base repeater is down, no other failures are assumed to occur until the failed base repeater has been repaired.

The number of tokens in Place P_{idle} represents the number of idle channels in the whole system, while the number of tokens in Place P_{talk} represents the number of talking channels in the whole system. As described before, $\#P_{idle} + \#P_{Talk} = MN - 1$ always holds for a working system.

Transition T_c represents an arrival of a new call, while Transition T_{hi} represents an arrival of a handoff call coming from outside. g guard channels are reserved for handoff calls. This means that when a new call arrives and finds less than g idle channels in the system, the call will be blocked. Handoff calls can only be dropped when all the channels are busy. This is recognized as an event of dropping handoff call. When one call terminates (or hands off) to the adjacent cell, Transition T_t (or T_{ho}) fires. The firing rates of transitions T_t and T_{ho} are proportional to the number of tokens in P_{talk}.

When either *sys_down* or *ctrl_down* occurs, i.e., the number of tokens in either Place P_{sys_down} or P_{ctrl_down} is 1, immediate transitions t_{d1} and t_{d2} fire, removing all the tokens in P_{idle} and P_{talk}, which is represented by the variable cardinalities of arcs respectively from P_{idle} to t_{d1}, and from P_{talk} to t_{d2}. When the system restores, t_{r1} fires, which removes all the tokens in P_{r1} indicating the repair for the system or the control channel, and returns $MN-1$ tokens to P_{idle}. In order to guarantee the correct action of the model, we assign t_{d1} and t_{d2} a higher priority over t_{r1}.

When *br_down* occurs, i and $M-i$ ($0 \leq i \leq M$) tokens will be removed from P_{talk} and P_{idle}, respectively, with probability a_i. This is realized with M immediate transitions, i.e., t_0 to t_M, each is assigned a probability to fire. When the failed base repeater restores, t_{r2} fires, which removes the token in P_{r2} and deposits M tokens to P_{idle}.

The firing rates, guards and probabilities of the transitions are listed in Table 1 and Table 2.

Note that the rate of T_{hi}, i.e., λ_h, is an **unknown** parameter. In order to solve the continuous time Markov chain underlying the SRN model, one additional constraint is needed. If we assume the cells in the whole system are homogeneous and the cell under study is generic, then the rate of T_{hi} should equal to the throughput of the transition T_{ho}. Using a fixed point iteration scheme [9], we can get λ_h.

The SRN model can be solved by using **SPNP** (Stochastic Petri Net Package), a software package for the automated generation and solution of Markovian stochastic systems developed by researchers in Duke University [3]. **SPNP** can calculate the throughput of each transition, which is used to find the fixed point. The blocking probability for the new calls and dropping probability for the handoff calls can be obtained by defining reward functions in the SRN model.

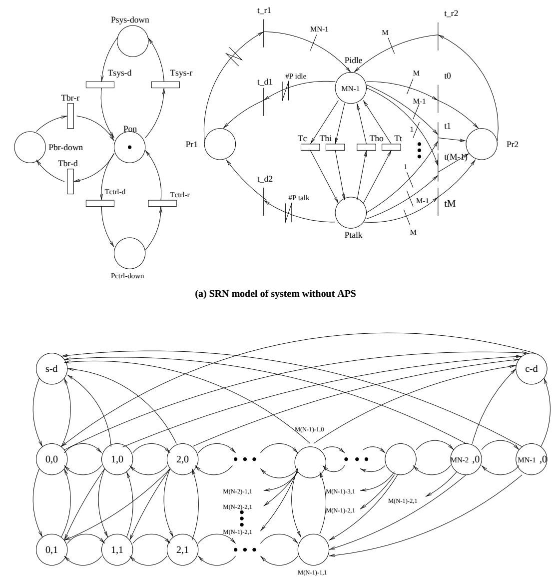
Table 1. Rates for Timed Trans. in Fig. 2

