

Transient Analysis of Minimum Duration Outage for RF Channel in Cellular Systems

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Abstract – Following Mandayam et al., we define outage events as the channel being attenuated for at least a deterministic period of time, τ_m . Compared with continuous time Markov chain or discrete time Markov chain, a semi-Markov process (SMP) is general enough that it allows the sojourn time to be any distribution function. In this paper, we develop a minimum duration outage model based on an SMP. Closed-form expressions are derived for the time dependent and steady state probabilities of outage. Describing the outage event in the transient regime, our model can reflect the channel fading behavior in a dynamic way.

1 Introduction

Traditionally, outage probability for a radio frequency (RF) link in cellular networks is defined as the probability that the signal-to-interference ratio (SIR) is below a certain threshold at a given time point. However, in a realistic cellular system, as pointed out by Mandayam et al. [10], an instantaneous drop of SIR below a threshold does not necessarily lead to the occurrence of an outage event. Rather, it is the *duration* of time that the SIR stays below a threshold that determines the channel outage event. To reflect the impact of this time duration, an RF channel outage event is defined [10] as the SIR staying below a threshold longer than the “minimum duration”, τ_m .

Existing models in the open literature [10, 21] are concerned with steady state measures of minimum duration RF channel outage. However, in a practical cellular system, an RF channel connection is of finite duration and steady states might never be reached. In addition, the quality of an RF channel may fluctuate due to temporary factors such as interference, fading and mobility. The channel behavior affected by these

dynamic factors cannot be adequately reflected by its steady state evaluation. In this paper, a dynamic channel behavior model is developed. The dynamic behavior of an RF channel is reflected through the time dependent outage probability.

The existence of the deterministic minimum duration τ_m gives rise to a stochastic process that is non-Markovian in nature. Because of its exponential distribution requirement, continuous time Markov chain (CTMC) model cannot be used as a computational tool under this scenario. Discrete time Markov chain (DTMC) can be used only to reflect a dynamic process at specific epochs in time.

In this paper, we derive an analytical model that describes the time dependent behavior of minimum duration outage in cellular networks. In order to explicitly accommodate the deterministic distribution in minimum duration outage, our model is based on a semi-Markov process (SMP) [9]. Because of its ability to represent general sojourn times, SMP is being used as a powerful technique in modeling complex processes that arise in computer [1, 2] and communication [11, 17] systems.

The results in this paper generalize those [10, 21] obtained for steady state behavior of minimum duration outage. Closed-form solutions are derived for both transient and steady state probabilities for the outage event. Through the transient analysis, the impact of SIR can be realistically reflected in a dynamic way. This provides a valuable aid [5] in selecting optimal system parameters and consequently allows an enhancement in network capacity and performance.

The remainder of this paper is organized as follows. In Section 2, an introduction of semi-Markov process is given. In Section 3, we derive our minimum duration outage model based on SMP. In Section 4, numerical

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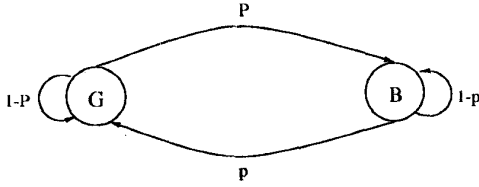


Figure 1: The Gilbert-Elliott channel.

results are presented. We conclude the paper in Section 5.

2 Semi-Markov Process

For years, Markov models have been used widely to describe the channel behaviors in communication systems [3, 4, 6, 12, 16, 19, 22]. A classic example is the Gilbert-Elliott channel (GEC) [3, 6] model, as shown in Figure 1. The GEC model is a DTMC with two states G (for good) and B (for bad or bursty). The state transition probabilities are P and p .

In [21], Zorzi proposed a DTMC model for minimum duration outage. The proposed analytical framework applies to a more general class of outage definition which was originally defined by Mandayam et al. [10]. However, their models do not capture the evolution of the outage event in continuous time. In a realistic cellular network, the quality of RF channel may fluctuate due to temporary factors such as interference, fading or mobility. An analytical model would be a helpful tool to reflect this dynamic nature. In this paper, an analytical model is derived that describes the time dependent behavior of minimum duration outage. The model is based on semi-Markov processes (SMP) [9]. Being able to release the stringent distribution assumptions that are required for DTMC and CTMC, SMP plays an important role in modeling stochastic systems. In the remainder of this section, a brief introduction to SMP is given.

