Performability Analysis of TDMA Cellular Systems Based on Composite and Hierarchical Markov Chain Models

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Abstract. Composite Markov chain models are built to study the performability of wireless TDMA system with and without automatic protection switching for the control channel. The models are then decomposed hierarchically. Measures of interest are explicitly given in closed form for the approximate hierarchical models, that are proven to be more robust and less time-consuming without losing accuracy. The presented models are of great interest in wireless network design and operation.

1 Introduction

With the increasing popularity of cellular communications systems nowadays, customers are expecting the same level of service, availability and performance from the wireless communications systems as provided by the traditional wire-line networks. The high degree of mobility enjoyed in wireless networks is in turn the cause of inherent unreliability. Compared with wired networks, wireless networks need to deal with disconnects due to the handoffs, noise and interference, blocked and weak signals and run-down batteries, etc. In addition, the performance and availability of a wireless system is affected by the outage-and-recovery of its supporting function units. Unplanned as well as planned outages of the equipments also contribute to the degradation of the system's availability and performance. From the designer and operator's point of view of the wireless network, it would be of great importance to take these factors into account integratively. Thus, a comprehensive model accounting for both performance and availability would be very useful in network design and operation. This motivates us to study performability of wireless systems.

In this paper, we consider a typical cellular system. Such a system consists of several operational areas, called *cells*. Cells are assumed to be statistically identical in this study. A cell has multiple base repeaters. Each base repeater provides a number of time-division-multiplexed channels for mobile stations to communicate with the system. Normally, one of the channels is dedicated to transmitting control messages. Such a channel is called the *control channel*.

Failure of the control channel will cause the whole system to fail. To avoid this undesirable situation, an automatic protection switching (APS) scheme is suggested in [1] so that the system automatically selects a channel from the rest of the available channels to substitute the failed control channel. If all channels are in use (talking), then one of them is forcefully terminated and is used as the control channel.

Two kinds of calls may arrive to a cell: *new calls* and *handoff calls* (from neighboring cells). A call is accepted only when the cell can find a channel not in use; otherwise, the call is dropped. Since dropping handoff calls is considered less desirable than blocking a new call, a guard channel scheme (GCS) ([2–4]) that reserves a number of channels for handoff calls is often used. Changing the number of guard channels results in different new call blocking probabilities and handoff call dropping probabilities.

A cell as a whole is subject to failures which will make all channels inaccessible, causing a full outage. In practice, this type of failures may occur when the communication links between base station controller and base repeaters do not function properly, or critical function units (such as base station controller) fail. In this paper, we will refer to this type of failure as the *platform failure*. Each base repeater is also subject to failure which disables the channels that it provides. In a system without APS, if a failed base repeater happens to be the one hosting the control channel, it results in a full outage, same as the situation caused by a platform failure. In this paper, we consider both systems: with and without APS. If APS is implemented, a failure of the base repeater hosting the control channel will only cause a partial outage. Readers interested in the impact of the control channel recovery scheme are encouraged to consult our previous work [1] based on stochastic Petri net models.

Performance analysis of the system has been carried out by several authors ([4,1,5]). In particular, [5] derived the "wireless Erlang-B formulae" and revealed their important properties. However, pure performance analysis tends to be optimistic since it ignores the failure-repair or transient failure-recovery behavior in the wireless communication networks. In this paper, our objective is to present comprehensive yet analytically tractable performability models in which not only performance but also availability are considered. We will first present accurate composite Markov chain models. We then apply decomposition technique [6] and use a two-level hierarchical model to approximate the composite models.

The remainder of the paper is organized as follows. In Section 2, the system specification is made for the performability analysis of the TDMA wireless system. In Section 3, exact composite models are developed for cellular systems with and without APS. In Section 4, the corresponding hierarchical models are presented to approximate the composite models. The numerical results from both the exact and approximate models are presented in Section 5. Section 6 concludes the paper.

2 Model Description

We consider a cell with N_b base repeaters. Each base repeater has M channels. Therefore, a total number of N_bM channels are available when the whole system is working properly. Since one of the channels has to be used as the control channel, the total number of available talking channels is $N_bM - 1$. We also assume that the control channel is selected randomly out of N_bM channels.

A channel can be either idle or occupied (talking). As mentioned earlier, when a handoff call arrives and an idle channel is available in the channel pool, the call is accepted and a channel is assigned to it. Otherwise, the handoff call is dropped. Let g be the number of guard channels. When a new call arrives, it is accepted if $g + 1$ or more idle channels are available in the channel pool; otherwise, the new call is blocked. We assume that the arrival stream of new calls and handoff calls are independent Poisson processes with rates λ_1 and λ_2, respectively. Ongoing call (new or handoff) completion times are exponentially distributed with parameter μ_1 and the time at which the mobile station engaged in the call departs the cell are exponentially distributed with parameter μ_2 which is independent of call completion times.