Transition	Firing rate	Guard function
T_{sys-d}	λ_s	
T_{sys-r}	μ_s	
T_{ctrl-d}	λ_{br}	
T_{ctrl-r}	μ_{br}	
T_{br-d}	$(N-1)\lambda_{br}$	
T_{br-r}	μ_{br}	
T_c	λ_c	$\#P_{idle} > g$
T_{hi}	λ_h	
T_{ho}	$\mu_h(\#P_{talk})$	
T_t	$\mu_t(\#P_{talk})$	

2.2 Markov Chain of System without APS

Fig. 2 also illustrates the corresponding continuous time Markov chain of the SRN model.

Let *s_d* be the state of the system after the occurrence of *sys_down* and let *c_d* be the state after the occurrence of



(a) SRN model of system without APS

(b) Underlying Markov Chain of the above SRN model

Figure 2. SRN and Markov Chain for System w/o APS

Table 2. Guards and Prob. for Immed. Trans. in Fig. 2

Transition	Guard	Probability
t_{r1}	$\#P_{on} == 1$	1
t_{r2}	$\#P_{on} == 1$	1
t_{d1}	$(\#P_{ctrl\_down} == 1$ or $\#P_{sys\_down} == 1)$ and $\#P_{r1} == 0$	1 1
t_{d2}	$(\#P_{ctrl\_down} == 1$ or $\#P_{sys\_down} == 1)$ and $\#P_{r1} == 0$	1
t_0 to t_M	$\#P_{br\_down} == 1$ and $\#P_{r2} == 0$	a_0 to a_M

ctrl_down. In state $(k,0)$, the system is working perfectly, with k busy traffic channels. In state $(k,1)$, one of the base repeaters where the control channel does not reside is down while k traffic channels are busy.

We denote P_{s_d}, P_{c_d}, $P_{k,0}$ and $P_{k,1}$ as the steady-state probability of the system being in state *s_d*, *c_d*, $(k,0)$ and $(k,1)$, respectively. The availability of the system, A, is defined as the probability of the system being working, including the case that one base repeater is down if the control channel does not reside in the failed base repeater. Then

$$A = 1 - P_{s\_d} - P_{c\_d} \tag{4}$$

The blocking probability for the new calls, P_b, is given by

$$P_b = \sum_{n=MN-g}^{MN-1} P_{n,0} + \sum_{n=M(N-1)-g}^{M(N-1)-1} P_{n,1} + P_{s\_d} + P_{c\_d} \tag{5}$$

The dropping probability for the handoff calls, P_d, is given by

$$P_d = P_{MN-1,0} + P_{M(N-1)-1,1} + P_{s\_d} + P_{c\_d} \qquad (6)$$

2.3 Hierarchical Model of System without APS

We notice that failure and repair events are rather rare compared with the new call and handoff call arrival and departure. Consequently, we can use hierarchical decomposition [2] to obtain an approximate solution. The model in Fig. 2 can be decomposed hierarchically, with the higher level focusing on availability (see Fig. 3), where state 0 and 1 are the aggregation of states $(k, 0)$ $(0 \le k \le MN-1)$ and $(k, 1)$ $(0 \le k \le MN-1)$ respectively, and the lower level focusing on performance (see Fig. 4).

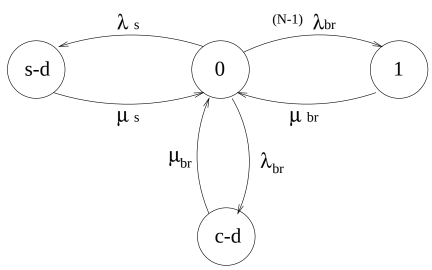


Figure 3. Availability Model for System w/o APS

Solving the Markov chain in Fig. 3, we can get

$$\begin{cases} P_0 = 1/(1+\lambda_s/\mu_s + N\lambda_{br}/\mu_{br}) \\ P_1 = \frac{(N-1)\lambda_{br}}{\mu_{br}} P_0,\ P_{s\_d} = \frac{\lambda_s}{\mu_s} P_0,\ P_{c\_d} = \frac{\lambda_{br}}{\mu_{br}} P_0, \end{cases} \qquad (7)$$

then the approximate solution of the availability is given as

$$A = P_0 + P_1 = \frac{1 + (N-1)\lambda_{br}/\mu_{br}}{1 + \lambda_s/\mu_s + N\lambda_{br}/\mu_{br}} \qquad (8)$$

