Loss Formulas and Their Application to Optimization for Cellular Networks

Guenter Haring, *Senior Member, IEEE*, Raymond Marie, Ramon Puigjaner, *Member, IEEE*, and Kishor Trivedi, *Fellow, IEEE*

***Abstract*—In this paper, we develop a performance model of a cell in a wireless communication network where the effect of handoff arrival and the use of guard channels is inlcuded. Fast recursive formulas for the loss probabilities of new calls and handoff calls are developed. Monotonicity properties of the loss probabilities are proven. Algorithms to determine the optimal number of guard channels and the optimal number of channels are given. Finally, a fixed-point iteration scheme is developed in order to determine the handoff arrival rate into a cell. The uniqueness of the fixed point is shown.**

***Index Terms*—Channel allocation, Markov models, optimization, performance modeling, wireless cellular networks.**

I. Introduction

The Erlang-B formula has been normally used to compute the loss probability in wireline networks. This formula cannot be used in cellular wireless networks due to the phenomenon of handoff. When a mobile station moves across a cell boundary the channel in the earlier cell is released and an idle channel is required in the target cell. This phenomenon is called handoff. If an idle channel is available in the target cell the handoff call is resumed nearly transparently to the user. Otherwise the handoff call is dropped. The dropping of a handoff call is generally considered more serious than blocking of a new call [2]. One way of reducing the dropping probability of a handoff call is to reserve a fixed number of channels (called guard channels) exclusively for the handoff calls [1], [3]. As a result, separate formulas for the dropping probability of handoff calls and the blocking probability of the new calls are required. Furthermore, as the number of guard channels is increased the dropping probability will be reduced while the blocking probability will increase. Thus, it is possible to derive an optimal number of guard channels subject to given constraints on the dropping and blocking probabilities.

Earlier efforts in this direction have been in the context of performability models including the effects of channel failures and recovery [4]. The objective of this paper is to derive the blocking and dropping probability formulas for a pure performance model. We also consider the optimal number of guard channels. We use a homogeneous continuous time Markov chain model for our derivations.

In Section II, we discuss the basic model and in Section III we consider the computational aspects. In Section IV we discuss properties of loss probabilities while in Section V we consider the optimization of the number of guard channels. In Section VI we discuss the use of fixed-point iteration to determine handoff call arrival rate. Finally, in Section VII we provide the conclusions.

Manuscript received September 28, 1999; revised April 25, 2000.

G. Haring is with the Universitaet Wien, A-1080 Wien, Austria (e-mail: haring@ani.univie.ac.at).

R. Marie is with IRISA, Campus de Beaulieu, 35042 Rennes cedex, France (e-mail: marie@irisa.fr).

R. Puigjaner is with the Universitat de les Illes Balears, 07071 Palma, Spain (e-mail: putxi@uib.es).

K. Trivedi is with the Electrical and Computer Engineering Department, CACC, Duke University, Durham, NC 27708 USA (e-mail: kst@ee.duke.edu).

Publisher Item Identifier S 0018-9545(01)06578-1.

II. Basic Model

We consider the performance model of a single cell in a cellular wireless communication network. Consider Poisson arrival stream of new calls at the rate λ_1 and the Poisson stream of handoff arrivals at the rate λ_2. An ongoing call (new or handoff) completes service at the rate μ_1 and the mobile engaged in the call departs the cell at the rate μ_2. There is a limited number of channels, N, in the channel pool. When a handoff call arrives and an idle channel is available in the channel pool, the call is accepted and a channel is assigned to it. Otherwise, the handoff call is dropped. When a new call arrives, it is accepted provided that $g+1$ or more idle channels are available in the channel pool; otherwise, the new call is blocked. Here, g is the number of guard channels. We assume that $g < N$ in order not to exclude new calls altogether.

Let $C(t)$ denote the number of busy channels at time t, then $\{C(t), t \geq 0\}$ is a birth–death process as shown in Fig. 1. We define $\lambda = \lambda_1 + \lambda_2$, $\mu = \mu_1 + \mu_2$. The state-dependent arrival and departure rates in the birth–death process are given by

$$\Lambda(n) = \begin{cases} \lambda, & n = 0, 1, \ldots, N-g-1 \\ \lambda_2, & n = N-g, \ldots, N-1; \quad g > 0 \end{cases}$$

and $M(n) = n\mu$, $n = 1, \ldots, N$.

Because of the structure of the Markov chain we can readily write down the solution to the steady–state balance equations as follows. Define the steady–state probability

$$p_n = \lim_{t \to \infty} \text{Prob}(C(t) = n), \quad C = 0, 1, 2, \ldots, N.$$

Let $A = \lambda/\mu$, $A_1 = \lambda_2/(\mu_1 + \mu_2)$. Then we have an expression for p_n

$$p_n = p_0 \begin{cases} \frac{A^n}{n!}, & n \leq N-g \\ \frac{A^{N-g}}{n!} A_1^{n-(N-g)}, & n \geq N-g \end{cases}$$

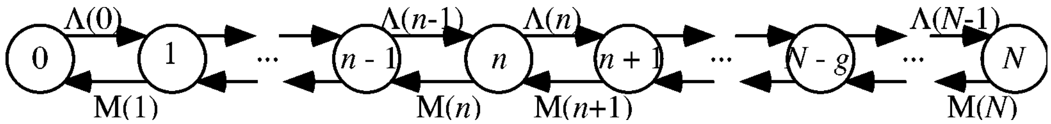

Fig. 1. Markov chain model of wireless handoff.

where

$$p_0 = \frac{1}{\sum_{n=0}^{N-g-1} \frac{A^n}{n!} + \sum_{n=N-g}^{N} \frac{A^{N-g}}{n!} A_1^{n-(N-g)}}.$$