All failure events are assumed to be mutually independent. Times to platform failures and repair are assumed to be exponentially distributed with mean $1/\lambda_s$ and $1/\mu_s$, respectively. Also assumed is that times to base repeater failures and repair are exponentially distributed with mean $1/\lambda_b$ and $1/\mu_b$ respectively, and that a single repair facility is shared by all the base repeaters.

The underlying stochastic process is thus a homogeneous continuous time Markov chain (CTMC), which we describe in the next section.

3 Exact Composite Model

In order to develop the Markov chain of the system, we need to know the distribution of the number of ongoing talking channels on a base repeater when it fails. Consider b $(0 < b \leq N_b)$ base repeaters available in the system. Let i denote the number of talking channels on the failing base repeater. Also let k denote the number of talking channels in the system, and for expository simplicity, we also use j $(j + k = bM - 1)$, the number of idle channels in the system. Assuming the channels are allocated randomly to arriving calls, we give the probability of i talking channels residing in the failing base repeater in the following two cases:

1. If the failing base repeater is not hosting the control channel, the probability, a_i, that i $(0 \leq i \leq M)$ talking channels are on the failing base repeater, is given as follows.

- if $k < M$,

$$a_i = \begin{cases} 0, & \text{if } i > k \\ \dfrac{\binom{(b-1)M-1}{k-i}\binom{M}{i}}{\binom{bM-1}{k}}, & \text{if } i \le k \end{cases} \tag{1}$$

- if $M \le k < (b-1)M - 1$,

$$a_i = \frac{\binom{(b-1)M-1}{k-i}\binom{M}{i}}{\binom{bM-1}{k}} \tag{2}$$

- if $k \ge (b-1)M - 1$,

$$a_i = \begin{cases} 0, & \text{if } i < M - j \\ \dfrac{\binom{(b-1)M-1}{j-M+i}\binom{M}{M-i}}{\binom{bM-1}{j}}, & \text{if } i \ge M - j \end{cases} \tag{3}$$

2. If the failing base repeater is hosting the control channel, the probability, a'_i, that i $(0 \le i \le M-1)$ talking channels are on the failing base repeater, is given in the similar way.
   - if $k < M - 1$,

$$a'_i = \begin{cases} 0, & \text{if } i > k \\ \dfrac{\binom{(b-1)M}{k-i}\binom{M-1}{i}}{\binom{bM-1}{k}}, & \text{if } i \le k \end{cases} \tag{4}$$

   - if $M - 1 \le k < (b-1)M - 1$,

$$a'_i = \frac{\binom{(b-1)M}{k-i}\binom{M-1}{i}}{\binom{bM-1}{k}} \tag{5}$$

   - if $k \ge (b-1)M - 1$,

$$a'_i = \begin{cases} 0, & \text{if } i < M - 1 - j \\ \dfrac{\binom{(b-1)M}{j-M+i}\binom{M-1}{M-i}}{\binom{bM-1}{j}}, & \text{if } i \ge M - 1 - j \end{cases} \tag{6}$$

3.1 System without APS

In a system without APS, if a base repeater fails and this base repeater happens to be hosting the control channel, then this failure will cause the whole system to go down. Let c denote this state in which the control channel is down. Also let s denote the state in which the system is down due to a platform failure.

We use a 2-tuple (b, k) to represent a state in which there is no platform failure or control channel failure. Clearly, $0 < b \leq N_b$[1] and $0 \leq k \leq bM - 1$. In addition to states s and c, they consist of the state space of the underlying Markov chain. It follows that the size of state space is $2 + M\frac{N_b(N_b + 1)}{2}$.

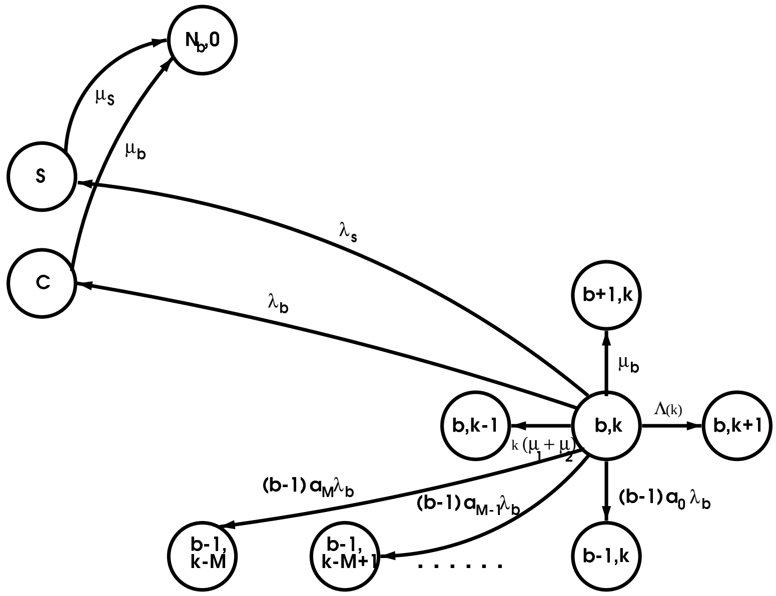

Fig. 1. State diagram of a system without APS

The state transitions of the Markov chain can be completely explored by iteratively enumerating *only* all outgoing transitions from all states. Hence, for a state, we only list its outgoing transitions. The outgoing transitions from state (b, k) are shown in Figure 1. From state (b, k), the following transitions can occur.