When the system is either perfectly working or working but having one base repeater down, the behavior of the system can be described by a birth-death process (see Fig. 4). We denote n as the total number of available channels. Clearly, $n = MN-1$, when the system is perfectly working; and $n = M(N-1)-1$, if the system is partially working having one base repeater down. We also denote $P_{b,0}$, and $P_{d,0}$ as the blocking probability of new calls and handoff calls when the system is perfectly working, while $P_{b,1}$ and $P_{d,1}$ as the blocking probability of new calls and handoff calls when the system is partially working with one base repeater down.

Solving the Markov chain, the blocking probability for the new calls and handoff calls are given by

$$P_b(n) = \frac{\sum_{k=n-g+1}^{n} a_k}{1 + \sum_{k=1}^{n} a_k} \qquad (9)$$

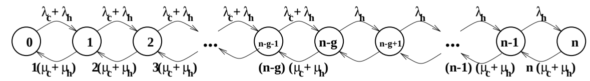


Figure 4. Birth-death process: $n = MN-1$ (system working) or $n = M(N-1)-1$ (one base repeater down)

and

$$P_d(n) = \frac{a_n}{1 + \sum_{k=1}^{n} a_k} \qquad (10)$$

where

$$a_k = \begin{cases} \frac{1}{k!}\left(\frac{\lambda_c+\lambda_h}{\mu_t+\mu_h}\right)^k, & 1 \le k \le n-g \\ \frac{1}{k!}\frac{(\lambda_c+\lambda_h)^{n-g}\lambda_h^{k-n+g}}{(\mu_t+\mu_h)^k}, & n-g+1 \le k \le n, \end{cases} \qquad (11)$$

As stated previously, the rate of incoming handoff calls, λ_h, is unknown in the model. We again apply a fixed point scheme to determine the parameter as we do in the exact SRN model. Under the assumption that cells of the system are homogeneous and the cell under study is generic, the incoming handoff rate equals to the throughput of outgoing handoffs. Given the new calls arrival rate λ_c, the handoff out rate μ_h and the call termination rate of calls μ_t, the fixed point equation to seek the handoff arrival rate, λ_h, is given by

$$\lambda_h = \mu_t \frac{\sum_{k=n-g+1}^{n} a_k}{1 + \sum_{k=1}^{n} a_k} \qquad (12)$$

By definition, we have

$$P_{b,0} = P_b(MN-1), \qquad (13)$$

$$P_{d,0} = P_d(MN-1), \qquad (14)$$

$$P_{b,1} = P_b(M(N-1)-1), \qquad (15)$$

$$P_{d,1} = P_d(M(N-1)-1). \qquad (16)$$

Then, the total blocking probability is given by assigning reward rates to states 0, 1, *c_d* and *s_d*, i.e., assigning a reward rate of 1 to *c_d* and *s_d*, $P_{b,0}$ to 0 and $P_{b,1}$ to 1.

$$P_b = P_0 P_{b,0} + P_1 P_{b,1} + P_{s\_d} + P_{c\_d} \qquad (17)$$

The total dropping probability is given by

$$P_d = P_0 P_{d,0} + P_1 P_{d,1} + P_{s\_d} + P_{c\_d} \qquad (18)$$

3 TDMA Wireless System *with* Automatic Protection Switching

With APS, when *br_down* occurs, the talking channels on the failed base repeater will be switched to other working base repeaters if excess idle channels are available. If the number of idle channels is not less than M when the base repeater failure occurs, then all the talking channels can be switched to the working base repeaters. Note that the talking channels can be switched to different base repeaters. If the number of idle channels, j, is less than M when the base repeater failure occurs, then (1) some of the talking channels can not be switched to the working base repeaters, (2) all the channels are busy after APS, and (3) $M - j$ ongoing calls will be dropped.