Now we can write expressions for the dropping probability for handoff calls

$$P_d(N,g) = p_N = \frac{\frac{A^{N-g}}{N!} A_1^g}{\sum_{n=0}^{N-g-1} \frac{A^n}{n!} + \sum_{n=N-g}^{N} \frac{A^{N-g}}{n!} A_1^{n-(N-g)}}. \tag{1}$$

Similarly, the expression for the blocking probability of new calls is

$$\begin{aligned} P_b(N,g) &= \sum_{n=N-g}^{N} p_n \\ &= \frac{\sum_{n=N-g}^{N} \frac{A^{N-g}}{n!} A_1^{n-(N-g)}}{\sum_{n=0}^{N-g-1} \frac{A^n}{n!} + \sum_{n=N-g}^{N} \frac{A^{N-g}}{n!} A_1^{n-(N-g)}} \\ &= A^{N-g} \frac{\sum_{k=0}^{g} \frac{A_1^k}{(k+N-g)!}}{\sum_{n=0}^{N-g-1} \frac{A^n}{n!} + \sum_{n=N-g}^{N} \frac{A^{N-g}}{n!} A_1^{n-(N-g)}}. \end{aligned} \tag{2}$$

Note that if we set $g = 0$ then expression (2) reduces to the classical Erlang-B loss formula. In fact, setting $g = 0$ in expression (1) also provides the Erlang-B loss formula. Note also that A is the total traffic in *Erlangs* as seen by a cell, while A_1 is the handoff traffic in *Erlangs*.

If the number of channels N is large, the two loss formulas we have developed are not convenient to use as overflow and underflow might occur due to factorials and large powers of A and A_1. In the next section, we show numerically stable computation for the loss probabilities.

III. Computational Aspects

The number of channels N, in most wireless systems is large leading to numerical difficulties in the direct use of the loss formulas (1) and (2). We show numerically stable methods of computation in this section that avoids the computation of factorials and large powers of loads in *Erlangs*. All computations are based on recursive relations we establish.

Let

$$E_B(A,N) = \frac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \cdots + \frac{A^N}{N!}} \tag{3}$$

be the Erlang-B formula.

Then we can show the following:

$$P_d(N,0) = P_b(N,0) = E_B(A,N). \tag{4}$$

Thus, to compute the loss probability in case there are no guard channels, we simply use the standard loss formula with total traffic A in *Erlangs*. Note that the traffic includes both new calls and handoff calls. The service rate includes both call completion and handoff out into adjacent cells.

Formula (3) (Erlang-B formula), if programmed as shown will lead to overflow problems. A recursive computation is used in [5].

- Let $\phi(1) = A$ and compute

$$\phi(k) = \phi(k-1)\frac{A}{k} \quad k = 2, 3, \ldots, N.$$

- Then let $G(0) = 1$ and compute

$$G(k) = G(k-1) + \phi(k) \quad k = 1, 2, \ldots, N.$$

- Finally, $E_B(A,N) = \phi(N)/G(N)$.

This computation is much more stable than the direct use of formula (3). Nevertheless, both the numerator and the denominator above can become very large for large values of A and N, leading to overflow. A much better recursion is the following [6]:

$$E_B(A,k) = \frac{\frac{A}{k} E_B(A,k-1)}{1 + \frac{A}{k} E_B(A,k-1)}, \quad k = 1, 2, \ldots, N \tag{5}$$

with $E_B(A,0) = 1.0$.

We have used this formula (5) for values of N as large as 1000 and have not encountered difficulties. Thus, formula (5) is recommended for computing Erlang-B loss probability.

Recall that if the number of guard channels, g is 0, we use formula (5) above to compute both, the dropping probability of handoff calls and the blocking probability of new calls.

In the case the number of guard channels $g > 0$, let $N_1 = N - g$ be the number of shared channels. Now we can use the following recursive formula (for a proof, see Appendix A.1):

Let $P_d(N_1, 0) = E_B(A, N_1)$ and compute

$$P_d(N_1 + k, k) = \frac{P_d(N_1 + (k-1), k-1)}{\frac{N}{\alpha A} + P_d(N_1 + (k-1), k-1)}, \quad k = 1, 2, \ldots, g. \tag{6}$$

Similarly for the blocking probability (again see Appendix A.1), let $P_b(N_1, 0) = E_B(A, N_1)$ and compute

$$\begin{aligned} &P_b(N_1 + k, k) \\ &= \frac{\frac{N}{\alpha A} P_b(N_1 + (k-1), k-1) + P_d(N_1 + (k-1), k-1)}{\frac{N}{\alpha A} + P_d(N_1 + (k-1), k-1)} \\ &\qquad k = 1, 2, \ldots, g \end{aligned} \tag{7}$$

where $\alpha A = A_1$ is the traffic in *Erlangs* due to handoff arrivals.