1. If $k < bM - 1$, the system is not fully-loaded and can accept handoff calls (and new calls if the number of idle channels in the channel pool is more than reserved channels for handoff calls). This corresponds to the transition from (b, k) to $(b, k + 1)$ with rate $\Lambda(k)$, where $\Lambda(k) = \lambda_1 + \lambda_2$ if $k < bM - 1 - g$ or $\Lambda(k) = \lambda_2$ if $bM - 1 - g \leq k < bM - 1$.
2. If $k > 0$, ongoing new calls and handoff calls will depart from the cell at rate $k(\mu_1 + \mu_2)$, *i.e.*, transition from (b, k) to $(b, k - 1)$. We note that the call departure rate depends on the number of talking channels, k, in the system.

[1] For $b = 0$, *i.e.*, there is no base repeater functioning in the system, a control channel failure must have occurred.

3. If $b > 0$, a base repeater may fail at rate $b\lambda_b$. Let i $(0 \leq i \leq M)$ be the number of ongoing calls on the failing base repeater. If the base repeater is not hosting the control channel, transition from (b, k) to $(b-1, k-i)$ will occur at rate $b\lambda_b a_i \left(1 - \frac{1}{b}\right) = (b-1)a_i\lambda_b$. Here $\frac{1}{b}$ is the probability that the failing base repeater hosts the control channel and the probability a_i can be calculated by (1), (2) and (3). If the base repeater happens to be hosting the control channel, transition from (b, k) to c occurs with rate $b\lambda_b \frac{1}{b} = \lambda_b$.
4. If $b < N_b$, a failed base repeater not hosting the control channel may be recovered at rate μ_b. This leads to the transition from (b, k) to $(b+1, k)$ with rate μ_b in view of the assumption that a single repair facility is shared.
5. For state (b, k), a platform failure may also occur in the system causing a full outage at rate λ_s. This corresponds to the transition from (b, k) to s with λ_s.

There is only one transition from state c, that is the failed base repeater hosting the control channel being recovered at rate μ_b, *i.e.*, the transition from c to $(N_b, 0)$. Similarly, only one transition from state s can occur, that is the platform failure is recovered at rate μ_s. This corresponds to the transition from s to $(N_b, 0)$. This completes the description of the underlying Markov chain of the non-APS system.

3.2 System with APS

An APS-capable system is able to switch the control channel from a failing base repeater to a functioning one. In case that the system is fully-loaded with currently available channels, one talking channel will be forced to terminate and is used as the control channel. Using the same notation introduced in Section 3.1, the underlying Markov chain, depicted in Figure 2, is very similar to that of a system without APS. The only differences are listed below.

1. If $b > 1$, a base repeater failure may occur at rate $b\lambda_b$. If the failing base repeater is not hosting the control channel, the same transition from (b, k) to $(b-1, k-i)$ will occur at rate $b\lambda_b a_i \left(1 - \frac{1}{b}\right) = (b-1)a_i\lambda_b$ as described in Section 3.1. However, due to the APS mechanism, if the failing repeater is the one hosting the control channel, the transitions are different: because in this case the base repeater provides only $M-1$ channels, instead of entering the control failure state c, the chain will move into state $(b-1, k-i)$ for $0 \leq i \leq M-1$ at rate $b\lambda_b a'_i \frac{1}{b} = a'_i\lambda_b$. Here a'_i is determined by (4), (5) and (6). Superposing the two cases, transition from (b, k) to $(b-1, k-i)$ has rate $(b-1)a_i\lambda_b + a'_i\lambda_b = [a_i(b-1) + a'_i]\lambda_b$ for $0 \leq i \leq M-1$ or $a_i(b-1)\lambda_b$ for $i = M$.

2. If $b = 1$, a base repeater failure will disable the control channel on the last base repeater and leads to state c. This corresponds to the transition from (b, k) to c with rate λ_b. We note that $(1, k)$ are the only states that can enter state c in a system with APS.