When *ctrl_down* occurs , the control channel will be switched to other base repeater even if there are no spare channels available, i.e., one ongoing talk will be preemptively dropped to make room for the control channel.

3.1 SRN Model of System with APS

The SRN model in Fig. 2 is modified as follows: (See Fig. 5)

- because *br_down* and *ctrl_down* have the same effect on the system performance, we aggregate them together as T_{p-d} with rate $\lambda_{ctrl} + \lambda_{br}$. The corresponding repair rate equals to μ_{br}.
- when *sys_down* occurs, immediate transitions t_{d1} and t_{d2} fire, removing all the tokens from P_{idle} and P_{talk}. When the system restores, t_{r1} fires, removing all the tokens from P_{r1} and depositing $MN - 1$ tokens to P_{idle}.
- when *br_down* or *ctrl_down* occurs, and the number of idle channels is not less than M, M tokens will be removed from P_{idle}. When *br_down* or *ctrl_down* occurs, and the number of idle channels is less than M, $M - l$ and l tokens will be removed from P_{idle} and P_{talk}, respectively. When the failed base repeater restores, t_{r1} fires and removes the token in P_{r1} and deposits M tokens in P_{idle}.

The expression of l is given by

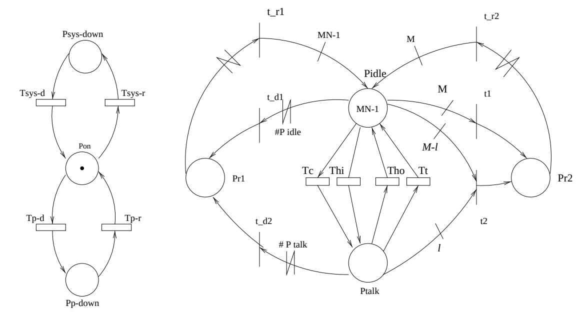
$$l = M - \#P_{idle} \tag{19}$$

The guards and firing probabilities of the immediate transitions in Fig. 5 are listed in Table 3.

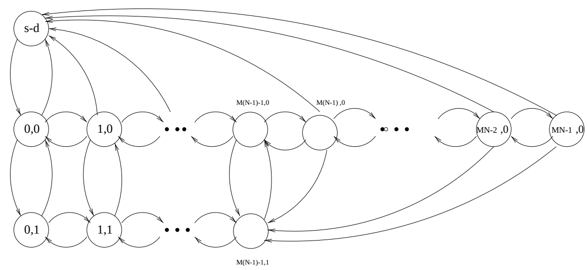
The blocking probability for the new calls and dropping probability for the handoff calls is obtained by defining reward functions on the SRN model.

3.2 Markov Chain of System with APS

Fig. 5 also illustrates the state diagram of the underlying continuous time Markov chain. State *c_d* in Fig. 2 does not exist in Fig. 5, because the base repeater where the control



(a) SRN model of the system with APS



(b) The underlying Markov Chain of the above model

Figure 5. SRN and Markov Chain for System w/ APS

channel resides shutting down does not cause the whole system down. State $(k, 0)$ represents that the system is working perfectly, with k busy traffic channels. State $(k, 1)$ represents that one of the base repeaters is down while k traffic channels are busy.