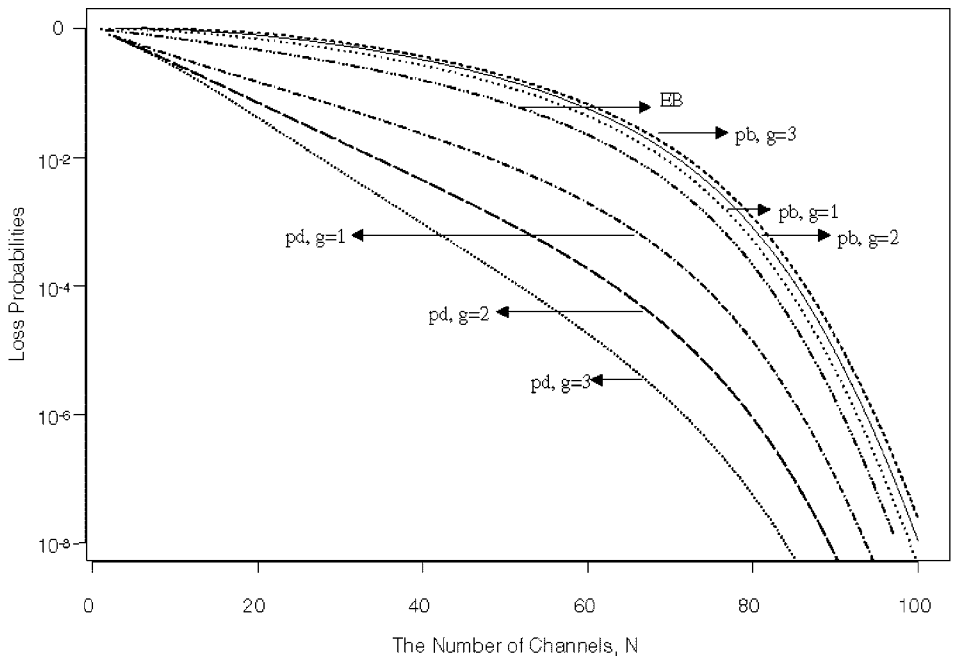

Fig. 2. Loss probabilities as functions of N.

Equations (6) and (7) can be easily programmed in a simple loop.

In Fig. 2, we have plotted the loss probabilities P_d, P_b and E_B as functions of the number of channels N for different values of g. We have assumed $A = 70$, $\alpha = 0.3$ and $\mu_1 + \mu_2 = 1$.

It is interesting to note that the ratio of the blocking probability to the dropping probability has a nice expression. Define $R(N, g) = (P_b(N, g))/(P_d(N, g))$. Then from expressions (6) and (7) we note that

$$R(N_1 + k, k) = \frac{N}{\alpha A} R(N_1 + (k-1), k-1) + 1, \quad k = 1, 2, \ldots, g \tag{8}$$

with $R(N_1, 0) = 1$.

Based on this recursion, we can also write

$$R(N_1 + g, g) = R(N, g) = \sum_{k=0}^{g} \left(\frac{N}{\alpha A}\right)^k = \frac{1 - \left(\frac{N}{\alpha A}\right)^{g+1}}{1 - \frac{N}{\alpha A}}. \tag{9}$$

IV. Properties of the Loss Formulas

Based on (1), (2), and (3), as well as on the recursive relations (5), (6), and (7), some important relations both for the blocking as well as for the dropping probability can be proven. At first, a relation for Erlang-B formula is given, which is used in the proofs of the subsequent relations for the loss probabilities.

Property 4.1: The loss probability $E_B(A, N)$ according to Erlang-B formula is a decreasing function of N, i.e., $E_B(A, N) < E_B(A, N-1)$.

Proof: The proof is given in Appendix A.2. QED

For the dropping probability $P_d(N, g)$ the following relations hold, assuming that all other system parameters are fixed.

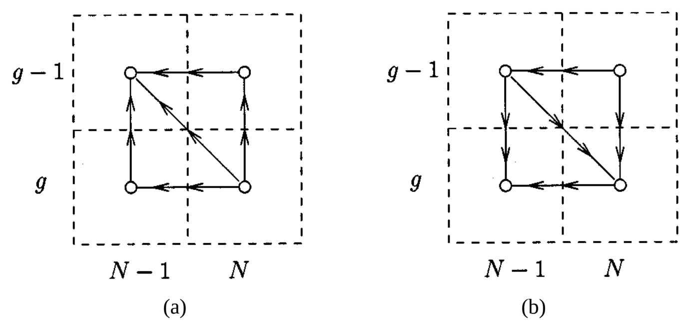

Fig. 3. Monotonicity properties of the loss probabilities. (a) For P_d. (b) For P_b.

Property 4.2: The dropping probability $P_d(N, g)$ is a decreasing function of g (for a fixed N), i.e., $P_d(N, g) < P_d(N, g-1)$.

Proof: The proof is given in Appendix A.3. QED

Property 4.3: The dropping probability $P_d(N, g)$ is a decreasing function of N (for a fixed g), i.e., $P_d(N, g) < P_d(N-1, g)$.

Proof: The proof is given in Appendix A.4. QED

Property 4.4: The dropping probability $P_d(N, g)$ is a decreasing function of N and g, i.e., if both N and g are increased by one at the same time, the following relation holds: $P_d(N, g) < P_d(N-1, g-1)$.

Proof: The relation follows directly from the two previous Propositions 4.3 and 4.2. QED

These three relations are summarized in Fig. 3(a).

For the blocking probability $P_b(N, g)$ the following relations hold, assuming that all other system parameters are fixed.

Property 4.5: The blocking probability $P_b(N, g)$ is a decreasing function of N (for a fixed g), i.e., $P_b(N, g) < P_b(N-1, g)$.

Proof: The proof is given in Appendix A.5. QED

TABLE I
RESULTS OF OPTIMIZATION PROBLEM $\mathbf{O_1}$

$P_{d\mathbf{0}}$	g^*	$P_d(g^*)$	$P_b(g^*)$
10^{-2}	0	0.003992	0.003992
10^{-3}	3	0.000504	0.012528
10^{-4}	6	0.000065	0.023195
10^{-5}	9	0.000008	0.038967
10^{-6}	13	0.00000058	0.069839

Property 4.6: The blocking probability $P_b(N,g)$ is an increasing function of g (for a fixed N), i.e., $P_b(N,g) > P_b(N,g-1)$.

Proof: The proof is given in Appendix A.6. QED

Property 4.7: The blocking probability $P_b(N,g)$ is an increasing function of N and g, i.e., if both N and g are increased by one at the same time, the following relation holds: $P_b(N,g) > P_b(N-1,g-1)$.