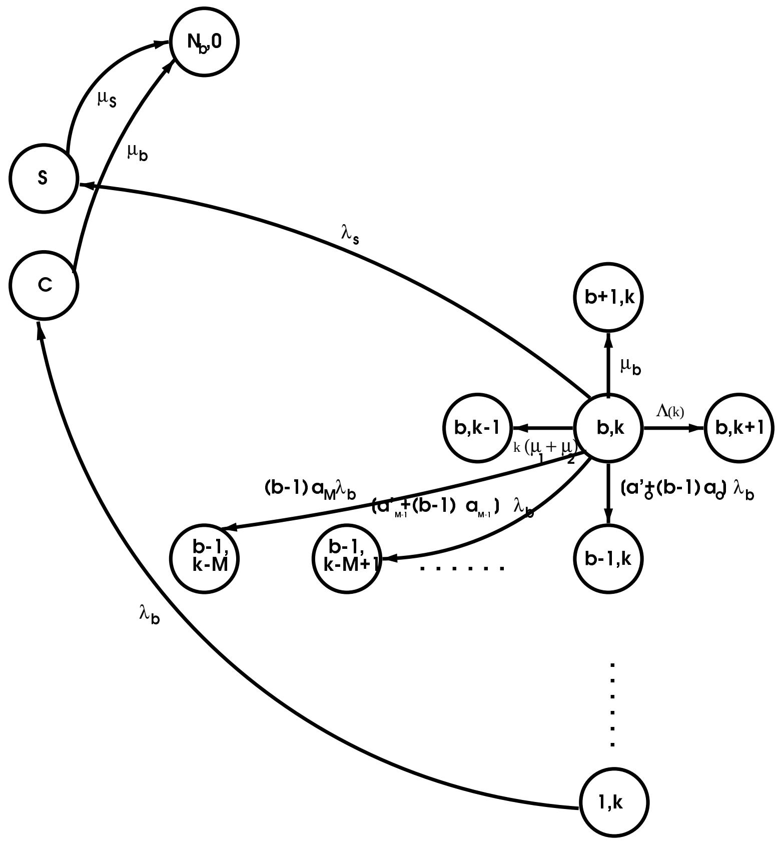

Fig. 2. State diagram of a system with APS

From the above discussion, it is clear that the gain by APS is achieved by the control channel switching ability upon failures to the hosting base repeater, lessening the chance of entering the control failure state, c.

3.3 Performability Indices

Three steady-state performability indices are considered in evaluating a cellular system: (I) the system unavailability ($\overline{A}$), (II) the overall new call blocking probability (P_b^o) and (III) the overall handoff call dropping probability (P_d^o).

Let p_c, p_s, and $p(b,k)$ be the steady-state state probabilities of the Markov chains described in Section 3.1 and 3.2. The three measures can be given as follows.

$$\overline{A}(N_b) = p_c + p_s \tag{7}$$

$$\begin{aligned} P_b^o(N_b, M, g) &= p_c + p_s + \sum_{b=1}^{N_b} \sum_{k=bM-1-g}^{bM-1} p(b,k) \\ &= \overline{A}(N_b) + \sum_{b=1}^{N_b} \sum_{k=bM-1-g}^{bM-1} p(b,k) \end{aligned} \tag{8}$$

$$\begin{aligned} P_d^o(N_b, M, g) &= p_c + p_s + \sum_{b=1}^{N_b} p(b, bM-1) \\ &= \overline{A}(N_b) + \sum_{b=1}^{N_b} p(b, bM-1) \end{aligned} \tag{9}$$

It is clear from the equations that, for both systems with and without APS, (1) the system unavailability $\overline{A}$ consists of one part of the overall new call blocking probability P_b^o and handoff call dropping probability P_d^o; (2) for system without GCS, *i.e.*, $g = 0$, $P_b^o(N_b, M, g) = P_d(N_b, M, g)$; and (3) $P_b^o(N_b, M, g) > P_d(N_b, M, g)$ holds for systems with one or more guard channels.

4 Approximate Hierarchical Model

For systems with large number of base repeaters, constructing the underlying Markov chain and seeking the solution is not trivial. Furthermore, the large difference between the time scales of reliability and performance parameters may cause the generator matrix to be highly ill-conditioned and may impose great *stiffness* problem in the steady-state solution. Hence it would be desirable to have well-behaved, less time-consuming and yet accurate approximate models.

We therefore use the decomposition method [6] to build a two-level performability model [7]: we first present an availability model which accounts for the failure-repair behavior of platform and base repeaters; second, we use a performance model to compute performance indices (new call blocking probability and handoff call dropping probability) given the number of non-failed base repeaters; finally, we combine them together and give performability measures of interest in closed forms.

4.1 Availability Model

Let $s \in S = \{0, 1\}$ denote a binary value indicating whether or not the system is down due to a platform failure (0: system down due to a platform failure; 1:

no platform failure has occurred). Also let $k \in B = \{0, 1, \cdots, N_b\}$ denote the number of non-failed base repeaters. The 2-tuple $(s, k), s \in S, k \in B$ defines a state in which the system is undergoing a (no) platform failure if $s = 0$ (if $s = 1$) and k base repeaters are up. The underlying stochastic process is a homogeneous continuous time Markov chain (CTMC) with state space $S \times B$. Let $P(s, k; N_b)$ be the corresponding steady state probability. The state diagram of this irreducible CTMC is depicted in Figure 3.