3.3 Hierarchical Model of System with APS

The model in Fig. 5 can be decomposed hierarchically with the higher level focusing on availability (see Fig. 6), where state 0 and 1 are the aggregation of states $(k, 0)$ $(0 \le k \le MN - 1)$ and $(k, 1)$ $(0 \le k \le MN - 1)$ respectively, and the lower level focusing on performance.

Solving the Markov chain in Fig. 6, we have

$$\begin{cases} P_0 = 1 \Big/ \left(1 + \frac{\lambda_s}{\mu_s} + \frac{N\lambda_{br}}{\mu_{br}}\right), \\ P_1 = P_0 \frac{N\lambda_{br}}{\mu_{br}}, \quad P_{s\_d} = P_0 \frac{\lambda_s}{\mu_s}. \end{cases} \tag{20}$$

The system availability is given as,

$$A = P_0 + P_1 = \frac{1 + N\lambda_{br}/\mu_{br}}{1 + \lambda_s/\mu_s + N\lambda_{br}/\mu_{br}}. \tag{21}$$

Applying the same derivation as in the preceding section, the total blocking probability is given by

$$P_b = P_0 P_{b,0} + P_1 P_{b,1} + P_{s\_d} \tag{22}$$

Table 3. Guards and Prob. for Immed. Trans. in Figure 4

Transition	Guard	Probability
t_{r1}	$\#P_{on} == 1$	1
t_{r2}	$\#P_{on} == 1$	1
t_{d1}	$(\#P_{ctrl\_down} == 1$ or $\#P_{sys\_down} == 1)$ and $\#P_{r1} == 0$	1
t_{d2}	$(\#P_{ctrl\_down} == 1$ or $\#P_{sys\_down} == 1)$ and $\#P_{r1} == 0$	1
t_1	$\#P_{p-down} == 1$ and $\#P_{r2} == 0$ and $\#P_{idle} \geq M$	1
t_2	$\#P_{p-down} == 1$ and $\#P_{r2} == 0$ and $\#P_{idle} < M$	1

where $P_{b,0}$ and $P_{b,1}$ are given by (13) and (15).The total dropping probability is given by

$$P_d = P_0 P_{d,0} + P_1 P_{d,1} + P_{s\_d} \qquad (23)$$

where $P_{d,0}$ and $P_{d,1}$ are given by (14) and (16).

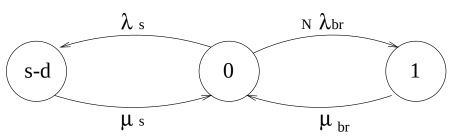


Figure 6. Availability Model for System w/ APS

4 Numerical Results

We report numerical results in this section. We consider a TDMA wireless system having 24 base repeaters with 8 RF channels available on each. Table 4 summarizes parameters used.

Parameter	Meaning	Value
N	Number of base repeaters	24
M	Number of channels on each base repeater	8
λ_s	System failure rate	1/year
μ_s	System repair rate	1/(4 hours)
λ_{br}	Single BR failure rate	1/year
μ_{br}	Single BR repair rate	1/(2 hours)

Table 4. Parameters used in numerical study

Before any further exploration, it is worthwhile to discuss fixed point iterations involved in both the exact SRN models and the hierarchically decomposed models to determine the rate of incoming handoff call, λ_h. The existence of fixed point can be proved by following the general procedure suggested in [9]. In our particular models, the proof of existence can be even simpler: we are dealing with iteration $\lambda_h = T(\lambda_h)$ as defined in (12), where T is the thoughput of outgoing handoffs. $T(\lambda_h) = \mu_h E[\#Talk]$, where $E[\#Talk]$ is the expected number of talking channels. It is obvious that $E[\#Talk] \leq n$ given n is the total number of available channels. Now, continuity of $G(\lambda_h)$ is guarenteed by the proof in [9]. Also, $G(\lambda_h)$ is bounded: $0 < G(\lambda_h) < n$, for $\lambda_h > 0$. So, there must be a positive fixed point existing for the equation. Although the proof of the uniqueness of the solutions remains open, experiments have not revealed cases with more than one feasible solution.