Proof: The proof is given in Appendix A.7. QED

These three relations are summarized in Fig. 3(b).

V. OPTIMIZATION PROBLEMS

For the problem on hand, we like to minimize both the blocking probability as well as the dropping probability. Hence, we have a multiobjective optimization problem [7]. The decision variables are the number of guard channels, g, and the number of channels, N. In a simpler version of the problem, we fix N and consider only g as the decision variable.

Given the two objectives, there are several different ways we can set up the optimization problem. We can pick either P_b or P_d as the objective function to be minimized and we impose a constraint on the other one. Thus, we consider two representative optimization problems below.

A. Optimal Number of Guard Channels

$\mathbf{O_1}$: Given A, N and α, determine the optimal integer value of g so as to

$$\text{minimize } P_b(g) \text{ such that } P_d(g) \leq P_{d\mathbf{0}}.$$

In order to solve the optimization problem above, we use the Properties 4.2 and 4.6 from Section IV.

Based on property 4.2, we first determine the smallest value of g such that $P_d(g) \leq P_{d\mathbf{0}}$. Then using the Property 4.6 we see that such a value of g will minimize $P_b(g)$. Thus the optimal value of g is obtained using a simple one-dimensional (1-D) search over the range $\{0,1,2,\ldots,N-1\}$ for g such that

$$g^* = \min\{g | P_d(g) \leq P_{d\mathbf{0}}\}. \tag{10}$$

As a numerical example, we take $A = 80$, $\mu_1 + \mu_2 = 1$, $\alpha = 0.5$ and $N = 100$. Table I gives the optimal values of g^* for different values of $P_{d\mathbf{0}}$.

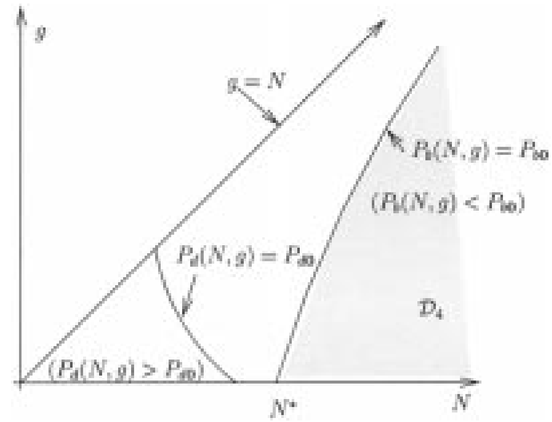

Fig. 4. Optimization problem $\mathbf{O_2}$; case 1: $P_{b\mathbf{0}} < P_{d\mathbf{0}}$.

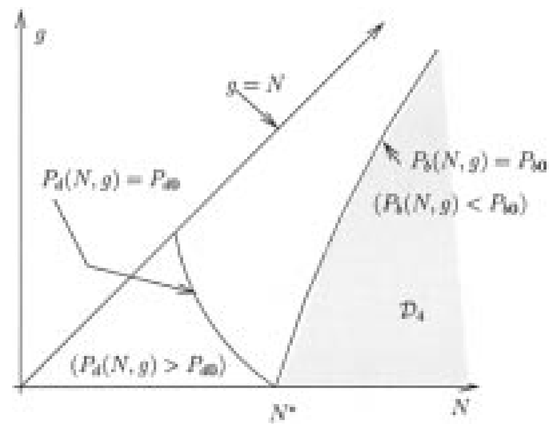

Fig. 5. Optimization problem $\mathbf{O_2}$; case 2: $P_{b\mathbf{0}} = P_{d\mathbf{0}}$.

B. Optimal Number of Channels

$\mathbf{O_2}$: Given A and α, determine the optimal integer values of N and g so as to

$$\text{minimize } N \text{ such that } \begin{cases} P_b(N,g) \leq P_{b\mathbf{0}} \\ P_d(N,g) \leq P_{d\mathbf{0}}. \end{cases}$$

In order to solve the optimization problem $\mathbf{O_2}$ above, we consider the first quadrant of the (N,g) plane shown in Figs. 4–6. In fact the region of interest is below the $g = N$ line. Further on this line, $P_b(N,N) = 1.0$. Also note that $P_d(N,0) = P_b(N,0) = E_B(A,N)$. This property enables us to distinguish three cases depending upon the values of $P_{d\mathbf{0}}$ and $P_{b\mathbf{0}}$. Note that although g and N are integers, for the sake of convenience the figures and much of the discussion below refer to them as if they were real variables.

The first case, when $P_{b\mathbf{0}} < P_{d\mathbf{0}}$, is shown in Fig. 4. In this case, we know that the active constraint will be $P_b(N,g) \leq P_{b\mathbf{0}}$ as the entire region to the right of the contour $P_d(N,g) = P_{d\mathbf{0}}$ and below the line $g = N$ will satisfy the constraint $P_d(N,g) \leq P_{d\mathbf{0}}$. Thus, the intersection of the two regions is the feasible region $\mathcal{D}_4$. It is clear that the minimum value of N, denoted by N^*, is the smallest value of N such that $P_b(N,0) = E_B(A,N) \leq P_{b\mathbf{0}}$, and $g^* = 0$.

In the second case, when $P_{b\mathbf{0}} = P_{d\mathbf{0}}$, we have the situation depicted in Fig. 5. In this case, $g^* = 0$ and N^* is obtained as the smallest value of N that satisfies $E_B(A,N) \leq P_{b\mathbf{0}} = P_{d\mathbf{0}}$.