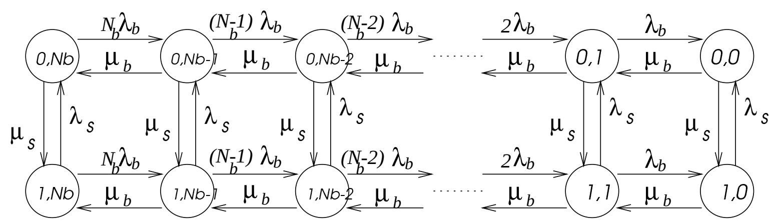

Fig. 3. Markov chain of Availability Model

Solving the Markov chain, we have

$$P(s,k;N_b) = \begin{cases} \dfrac{\lambda_s}{\lambda_s + \mu_s} \dfrac{1}{k!} \left(\dfrac{\mu_b}{\lambda_b}\right)^k \left[1 + \displaystyle\sum_{j=1}^{N_b} \dfrac{1}{j!} \left(\dfrac{\mu_b}{\lambda_b}\right)^j\right]^{-1}, & \text{if } s = 0, \\ \dfrac{\mu_s}{\lambda_s + \mu_s} \dfrac{1}{k!} \left(\dfrac{\mu_b}{\lambda_b}\right)^k \left[1 + \displaystyle\sum_{j=1}^{N_b} \dfrac{1}{j!} \left(\dfrac{\mu_b}{\lambda_b}\right)^j\right]^{-1}, & \text{if } s = 1. \end{cases} \tag{10}$$

The system unavailability corresponds to all the states in which either the system has a platform failure that brings the whole system down, or in a system without APS, a base repeater hosting the control channel fails, or the system even without platform failure has no working base repeater left. For a system without APS, the probability that one of the $(N_b - k)$ failed base repeaters happens to host the control channel is $(N_b - k)/N_b$. Let $\overline{A}(N_b)$ denote the steady state system unavailability. For both systems with and without APS, we thus write unavailability as

$$\overline{A}(N_b) = \begin{cases} \displaystyle\sum_{k=0}^{N_b} P(0, k; N_b) + \sum_{k=0}^{N_b} P(1, k; N_b) \frac{N_b - k}{N_b}, & \text{w/o APS} \\ \displaystyle\sum_{k=0}^{N_b} P(0, k; N_b) + P(1, 0; N_b), & \text{w/ APS.} \end{cases} \tag{11}$$

4.2 Performance Model

For each of the states of the availability model of Figure 3, we now seek to obtain key performance indices. Performance indices of interests are the steady state new call blocking probability and handoff call dropping probability. Given the number of available channels, the previous work in [5,1] provided formulae for these indices. We recall the results here. For a system having k non-failed channels and g guard channels, based on a birth-and-death process, the new call blocking probability is given as

$$P_b(k,g) = \frac{\sum_{n=k-g}^{k} \frac{A^{k-g}}{n\,!} A_1^{n-(k-g)}}{\sum_{n=0}^{k-g-1} \frac{A^n}{n\,!} + \sum_{n=k-g}^{k} \frac{A^{k-g}}{n\,!} A_1^{n-(k-g)}} \tag{12}$$

and the handoff call dropping probability is given as

$$P_d(k,g) = \frac{\frac{A^{k-g}}{k\,!} A_1^{g}}{\sum_{n=0}^{k-g-1} \frac{A^n}{n\,!} + \sum_{n=k-g}^{k} \frac{A^{k-g}}{n\,!} A_1^{n-(k-g)}}, \tag{13}$$

where $A = \frac{\lambda_1 + \lambda_2}{\mu_1 + \mu_2}$, $A_1 = \frac{\lambda_2}{\mu_1 + \mu_2}$. It should be noted that $P_b(k,g) > P_d(k,g)$ holds for $g \geq 1$ and when $g = 0$, $P_b(k,0) = P_d(k,0)$ becomes the Erlang B formula. In practice, the incoming handoff rate λ_2 is an unknown parameter which needs to be determined as a function of k, g, λ_1, μ_1 and μ_2. For a generic cell, assuming all cells are statistically identical, λ_2 can be determined by the following fixed point: the steady-state throughput of incoming handoff calls should be equal to the throughput of outgoing handoff calls. The existence and uniqueness of the fixed point are proved in [5].

4.3 Performability

From the last two sections, we notice that calls can be blocked (or dropped) due to system *being down* or *being full*. The former type of loss is captured by the pure availability model while the latter type of loss is captured by the pure performance model. We now wish to combine the two types of losses. The primary vehicle for doing this is to determine pure performance losses for each of the availability model states and attach these loss probabilities as reward rates (or weights) to these states. Such a Markov reward model has been called a performability model ([7,6]). We list reward rates for the states of the availability model in Table 1 for systems without APS and Table 2 for system with APS. Let us first consider states of system being down.