In what follows, we first confirm that the accuracy of our hierarchical models is high enough to be used to quickly conduct the evaluations of system availability and performance. We then report (1) the improvement of system availability and performance gained by introducing the APS mechanism; (2) the effect of increasing the number of reserved handoff channels; and (3) the optimal g for a weighted objective function of new call blocking probablity and handoff dropping probabilty.

4.1 Accuracy of hierarchical models

To confirm the accuracy of our hierarchical models, we compare system availabilities A, new call blocking probabilities P_b and handoff dropping probabilities P_d obtained from the SRN models and the corresponding hierarchical models with different sets of parameters. Observe from Table 5 (for $\mu_t = 1/5.0$ and $\mu_h = 1/2.5$) that availabilities obtained from SRN models and hierarchical models match very well, providing a very small relative error of less than (2e-4)%. We also observe that new call blocking probabilities P_b and handoff dropping probabilities P_d solved from SRN models and hierarchical models show very small mismatch, a relative error of less than 2.2% for all cases tested.

Parameters	APS	A (SPNP)	A (Hiera)	Δ (%)
$\lambda = 15, 30, 45$	no	0.9993191	0.9993191	-
$\lambda = 15, 30, 45$	yes	0.9995460	0.9995448	1.2e-4

Table 5. Availabilities from SRN and hierarchical models

The proven high accuracy of hierarchical model with respect to accurate SRN models allows us to carry out a variety of tests with much higher computational efficiency. The following study is therefore conducted by solving these hierarchical models.

4.2 Performability Improvement by APS

4.2.1 System Availability

We notice that system availability of our models is a function of the following parameters: N (total number of BRs),

λ_s, μ_s, λ_s, and μ_{br}. The improvement of availability by APS is achieved by eliminating the *ctrl_down* outage. Table 5 shows the availability gain of (2.257e-04) due to APS, a reduction of 118.6 minutes down time per year.

4.2.2 Blocking probability of new calls and Dropping probability of Handoff calls

In this section, we study the effectiveness of APS in terms of reduction in new call blocking probability P_b and the handoff dropping probability P_d. We evaluate performance with $\mu_t = 1/2.5$ (*i.e.*, the average holding time of a new call is 2.5 minutes), $\mu_h = 1/1.25$ (*i.e.*, outgoing handoff happens at a rate of once per 1.25 minutes). In Figure 7, we plot P_b and P_d of systems with and without APS against new call arrival rate. We find that (I) Both P_b and P_d in systems with and without APS are monotonically increasing functions with respect to new call arrival rates, λ_c. In the tests with the given parameters, both probabilities increase but stay nearly flat when new call traffic is low (within 10 - 60 calls/minute). The probabilities then increase sharply after λ_c exceeds 60 calls/minute; (II) Improvement by APS is seen as reductions of P_b and P_d. Improvement remains steady (a $> 30\%$ relative reduction of both P_b and P_d) given low traffic but diminishes rapidly as traffic becomes heavier.