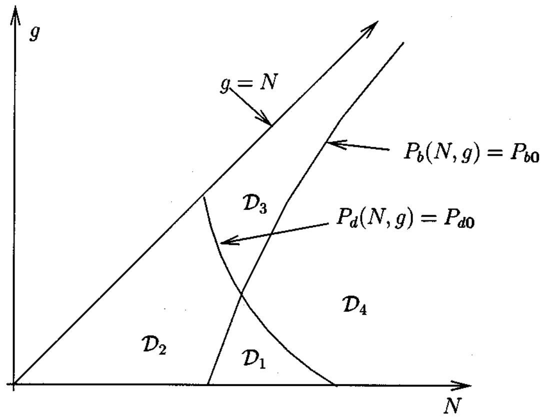

Fig. 6. Optimization problem $\mathbf{O_2}$; case 3: $P_{b0} > P_{d0}$.

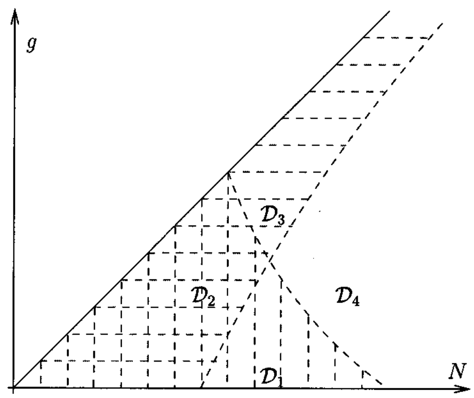

Fig. 7. Optimization problem $\mathbf{O_2}$; regions of interest.

The third case, when $P_{b\mathbf{0}} > P_{d\mathbf{0}}$, is the most interesting one ; it is shown in Fig. 6. Since we have $P_d(N,g) > P_{d\mathbf{0}}$ on $\{\mathcal{D}_1 \cup \mathcal{D}_2\}$ and $P_b(N,g) > P_{b\mathbf{0}}$ on $\{\mathcal{D}_2 \cup \mathcal{D}_3\}$, the feasible region is labeled $\mathcal{D}_4$ in this figure. In order to explain the algorithm that will follow, we show the regions $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_3$ and $\mathcal{D}_4$ in detail in Fig. 7. In the figure, we show vertical dotted lines in (N,g) plane for the regions when the constraint $P_d(N,g) \le P_{d\mathbf{0}}$ is violated and horizontal dotted lines are used for the regions where $P_b(N,g) \le P_{b\mathbf{0}}$ is violated. The following algorithm $\mathbf{O_2}$ which consists of three parts (A, B, C), is based on bisection and utilizes the fast recursive formulas for the computation of $P_d(N,g)$ and $P_b(N,g)$ we have developed earlier.

Algorithm $\mathbf{O_2}$*: Part A: Calculate* $E_B(A,N)$ *for* $N \in [1, N_{\max}]$*, where* $N_{\max}$ *is the smallest value of* N *such that* $E_B(A, N_{\max}) \le P_{d\mathbf{0}}$.

Determine $N_{\min}$ *as the smallest value of* $N \in [1, N_{\max}]$ *such that* $E_B(A, N_{\min}) \le P_{b0}$.

Step 1: $g := 0$
$N_{\text{mid}} := (N_{\max} + N_{\min})/2$
$N := N_{\text{mid}}$
Step 2: $g := g + 1$
$N := N + 1$
calculate $P_d(N,g)$ *and* $P_b(N,g)$ *from* $P_d(N-1, g-1)$ *and* $P_b(N-1, g-1)$
using (6) and (7)
case of:
$(N,g) \in \mathcal{D}_1$: goto step 2
$(N,g) \in \mathcal{D}_2$: $N_{\min} := N_{\text{mid}}$
goto step 1
$(N,g) \in \mathcal{D}_3$: $N_{\min} := N_{\text{mid}}$
find smallest value of $N (= N_b) \in [N_{\min} + g, N_{\max} + g]$
such that $P_b(N_b, g) \le P_{b\mathbf{0}}$ using part **B.**
find smallest value of $N (= N_d) \in [N_{\min} + g - 1, N_{\max} + g - 1]$
such that $P_d(N_d, g-1) \le P_{d\mathbf{0}}$ using part **C.**
case of:
$N_b = N_d$: $N^* := N_b$, $g^* := g$ or $g - 1$
$N_b < N_d$: $N^* := N_b$, $g^* := g N_b > N_d$
$N_b > N_d$: $N^* := N_d$, $g^* := g - 1$
endcase.
$(N,g) \in \mathcal{D}_4$: $N_{\max} := N_{\text{mid}}$
goto step 1.
endcase.

Part B:

Part **B**:
$\min := N_{\min}$
$\max := N_{\max}$
while $(\max - \min > 1)$ **do**
$\text{mid} := (\min + \max)/2$
$n := \text{mid}$
for $g' \in [1, g]$ do
$n := n + 1$
Calculate $P_d(n, g')$ and $P_b(n, g')$ from $P_d(n-1, g'-1)$ and
$P_b(n-1, g'-1)$ *using (6) and (7)*
endfor
if $(P_b(n,g) < P_{b\mathbf{0}})$
then $\max := \text{mid}$
else $\min := \text{mid}$
endwhile
$N_b := \max + g$

Part **C** has basically the same structure as Part **B**, except that:

1) the for-loop runs up to $g - 1$ instead of g;
2) the calculation within the for-loop is replaced by: calculate $P_d(n, g')$ from $P_d(n-1, g'-1)$ using (6);
3) the if-condition is $(P_d(n, g-1) < P_{d\mathbf{0}})$;
4) the last statement is $N_d := \max + g - 1$.

As a numerical illustration, we take $A = 80$, $\mu_1 + \mu_2 = 1$, and $\alpha = 0.5$. Table II gives the results of optimization problem $\mathbf{O_2}$ for various pairs of $P_{d\mathbf{0}}$ and $P_{b\mathbf{0}}$ values.