Consider a discrete state-space process $X(t)$ with the state-space $\Omega \subseteq \mathcal{S} = \{0, 1, 2, \dots\}$. Suppose that the system is observed at times $T_0 = 0, T_1, T_2, \dots$, etc. Thus the number of observations of $X(t)$ up to time t is

$$N(t) = \sup\{n \geq 0 : T_n \leq t\}. \quad (1)$$

Let Y_n be the state of the system at time T_n .

Definition 2.1 A sequence of bivariate random variables $\{(Y_n, T_n), n \geq 0\}$ is called a Markov renewal sequence (MRS) if

1. $T_0 = 0, T_{n+1} \geq T_n; Y_n \in \mathcal{S} = \{0, 1, 2, \dots\}$ and

2. $\forall n \geq 0,$

$$\begin{aligned} & P[Y_{n+1} = j, T_{n+1} - T_n \leq t \mid Y_n = i, T_n, \\ & \quad Y_{n-1}, T_{n-1}, \dots, Y_0, T_0] \\ &= P[Y_{n+1} = j, T_{n+1} - T_n \leq t \mid Y_n = i] \\ &= P[Y_1 = j, T_1 \leq t \mid Y_0 = i]. \end{aligned} \quad (2)$$

The conditional probability in (2) is denoted by $k_{ij}(t)$. The matrix $\mathbf{K}(t)$ formed by the elements $k_{ij}(t)$ is called the *kernel* of the MRS. From Equation (2), it can be seen that $\{Y_n, n \geq 0\}$ is an embedded DTMC with transition probability matrix $\mathbf{K}(\infty)$. Given that $Y_0 = i$, the distribution of the sojourn time in state i is defined as:

$$\begin{aligned} H_i(t) &\equiv P[T_{n+1} - T_n \leq t \mid Y_n = i, T_n, \\ & \quad Y_{n-1}, T_{n-1}, \dots, Y_0, T_0] \\ &= P[T_1 \leq t \mid Y_0 = i] \\ &= \sum_{j \in \Omega} k_{ij}(t), \end{aligned} \quad (3)$$

where $\Omega \subseteq \mathcal{S} = \{0, 1, 2, \dots\}$.

Definition 2.2 Let $\{(Y_n, T_n), n \geq 0\}$ be an MRS and $N(t)$ be defined as in Equation (1). The stochastic process $\{X(t), t \geq 0\}$ is called a semi-Markov process (SMP) if

$$X(t) = Y_{N(t)}, \text{ where } t \geq 0. \quad (4)$$

Now define the conditional probability $v_{ij}(t)$ as

$$v_{ij}(t) = P[X(t) = j \mid X(0) = i], \text{ where } t \geq 0. \quad (5)$$

It is shown [9] that v_{ij} satisfies the following equation:

$$v_{ij}(t) = (1 - H_i(t))\delta_{ij} + \sum_{m \in \Omega} k_{im} * v_{mj}(t), \quad (6)$$

where δ_{ij} is the Kronecker delta function and $k_{im} * v_{mj}(t)$ is the Stieltjes convolution integral:

$$k_{im} * v_{mj}(t) = \int_0^t dk_{im}(x)v_{mj}(t-x).$$

Equation (6) is referred as the *Markov renewal equation*, where the matrix form is

$$\mathbf{V}(t) = \mathbf{E}(t) + \mathbf{K} * \mathbf{V}(t), \quad (7)$$

with

$$e_{ij}(t) = \begin{cases} 1 - H_i(t) & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases}$$