State (s, k)	Reward rate	
	New call blocking	Handoff call dropping
$(0, k)$, for $k = 0, \cdots, N_b$	1	1
$(1, 0)$	1	1
$(1, k)$, for $k = 1, \cdots, N_b$	1, if $kM - 1 \leq g$ $\frac{N_b - k}{N_b} + P_b(kM - 1, g)\frac{k}{N_b}$, o.w.	$\frac{N_b - k}{N_b} + P_d(kM-1, g)\frac{k}{N_b}$

Table 1. Reward rates for systems without APS

Clearly, for both systems without and with APS, a cell is not able to accept any new calls or handoff calls if it has platform failure which corresponds to the states $(0, k)$ for $k = 0, \cdots, N_b$, or all base repeaters are down which corresponds to the state $(1, 0)$, Therefore, reward rates of both overall new call blocking and handoff call dropping are 1's.

In addition, for a system without APS, control channel down may occur in states $(1, k)$ for $k = 1, \cdots, N_b$ with probability $(N_b - k)/N_b$ and cause new call blocking and handoff call dropping. This corresponds to the rates with $(N_b - k)/N_b$ in the last row of Table 1.

State (s, k)	Reward rate	
	New call blocking	Handoff call dropping
$(0, k)$, for $k = 0, \cdots, N_b$	1	1
$(1, 0)$	1	1
$(1, k)$, for $k = 1, \cdots, N_b$	1, if $kM - 1 \leq g$ $P_b(kM - 1, g)$, o.w.	$P_d(kM - 1, g)$

Table 2. Reward rates for systems with APS

All cases mentioned above contribute to system unavailability, $\overline{A}(N_b)$, discussed in Section 4.1. Hence, system unavailability, $\overline{A}(N_b)$, also consists of one of the parts of the overall new call blocking probability and handoff call dropping probability.

We now consider states in which the system is not undergoing a full outage caused by failures of platform, control channel (if system w/o APS) or all base repeaters being down.

The corresponding states are $(1, k)$ for $k = 1, \cdots, N_b$. The total number of available channels for state $(1, k)$ is $kM - 1$. From Section 4.3, new call blocking probability and handoff call dropping probability in these states are $P_b(kM - 1, g)$ and $P_d(kM - 1, g)$, respectively. Thus, these probabilities are used as reward rates to these states for overall new call blocking and handoff call dropping.

For a system without APS, we note that the probability of not having the control channel down in state $(1,k)$ for $k>0$ is k/N_b. Therefore, the reward rates, $P_b(kM-1,g)$ and $P_d(kM-1,g)$, are also weighted by k/N_b (shown in the last row of Table 1).

Also, in case that the number of idle channels is less than the number of guard channels, *i.e.*, $kM-1<g$ for states $(1,k)$, $k=1,\cdots,N_b$, a cell is not able set up any new calls because all available channels are reserved for handoff calls. Hence, the reward rates for new call blocking assigned to the corresponding states are 1's.

Now let $G=\lfloor (g+1)/M \rfloor$. Summarizing Table 1 and Table 2, the overall call blocking probability can be written as the expected steady state reward rate,

$$P_b^o(N_b,M,g)=\overline{A}(N_b) + \begin{cases} \mathbf{1}(G>0)\displaystyle\sum_{k=1}^{G} P(1,k;N_b)\left(\frac{k}{N_b}\right) \\ \quad + \displaystyle\sum_{k=G+1}^{N_b} P(1,k;N_b)P_b(kM-1,\ g)\left(\frac{k}{N_b}\right), & \text{w/o APS} \\ \mathbf{1}(G>0)\displaystyle\sum_{k=1}^{G} P(1,k;N_b) \\ \quad + \displaystyle\sum_{k=G+1}^{N_b} P(1,k;N_b)P_b(kM-1,\ g), & \text{w/ APS} \end{cases} \tag{14}$$

where $\mathbf{1}(e)$ is the indicator function: $\mathbf{1}(e)=1$ if expression e is true; $\mathbf{1}(e)=0$, otherwise. Similarly the overall handoff call dropping probability can be given as

$$P_d^o(N_b,M,g)=\overline{A}(N_b) + \begin{cases} \displaystyle\sum_{k=1}^{N_b} P(1,k;N_b)P_d(kM-1,\ g)\frac{k}{N_b}, & \text{w/o APS} \\ \displaystyle\sum_{k=1}^{N_b} P(1,k;N_b)P_d(kM-1,\ g), & \text{w/ APS.} \end{cases} \tag{15}$$

5 Numerical Results and Discussion

We present numerical results in this section. Table 3 summarizes parameters used.

5.1 Accuracy of the hierarchical models

We tabulate some results from composite models and hierarchical models to show the accuraracy of hierarchical models. Tables 4, 5 and 6 compare system

Parameter	Meaning	Value
N_b	Number of base repeaters	10
M	Number of channels/base repeater	8
λ_1	New call arrival rate	20 calls/minute
$1/\mu_1$	Mean call holding time	2.5 minutes
$1/\mu_2$	Mean time to handout	1.25 minutes
λ_s	Platform failure rate	1/year
μ_s	Mean repair time of platform	8 hours
λ_b	Base repeater failure rate	2/year
μ_b	Mean repair time of base repeater	2 hours

Table 3. Parameters used in numerical study

unavailability $\overline{A}$, overall new call blocking probability P_b^o and overall handoff call dropping probability P_d^o, respectively, from both models. The presented results from the two models show negligible difference, with the maximum relative error for all three measures less than 0.2%.