VI. Fixed Point Iteration

In the earlier sections of this paper, we assumed that handoff call arrival rate λ_2 is given. In practice, the value of λ_2 needs to be determined as a function of λ_1, μ_1, μ_2, N and g. We assume that all cells are statistically identical. Thus the rate of handoff out from a cell equals the rate at which handoff calls arrive into

TABLE II
RESULTS OF OPTIMIZATION PROBLEM $\mathbf{O_2}$

P_{d0}	P_{b0}	N^*	g^*	$P_d(N^*, g^*)$	$P_b(N^*, g^*)$
10^{-3}	10^{-2}	101	2	0.000791455	0.0077859
10^{-4}	10^{-3}	109	2	0.000085555	0.0009482
10^{-5}	10^{-4}	116	2	0.000007625	0.0000933
10^{-6}	10^{-5}	122	2	0.000000687	0.0000091

a cell. In other words, the steady–state *handoff-out* throughput should equal the arrival rate (λ_2) of the *handoff-in* traffic.

Let $T(x)$ denote the throughput of *handoff-out* for $\lambda_2 = x$ when the other parameters λ_1, μ_1, μ_2, N and g are fixed. By definition

$$T(x) = \mu_2 \sum_{n=1}^{N} n p_n. \tag{11}$$

In Appendix A.8, we show that

$$T(x) = \frac{\mu_2}{\mu}\lambda_1(1 - P_b(x)) + x\frac{\mu_2}{\mu}(1 - P_d(x)). \tag{12}$$

If we consider $x = T(x)$, we get

$$x = \frac{\mu_2\lambda_1(1 - P_b(x))}{\mu - \mu_2(1 - P_d(x))}. \tag{13}$$

Consider the function $f(x)$, the right-hand side of (13)

$$f(x) = \frac{\mu_2\lambda_1(1 - P_b(x))}{\mu - \mu_2(1 - P_d(x))}. \tag{14}$$

It is easy to see that $f(x)$ is a decreasing function of x on $[0, +\infty]$ (see Appendix, Sections A.9 and A.10). Moreover

$$\begin{aligned} f(0) =& \frac{\mu_2\lambda_1}{\mu_1}(1 - E_B(A_0, N - g)) \text{ with} \\ A_0 =& \frac{\lambda_1}{\mu}, \text{ and } \lim_{x \to +\infty} f(x) = 0. \end{aligned} \tag{15}$$

So the solution of $x = f(x)$ on $[0, +\infty]$ is unique. Denote this unique solution by $\tilde{x}$. This value can be obtained using the iterative procedure

$$x^k = f(x^{k-1}) \quad k = 1, 2, \ldots \tag{16}$$

with for example $x^0 = \mu_2\lambda_1/\mu_1$.

Because of the monotonicity of $f(x)$, $P_b(x)$ and $P_d(x)$, it is also easy to see that $P_b(\tilde{x}) \leq P_b(x^0)$ and $P_d(\tilde{x}) \leq P_d(x^0)$. So x^0 can be used as an approximation to $\tilde{x}$. In fact, since $P_b(x)$ and $P_d(x)$ are expected to be small values, the approximation x^0 should be very good in most situations. We also found that the iteration above converges very fast (within a few iterations) in practice.

We can combine the fixed point iteration with the optimization discussed in the previous section. Consider for instance

TABLE III
RESULTS OF OPTIMIZATION PROBLEM $\mathbf{FO_1}$

iteration #	x_{i-1}	g_i	$10^4 P_d(g_i)$	$10^3 P_b(g_i)$
1	40.000000	3	5.042086	12.252801
2	39.739356	2	9.323258	9.123114
⋮	⋮	⋮	⋮	⋮
9	39.611338	2	8.839696	8.988869
10	39.611072	2	8.839114	8.988184

combining problem $\mathbf{O_1}$ and the fixed point iteration to determine the optimal value of g given a constraint on the dropping probability. We can proceed as per the following algorithm:

Algorithm $\mathbf{FO_1}$*:*

$x_0 := \mu_2\lambda_1/\mu_1$
$x_{-1} := 2x_0$
$i := 0$
while$(|x_i - x_{i-1}|/x_{i-1} > \epsilon)$**do**
 step 1: $i := i + 1$
 step 1: Determine g_i as the optimal value of g in the $\mathbf{O_1}$ problem given $\lambda_2 = x_{i-1}$, $A = (\lambda_1 + \lambda_2)/\mu$ and $\alpha = \lambda_2/(\lambda_1 + \lambda_2)$.
 step 3: Determine x_i as the solution of the fixed point iteration (16) using $g = g_i$.
endwhile
$g^* = g_i$
$\lambda_2 = x_i$
$P_d = P_d(g^*, \lambda_2)$
$P_b = P_b(g^*, \lambda_2)$

As a numerical example, we use $N = 100$, $\lambda_1 = 40$, $\mu_1 = \mu_2 = 0.5$ and $P_{d\mathbf{0}} = 10^{-3}$. For $\epsilon = 10^{-5}$, the iterative procedure converged in ten steps. Table III gives the values for the first and last steps. Proof of convergence of the algorithm $\mathbf{FO_1}$ is still an open problem.

VII. CONCLUSION

We have developed a performance model of wireless handoff scheme using a Markov chain. We derived fast recursive formulas for the blocking probability of new calls and dropping probability of handoff calls. We proved useful monotonicity properties of these loss probabilities. We developed optimization problems to determine the optimal number of guard channels and the optimal number of total channels. Effective algorithms to solve the optimization problems are provided. We developed a fixed-point iteration-based scheme to determine handoff arrival rate into a cell and showed the uniqueness of the fixed point.