We consider the steady state analysis of an SMP by studying its limiting behavior. Let π be the steady

state probability vector of the embedded DTMC for an SMP:

$$\boldsymbol{\pi} = \boldsymbol{\pi} \cdot \mathbf{P} \quad \text{and} \quad \boldsymbol{\pi} \cdot \mathbf{I} = 1,$$

where $\mathbf{P} = \mathbf{K}(\infty)$. It is shown [9, 14] that if there is a positive solution for $\boldsymbol{\pi}$ and the SMP is aperiodic, irreducible, and positive recurrent, then the steady state probability p_j of the SMP is given by

$$\begin{aligned} p_j &= \lim_{t \rightarrow \infty} P[X(t) = j \mid Y_0 = i] \\ &= \frac{\pi_j \mu_j}{\sum_{k \in \Omega} \pi_k \mu_k}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \mu_i &= E[\text{time spent in state } j] \\ &= \int_0^{\infty} (1 - H_i(t)) dt. \end{aligned}$$

3 The SMP Model for Minimum Duration Outage

The state transition diagram for minimum duration outage is shown in Figure 2. State *Up* represents that the SIR of an RF channel is above a predetermined threshold. When the SIR goes below the threshold, the channel status is represented by the state *Tmp*. After the SIR stays below the threshold over a deterministic period of time, τ_m , the outage event is triggered. This is represented by state *Out*. Solid arcs represent transition firings in accordance to exponential distribution [13, 18], while the dashed arc represents the transition with deterministic distribution. Because of the memoryless property of exponential distribution [15], the transition rates from states *Tmp* and *Out* to state *Up* have the same value, denoted by λ_2 .

Let the state probability $p_j(t)$, where $j = 1, 2, 3$, be defined as

$$p_j(t) = \begin{cases} 1 & \text{the system is in state } Up \text{ at time } t, \\ 2 & \text{the system is in state } Tmp \text{ at time } t, \\ 3 & \text{the system is in state } Out \text{ at time } t. \end{cases}$$

Suppose that the system is initially in state *Up*. We are interested in finding the outage probability $p_3(t)$ and $p_3(t) = v_{13}(t)$. In order to solve Equation (7), the crux is to find kernel $\mathbf{K}(t)$. According to Equation (2), the kernel $\mathbf{K}(t)$ has the following structure:

$$\mathbf{K}(t) = \begin{bmatrix} 0 & k_{12}(t) & 0 \\ k_{21}(t) & 0 & k_{23}(t) \\ k_{31}(t) & 0 & 0 \end{bmatrix},$$

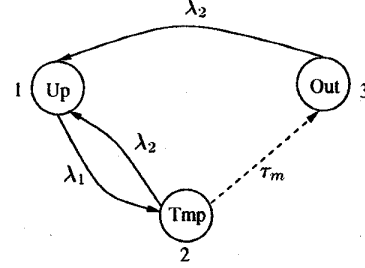


Figure 2: State transition diagram for minimum duration outage.

where

$$\begin{aligned} k_{12}(t) &= 1 - e^{-\lambda_1 t}, \\ k_{21}(t) &= \begin{cases} 1 - e^{-\lambda_2 t}, & t < \tau_m \\ 1 - e^{-\lambda_2 \tau_m}, & t \geq \tau_m \end{cases}, \\ k_{23}(t) &= \begin{cases} 0, & t < \tau_m \\ e^{-\lambda_2 \tau_m}, & t \geq \tau_m \end{cases}, \\ k_{31}(t) &= 1 - e^{-\lambda_2 t}. \end{aligned}$$

From Equation (6), the individual entries for the diagonal matrix $\mathbf{E}(t)$ are found to be:

$$\begin{aligned} e_{11}(t) &= e^{-\lambda_1 t}, \\ e_{22}(t) &= \begin{cases} e^{-\lambda_2 t}, & t < \tau_m \\ 0, & t \geq \tau_m \end{cases}, \\ e_{33}(t) &= e^{-\lambda_2 t}. \end{aligned}$$