Parameters	APS	$\overline{A}_C$ Composite	$\overline{A}_H$ Hierarchical	$100\frac{\overline{A}_H - \overline{A}_C}{\overline{A}_C}$ (%)
$N_b = 8$	no	0.00136799	0.00136987	+0.1374
$N_b = 8$	yes	0.00091280	0.00091241	-0.0427
$N_b = 10$	no	0.00136799	0.00137028	+0.1674
$N_b = 10$	yes	0.00091241	0.00091241	0
$N_b = 12$	no	0.00136799	0.00137070	+0.1981
$N_b = 12$	yes	0.00091241	0.00091241	0

Table 4. Comparison of $\overline{A}$ from composite and hierarchical models

Parameters	APS	$P_{b\,C}^o$ Composite	$P_{b\,H}^o$ Hierarchical	$100\frac{P_{b\,H}^o - P_{b\,C}^o}{P_{b\,C}^o}$ (%)
$\lambda_1 = 12$	no	0.00136799	0.00137028	+0.1674
$\lambda_1 = 12$	yes	0.00091241	0.00091241	0
$\lambda_1 = 20$	no	0.00174871	0.00175125	+0.1452
$\lambda_1 = 20$	yes	0.00129612	0.00129638	+0.0201
$\lambda_1 = 32$	no	0.13844626	0.13845285	+0.0048
$\lambda_1 = 32$	yes	0.13807912	0.13808954	+0.0075

Table 5. Comparison of P_b^o from composite and hierarchical models

The proven high accuracy of hierarchical models with respect to accurate composite models allows us to carry out a variety of experiments with much higher computational efficiency. The following study is therefore conducted by solving hierarchical models.

Parameters	APS	$P^o_{d\,C}$ Composite	$P^o_{d\,H}$ Hierarchical	$100\frac{P^o_{d\,H}-P^o_{d\,C}}{P^o_{d\,C}}$ (%)
$\lambda_1 = 12$	no	0.00136799	0.00137028	+0.1674
$\lambda_1 = 12$	yes	0.00091241	0.00091241	0
$\lambda_1 = 20$	no	0.00137566	0.00137796	+0.1672
$\lambda_1 = 20$	yes	0.00092016	0.00092017	+0.0011
$\lambda_1 = 32$	no	0.00846951	0.00847205	+0.0300
$\lambda_1 = 32$	yes	0.00801872	0.00801933	+0.0076

Table 6. Comparison of P^o_d from composite and hierarchical models

5.2 Improvement by APS

In Figure 4 we plot the overall new call blocking probability, P^o_b, and handoff call dropping probability, P^o_d, against new call arrival rate, λ_1, for both systems with APS and without APS. The plots show that both probabilities increase but stay nearly flat when new call traffic is low ($<$ 20 calls/minute). The probabilities then increase sharply after λ_1 exceeds 20 calls/minute. The improvement by APS can be seen as reductions of P^o_b and P^o_d. Improvement remains steady (a $>$ 30% relative reduction of both P^o_b and P^o_d) given low traffic but diminishes rapidly as traffic load becomes heavier.

5.3 Percentage of $\overline{A}$ in P^o_b and P^o_d

In the same figure, we also plot the percentage of unavailability $\overline{A}$ in P^o_b and P^o_d to see how much failures of platform, base repeaters and control channel (*i.e.*, system being down) contribute to overall performability measures. It can be seen from the plots that system unavailability dominates under light traffic and becomes a less important factor under intense traffic when heavy traffic with limited system capacity becomes the major factor causing blocking and dropping.

5.4 Optimization of N_b and g

We also plot curves of both P^o_b and P^o_d against the number of guard channels, g, for $N_b = 8, 9$ and 10 for an APS-capable system[2] in Figure 5. The figure shows that, for each N_b, (I) each curve of P^o_b and P^o_d starts with the same values, *i.e.*, $P^o_b = P^o_d$ when $g = 0$ and (II) increasing g results in a decrease in P^o_d and an increase in P^o_b. From Figure 5, it is also clear that when increasing the number of base repeaters, N_b, both curves of P^o_b and P^o_d move down, indicating the performability improvement.

In practice, we may face problems to minimize the overall new call blocking probability P^o_b and handoff call dropping probability P^o_d. The number of channels M that a base repeater provides is normally fixed after the repeater

[2] Similar curves can also be plotted for systems w/o APS (see [8]).