APPENDIX

We will use the following notation throughout the Appendix:

$$Y_1(N, g) = \sum_{n=0}^{N-g-1} \frac{A^n}{n!}, \quad Y_2(N, g) = \sum_{n=N-g}^{N} \frac{A^n}{n!}\alpha^{n-(N-g)}.$$

A. *Recursive Formulas for* $P_d(N,g)$ *and* $P_b(N,g)$

In addition to the previous notation, let us define

$$\begin{aligned} G(N,g) =& Y_1(N,g) + Y_2(N,g), \\ \phi(N,g) =& \frac{A^{N-g}}{N!} A_1^g \text{ and } \alpha A = A_1. \end{aligned}$$

Then $\phi(N,g) = \alpha A/N \phi(N-1, g-1)$ and $Y_2(N,g) = Y_2(N-1, g-1) + \phi(N,g)$.

Since $Y_1(N,g) = Y_1(N-1,g-1)$, we may write

$$\begin{aligned} P_d(N,g) =& \frac{\phi(N,g)}{G(N,g)} \\ =& \frac{\frac{\alpha A}{N}\phi(N-1,g-1)}{G(N-1,g-1) + \frac{\alpha A}{N}\phi(N-1,g-1)} \\ =& \frac{P_d(N-1,g-1)}{\frac{N}{\alpha A} + P_d(N-1,g-1)} \end{aligned}$$

and

$$\begin{aligned} P_b(N,g) =& \frac{Y_2(N,g)}{G(N,g)} \\ =& \frac{Y_2(N-1,g-1) + \phi(N,g)}{G(N,g)} \\ =& \frac{Y_2(N-1,g-1) + \frac{\alpha A}{N}\phi(N-1,g-1)}{G(N-1,g-1) + \frac{\alpha A}{N}\phi(N-1,g-1)} \\ =& \frac{\frac{N}{\alpha A} P_b(N-1,g-1) + P_d(N-1,g-1)}{\frac{N}{\alpha A} + P_d(N-1,g-1)}. \end{aligned}$$

B. *Proof of Property 4.1* $E_B(A,N) < E_B(A,N-1)$

From (5) we see that $E_B(A,N) < E_B(A,N-1)$ is equivalent to showing that $E_B(A,N-1) + N/A > 1$. This is obvious if $A \le N$. More generally

$$\begin{aligned} E_B(A,N-1) + \frac{N}{A} =& \frac{\frac{A^{N-1}}{(N-1)!}}{\sum_{i=0}^{N-1} \frac{A^i}{i!}} + \frac{N}{A} \\ =& \frac{\frac{A^N}{(N-1)!} + N\sum_{i=0}^{N-1} \frac{A^i}{i!}}{\frac{A^N}{(N-1)!} + \sum_{i=0}^{N-2} \frac{A^{i+1}}{i!}} \\ =& \frac{\frac{A^N}{(N-1)!} + N\sum_{i=0}^{N-1} \frac{A^i}{i!}}{\frac{A^N}{(N-1)!} + \sum_{i=1}^{N-1} \frac{A^i}{(i-1)!}}. \end{aligned}$$

Because $(N/i)(A^i/(i-1)!) > (A^i/((i-1)!)$, we conclude that $E_B(A,N-1) + (N/A) > 1$.

C. *Proof of Property 4.2* $P_d(N,g) < P_d(N,g-1)$

Since the dropping probability for handoff calls can be written as

$$P_d(N,g) = \frac{\frac{A^N}{N!}\alpha^g}{Y_1(N,g) + Y_2(N,g)} \tag{17}$$

it is equivalent to show that

$$\begin{aligned} & \frac{A^N}{N!}\alpha^g [Y_1(N,g-1) + Y_2(N,g-1)] \\ & \quad < \frac{A^N}{N!}\alpha^{g-1}[Y_1(N,g) + Y_2(N,g)] \\ & \Leftrightarrow \alpha[Y_1(N,g-1) + Y_2(N,g-1)] \\ & \quad < [Y_1(N,g) + Y_2(N,g)] \\ & \Leftrightarrow \alpha\left[Y_1(N,g) + \frac{A^{N-g}}{(N-g)!}\right] + \alpha \sum_{n=N-g+1}^{N} \frac{A^n}{n!}\alpha^{n-(N-g+1)} \\ & \quad < [Y_1(N,g) + Y_2(N,g)] \\ & \Leftrightarrow \alpha Y_1(N,g) + \sum_{n=N-g}^{N} \frac{A^n}{n!}\alpha^{n-(N-g)} \\ & \quad < [Y_1(N,g) + Y_2(N,g)] \\ & \Leftrightarrow 0 < (1-\alpha) Y_1(N,g) \end{aligned}$$

which is always true since $0 < \alpha < 1$ and $Y_1(N,g) > 0$.

D. *Proof of Property 4.3* $P_d(N,g) < P_d(N-1,g)$

This is equivalent to show that

$$\begin{aligned} & \frac{A^N}{N!}\alpha^g [Y_1(N-1,g) + Y_2(N-1,g)] \\ & \quad < \frac{A^{N-1}}{(N-1)!}\alpha^g [Y_1(N,g) + Y_2(N,g)] \\ & \Leftrightarrow \frac{A}{N}[Y_1(N-1,g) + Y_2(N-1,g)] \\ & \quad < [Y_1(N,g) + Y_2(N,g)] \\ & \Leftrightarrow \frac{A}{N}\left[\sum_{n=0}^{N-g-2} \frac{A^n}{n!} + \sum_{n=N-g-1}^{N-1} \frac{A^n}{n!}\alpha^{n-(N-1-g)}\right] \\ & \quad < \sum_{n=0}^{N-g-1} \frac{A^n}{n!} + \sum_{n=N-g}^{N} \frac{A^n}{n!}\alpha^{n-(N-g)} \\ & \Leftrightarrow \sum_{n=0}^{N-g-2} \frac{A}{N}\frac{A^n}{n!} + \sum_{n=N-g-1}^{N-1} \frac{A}{N}\frac{A^n}{n!}\alpha^{n-(N-1-g)} \\ & \quad < 1 + \sum_{n=0}^{N-g-2} \frac{A}{(n+1)}\frac{A^n}{n!} \\ & \qquad + \sum_{n=N-g-1}^{N-1} \frac{A}{(n+1)}\frac{A^n}{n!}\alpha^{n-(N-1-g)} \end{aligned}$$

which is always true since α and A are positive.