Next, we use Laplace-Stieltjes transformation to solve Equation (7). The Laplace transform (LT) of a function $F(t)$ is defined [7, 20] as $\bar{F}(s) = \int_0^{\infty} e^{-st} F(t) dt$ and the Laplace-Stieltjes transform (LST) of a function $F(t)$ is defined [20] as $\tilde{F}(s) = \int_0^{\infty} e^{-st} dF(t)$. The relationship between $\bar{F}(s)$ and $\tilde{F}(s)$ is $\tilde{F}(s) = s\bar{F}(t)$. The LST for Equation (7) is of form:

$$\begin{aligned} \tilde{\mathbf{V}}(s) &= \tilde{\mathbf{E}}(s) + \tilde{\mathbf{K}}(s)\tilde{\mathbf{V}}(s) \\ &= [\mathbf{I} - \tilde{\mathbf{K}}(s)]^{-1} \tilde{\mathbf{E}}(s). \end{aligned} \quad (9)$$

The expressions of the LSTs for $\mathbf{K}(t)$ and $\mathbf{E}(t)$ are obtained as following:

$$\tilde{k}_{12}(s) = \frac{\lambda_1}{s + \lambda_1}, \quad (10)$$

$$\tilde{k}_{21}(s) = (1 - e^{-(s+\lambda_2)\tau_m}) \left(1 - \frac{s}{s + \lambda_2}\right), \quad (11)$$

$$\tilde{k}_{23}(s) = e^{-(s+\lambda_2)\tau_m}, \quad (12)$$

$$\tilde{k}_{31}(s) = \frac{\lambda_2}{s + \lambda_2}; \quad (13)$$

and

$$\tilde{e}_{11}(s) = \frac{s}{s + \lambda_1}, \quad (14)$$

$$\tilde{e}_{22}(s) = (1 - e^{-(s+\lambda_2)\tau_m}) \frac{s}{s + \lambda_2}, \quad (15)$$

$$\tilde{e}_{33}(s) = \frac{s}{s + \lambda_2}. \quad (16)$$

After some manipulations by substituting Equations (10) - (16) into Equation (9), we obtain:

$$\tilde{v}_{13}(s) = \frac{\lambda_1 s}{e^{(\lambda_2+s)\tau_m} (\lambda_1 + s) (\lambda_2 + s)} \times \frac{1}{A}, \quad (17)$$

where

$$A = \left(1 - \frac{\lambda_1}{\lambda_1 + s} + \frac{\lambda_1}{e^{(\lambda_2+s)\tau_m} (\lambda_1 + s)} \frac{\lambda_1 \lambda_2}{e^{(\lambda_2+s)\tau_m} (\lambda_1 + s) (\lambda_2 + s)} + \frac{\lambda_1 s}{(\lambda_1 + s) (\lambda_2 + s)} - \frac{\lambda_1 s}{e^{(\lambda_2+s)\tau_m} (\lambda_1 + s) (\lambda_2 + s)} \right).$$

After further simplification of Equation (17), the LT of $v_{13}(t)$ is obtained as:

$$\bar{v}_{13}(s) = \frac{\lambda_1 e^{-\lambda_2 \tau_m} e^{-\tau_m s}}{s(s + \lambda_1 + \lambda_2)}. \quad (18)$$

The outage probability $p_3(t)$ is then found through the inversion of Equation (18):

$$v_{13}(t) = \begin{cases} 0 & t < \tau_m, \\ \frac{\lambda_1 e^{-\lambda_2 \tau_m}}{\lambda_1 + \lambda_2} (1 - e^{-(\lambda_1 + \lambda_2)(t - \tau_m)}) & t \geq \tau_m. \end{cases} \quad (19)$$

When $\tau_m = 0$, states 2 and 3 in Figure 2 merge into one state. The three-state outage model is simplified into a two-state CTMC model. Equation (19) then turns out to be

$$v_{12}(t)|_{\tau_m=0} = \frac{\lambda_1}{\lambda_1 + \lambda_2} (1 - e^{-(\lambda_1 + \lambda_2)t}), \quad (20)$$

which is equal to the result obtained by solving the CTMC model directly. If we further take $t \rightarrow \infty$, from Equation (20), we obtain $\pi_2 = \lambda_1/(\lambda_1 + \lambda_2)$, which is consistent with the result obtained from the GEC model.