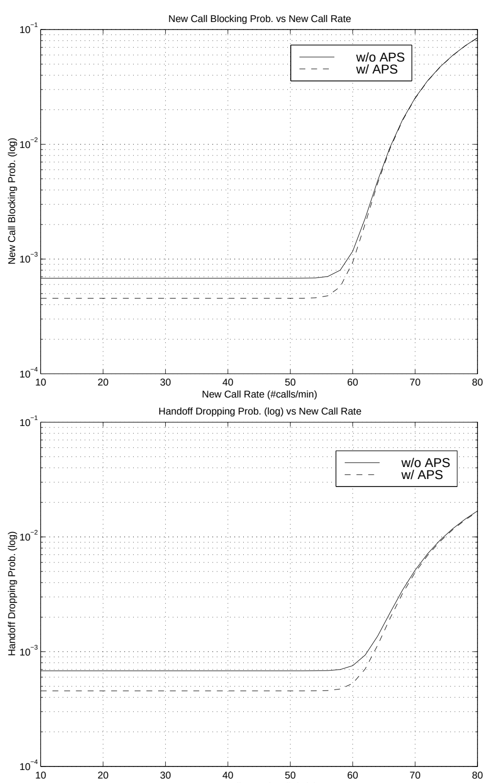


Figure 7. P_b (top) and P_d (bottom): Improvement by APS

4.3 Effect of channel reservation for handoff calls: g

Intuitively, it is expected that increasing the number of channels reserved for handoff calls, g, would decrease handoff drops (P_d) with the price of the increased new call blockings (P_c). It is therefore a trade-off for a network designer to decide the number of reserved channels. The models proposed enable us to provide quantitative results which are practically useful for such a decision. We visualize the effect of increasing g on P_b and P_d in Fig. 8. P_b and P_d in logarithmic scale for different new call arrival rates (20, 40, 60 and 80 calls/minute) are plotted against g. It is indeed interesting to note that, under high traffic condition, sacrificing a small portion of new calls may bring a noticeable reduction in handoff dropping.

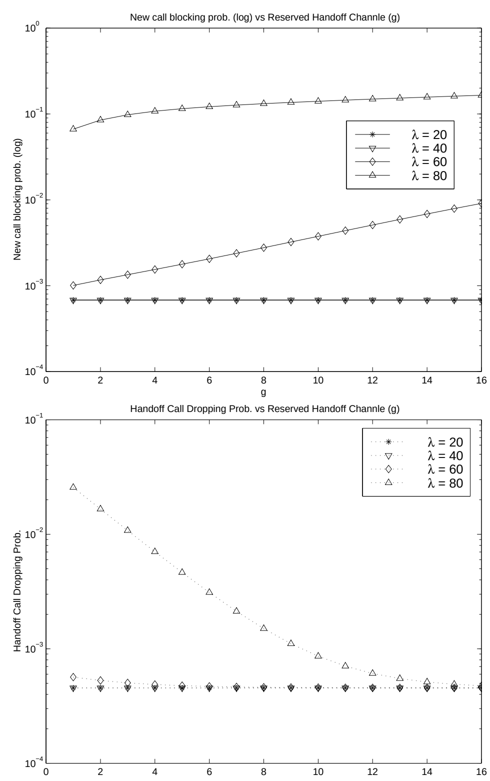


Figure 8. P_b (top) and P_d (bottom) vs. g (w/ APS)

4.4 Optimization of g

We have pointed out in the introduction and shown quantitatively that because incoming new calls and handoff calls share the common channels of a limited number, new call blocking probability, P_b, and handoff call dropping probability, P_d, are conflicting with each other. It is therefore of great interest to introduce an objective function in terms of both probabilities and then to optimize it by varying g.

From a system designer's point of view, such an objective function should encompass the trade-off between these two quantities. A handy objective function is in the form of a weighted function, $G(g) = \alpha P_b + (1 - \alpha) P_d, \ \ 0 \le \alpha \le 1$. Here $\alpha/(1 - \alpha)$ is the relative importance of handoff dropping probability over new call blocking probability. Generally, $\alpha < 0.5$. The problem now is to find g such that the weighted function is minimized with other system parameters given. As an example, we plot $G(g)$ for system with APS given an incoming new call rate of 60 calls/minutes, a call duration of 2.5 minutes and an outgoing handoff rate of once per 1.25 minutes. We find that for certain values of α, the objective function, $G(g)$, is a concave function. Minima of $G(g)$ can be found for different valuess of α as demonstrated in the plots in Fig. 9.