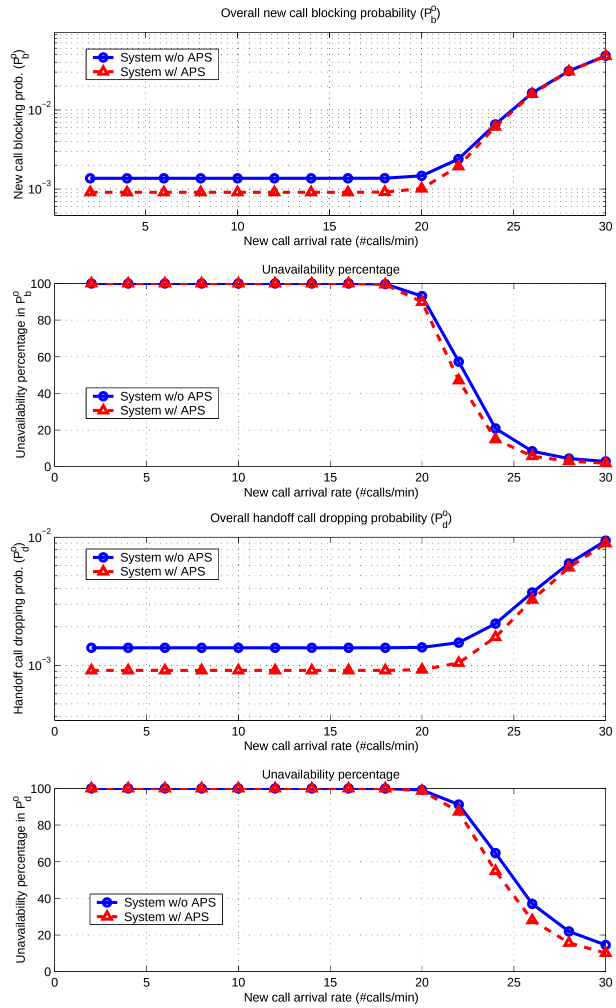

Fig. 4. $P_b^o(N_b, M, g)$ (1st from top) and $P_d^o(N_b, M, g)$ (3rd) versus g for systems w/o APS and w/ APS; Percentage of unavailability $\overline{A}(N_b)$ in $P_b^o(N_b, M, g)$ (2nd) and $P_d^o(N_b, M, g)(4th)$

is manufactured. Given that the reliability parameters $(\lambda_s, \mu_s, \lambda_b, \mu_b)$ and traffic parameters (A, A_1) are fixed, the decision variables are the number of guard channels, g, and the number of base repeaters, N_b. For example, the following optimization problem may occur to a network designer,

O: Given reliability parameters $(\lambda_s, \mu_s, \lambda_b, \mu_b)$, traffic parameters (A, A_1) and the number of channels M on a base repeaters, determine the optimal

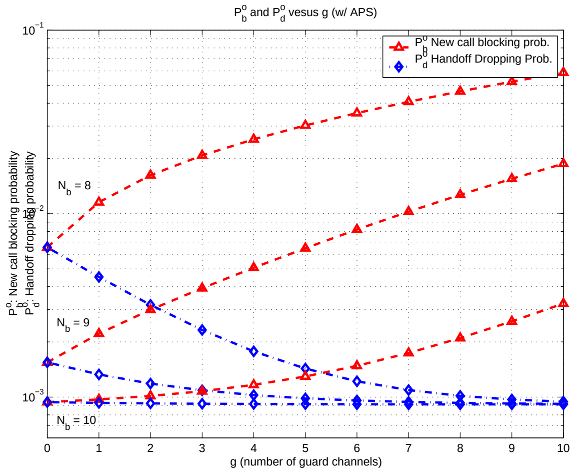

Fig. 5. P_b^o and P_d^o versus N_b and g

integer values of N_b and g so as to

$$\text{minimize } N_b$$
$$\text{such that } \begin{cases} P_b^o(N_b, M, g) \le P_{b0} \\ P_d^o(N_b, M, g) \le P_{d0}. \end{cases}$$

We now show how the plot in Figure 5 also provides a graphical way to solve the optimization problems concerning N_b and g. We may draw two lines, $P_b^o = P_{b0}$ and $P_d^o = P_{d0}$. Pairs of triangle marks ($\triangle$) for P_b^o under line $P_b^o = P_{b0}$ and diamond marks ($\diamond$) for P_d^o under line $P_d^o = P_{d0}$ consist of the set of possible solutions. We then choose the minimum N_b. For $P_{b0} = 0.003$ and $P_{d0} = 0.002$, $N_b^* = 9$ and $g^* = 0$ or 1.

6 Conclusion

We have presented the exact composite Markov chain models for performbability study of TDMA wireless systems with and without automatic protection switch. We then have followed a reward-assigning approach to develop the two-level hierarchical performability models. Measures of interest, such as system unavailability, overall new call blocking probability and handoff call dropping probability are explicitly given in closed form. This enables the approximate models to possess excellent scalability. Compared with composite models, the more robust and less time-consuming hierarchical models are proven to be able to provide high accuracy. Numerical results are given under realistic parameter settings. It is expected that the models presented in this paper will be useful in wireless network design and operation.

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