Using Equation (8), we obtain the steady state probabilities as following:

$$p_1 = \frac{\lambda_2}{\lambda_1 + \lambda_2},$$

$$p_2 = \frac{\lambda_1(1 - e^{-\lambda_2 \tau_m})}{\lambda_1 + \lambda_2} \quad \text{and} \quad p_3 = \frac{\lambda_1 e^{-\lambda_2 \tau_m}}{\lambda_1 + \lambda_2}.$$

Combining p_2 and p_3 , we get $p_{(2,3)} = \lambda_1/(\lambda_1 + \lambda_2)$, which equals to π_2 in the GEC model.

4 Numerical Results

In dB scale, the SIR going below a threshold T can be written as $S(t) < T + I(t)$. Assuming that interference $I(t)$ is fixed [10], we convert the problem of $SIR(t)$ going below a threshold into the problem that signal strength $S(t)$ is going below a level R . For the purpose of discussion, we consider a cellular system of 900 MHz. The vehicle speed is 37 km/h and the signal level threshold is -10 dB. The free-space speed (3×10^8 m/s) is assumed for the propagation of electromagnetic waves.

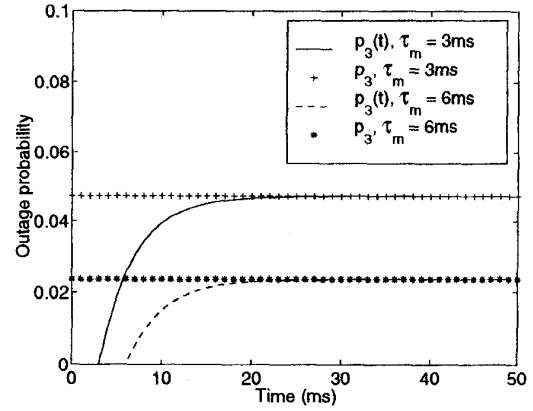


Figure 3: Outage probabilities vs τ_m , $v = 37$ km/h.

It has been shown [5, 8] that the average duration of signal strength above or below a level R is respectively:

$$t_a = \frac{1}{\sqrt{2\pi} f_m \rho} \quad \text{and} \quad t_b = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho},$$

where $\rho = R/(\sqrt{2}\sigma)$, $f_m = v/\lambda$, $\sqrt{2}\sigma$ is the root mean square (RMS) value, v is the speed of the vehicle and λ is the carrier wavelength. The state transition rates are then obtained as $\lambda_1 = 1/t_a = 0.0244/\text{ms}$, and $\lambda_2 = 1/t_b = 0.2324/\text{ms}$. Figure 3 shows the outage probabilities for two different values of τ_m . As expected, the transient values approach the steady state results as time goes to infinity. In addition, the outage probability decreases as τ_m increases. From the figure, we can also estimate the rate at which the system reaches a steady state. Figure 4 demonstrates the effects of velocity on the outage probabilities. We notice that when all the other parameters are fixed, the

mobile with lower velocity has a higher probability of outage. This is consistent with the results obtained from [10].

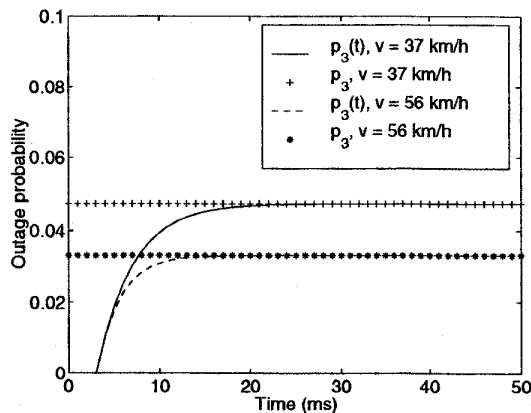


Figure 4: Outage probabilities vs v , $\tau_m = 3$ ms.

5 Conclusion

In this paper, an SMP model is developed for transient analysis of minimum duration outage in cellular networks. Closed-form solutions are derived for both transient and steady state probabilities of outage. Through the transient analysis, the dynamic impact of SIR on an RF channel is reflected in a realistic way. This provides a helpful computational tool in selecting optimal transmission bit rates, word lengths and coding schemes in a digital cellular network.

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