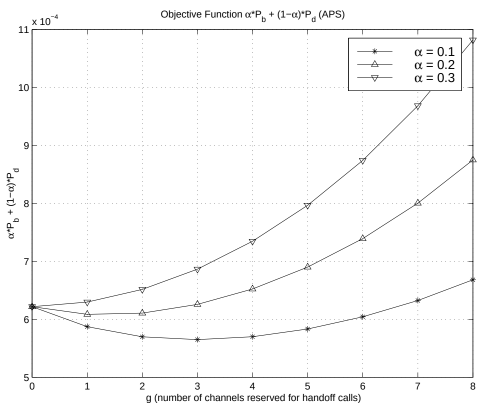


Figure 9. $G(g) = \alpha P_b + (1 - \alpha) P_d$ **(w/ APS)**

5 Conclusion

Composite SRN models are built to study the performability of wireless TDMA system with/without automatic protection switching. In order to simplify the analysis, the models are decomposed hierarchically. From this paper, it is concluded that the automatic protection switching can enhance the availability of the wireless TDMA system significantly. It also improves the performance in terms of new call blocking probability and handoff call dropping probability. The effect of increasing the number of reserved channels for handoff calls is also investigated. A related optimization problem has also been raised by introducing a weighted objective function of new call blocking probability and handoff call dropping probability. Minimum of the objective function can be found by varying the number of reserved channels for handoff calls. Further studies include evaluating the performability of wireless TDMA system with multiple control channels and integrated services.

References

[1] R. Sahner, K. S. Trivedi, A. Puliafito, *Performance and Reliability Analysis of Computer Systems: An Example-Based Approach Using the SHARPE Software Package*, Kluwer Academic Publishers, 1995.

[2] K. S. Trivedi, J. Muppala, S. Woolet, Boudewijn R. Haverkort, "Composite Performance and Dependability Analysis," *Performance Evaluation*, Vol. 14, Nos. 3-4, pp. 197-216, February 1992.

[3] G. Ciardo, J. Muppala and K. Trivedi, "SPNP: Stochastic Petri Net Package," *Intl. Conf. on Petri Nets and Perf. Models*, Kyoto, Japan, December 1989.

[4] Y. Ma, C. W. Ro, K. S. Trivedi, "Performability Analysis of Channel Allocation with Channel Recovery Strategy in Cellular Networks", *ICUPC '98*, Oct. 5-9, 1998, Florence, Italy.

[5] Y. Ma, James J. Han, K. S. Trivedi, "A Channel Recovery Method for RF Channel Failure in TDMA Wireless Systems", *Proc. of 50th IEEE Intl. Vehi. Tech. Conf.*, Amsterdam, Netherland, Sept. 1999.

[6] S. Tekinay, B. Jabbari, "Handover Policies and Channel Assignment Strategies in Mobile Cellular Networks", *IEEE Comm. Mag.*, Vol. 29, No. 11, pp.42-46, 1991.

[7] I. Katzela, M. Naghshineh, "Channel Assignment Schemes for Cellular Mobile Telecommunication System: A Comprehensive Survey", *IEEE Personnal Comm.*, Vol.3, pp.10-31, June, 1996.

[8] D. Hong, S. S. Rappaport, "Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Non-prioritized Handoff Procedures", *IEEE Trans. on Vehi. Tech.*, Vol.35, No.3, pp.77-92, 1986.

[9] V. Mainkar, K. S. Trivedi, "Sufficient Conditions for the Existence of a Fixed Point in Stochastic Reward Net-Based Iterative Models," *IEEE Trans. on Soft. Eng.*, Vol. 22, No. 9, Sept. 1996, pp. 640-653.

[10] J. E. Padgett, C. G. Gunther, T. Hattori, "Overview of wireless personal comunications," *IEEE Comm. Mag.*, pp.28-41, January, 1995.

[11] D. D. Falconer, F. Adachi, B. Gudmundson, "Time division multiple access methods for wireless personal communications," *IEEE Comm. Mag.*, pp.50-57, January, 1995.