E. *Proof of Property 4.5* $P_b(N,g) < P_b(N-1,g)$

Since the blocking probability of new calls can be written as

$$P_b(N,g) = \frac{Y_2(N,g)}{Y_1(N,g) + Y_2(N,g)} \tag{18}$$

it is equivalent to show that

$$\begin{aligned}
&Y_2(N,g)[Y_1(N-1,g)+Y_2(N-1,g)]\\
&\quad< Y_2(N-1,g)[Y_1(N,g)+Y_2(N,g)]\\
&\Leftrightarrow Y_2(N,g)Y_1(N-1,g) < Y_2(N-1,g)Y_1(N,g)\\
&\Leftrightarrow Y_2(N-1,g) > Y_2(N,g)\frac{Y_1(N-1,g)}{Y_1(N,g)}\\
&\Leftrightarrow Y_2(N-1,g) > Y_2(N,g)[1-E_B(A,N-g-1)]\\
&\Leftrightarrow \sum_{n=N-g-1}^{N-1}\frac{A^n}{n!}\alpha^{n-(N-g-1)}\\
&\quad-\sum_{n=N-g}^{N}\frac{A^n}{n!}\alpha^{n-(N-g)}[1-E_B(A,N-g-1)]>0\\
&\Leftrightarrow \sum_{k=0}^{g}\alpha^k\frac{A^{N-g-1+k}}{(N-g-1+k)!}\\
&\quad-\sum_{j=0}^{g}\alpha^j\frac{A^{N-g+j}}{(N-g+j)!}[1-E_B(A,N-g-1)]>0\\
&\Leftrightarrow \sum_{k=0}^{g}\alpha^k\frac{A^{N-g-1+k}}{(N-g-1+k)!}\\
&\quad\times\left[1-\frac{A}{(N-g+k)}[1-E_B(A,N-g-1)]\right]>0.
\end{aligned}$$

Now, from (5) we can show that

$$\frac{A}{k}[1-E_B(A,k)]=\frac{E_B(A,k)}{E_B(A,k-1)},\quad k=1,2,\ldots \tag{19}$$

and $(E_B(A,k))/(E_B(A,k-1))<1$ by property 4.1.

So, since $A/(N-g+k)<A/(N-g-1), k=0,1,\ldots,g$, this implies that

$$\left[1-\frac{A}{(N-g+k)}[1-E_B(A,N-g-1)]\right]>0,\quad k=0,1,(g-1).$$

Therefore, $P_b(N,g)$ is decreasing with respect to N.

F. *Proof of Property 4.6* $P_b(N,g)>P_b(N,g-1)$

From (7) we can write

$$P_b(N,g)=\frac{\frac{N}{\alpha A}P_b(N-1,g-1)+P_d(N-1,g-1)}{\frac{N}{\alpha A}+P_d(N-1,g-1)},\quad k=1,2,\ldots,g \tag{20}$$

and by property 4.5

$$\frac{\frac{N}{\alpha A}P_b(N-1,g-1)+P_d(N-1,g-1)}{\frac{N}{\alpha A}+P_d(N-1,g-1)} > \frac{\frac{N}{\alpha A}P_b(N,g-1)+P_d(N-1,g-1)}{\frac{N}{\alpha A}+P_d(N-1,g-1)}.$$

Therefore

$$\begin{aligned}
&\frac{N}{\alpha A}P_b(N,g)+P_d(N-1,g-1)P_b(N,g)\\
&\quad>\frac{N}{\alpha A}P_b(N,g-1)+P_d(N-1,g-1)\\
&\Leftrightarrow \frac{N}{\alpha A}(P_b(N,g)-P_b(N,g-1))\\
&\quad>P_d(N-1,g-1)(1-P_b(N,g))\\
&\Leftrightarrow \frac{N}{\alpha A}(P_b(N,g)-P_b(N,g-1))>0.
\end{aligned}$$

Then since α and A are positive, $P_b(N,g)$ is increasing with respect to g.

G. *Proof of Property 4.7* $P_b(N,g)>P_b(N-1,g-1)$

From (7) we can write

$$P_b(N,g)=\frac{\frac{N}{\alpha A}P_b(N-1,g-1)+P_d(N-1,g-1)}{\frac{N}{\alpha A}+P_d(N-1,g-1)}. \tag{21}$$

Therefore

$$\begin{aligned}
&\left[\frac{N}{\alpha A}+P_d(N-1,g-1)\right](P_b(N,g)-P_b(N-1,g-1))\\
&\quad=\frac{N}{\alpha A}P_b(N-1,g-1)+P_d(N-1,g-1)\\
&\qquad-\frac{N}{\alpha A}P_b(N-1,g-1)\\
&\qquad-P_b(N-1,g-1)P_d(N-1,g-1)\\
&\quad=(1-P_b(N-1,g-1))P_d(N-1,g-1)>0.
\end{aligned}$$

Thus, $(P_b(N,g)-P_b(N-1,g-1))$ is always positive since α and A are positive.

H. *Proof of the Expression of* $T(x)$

Remember that here λ_2 is the variable and let us denote

$$A_x=\frac{\lambda_1+x}{\mu},\quad \alpha_x=\frac{x}{\lambda_1+x} \tag{22}$$

$$\begin{aligned}
T(x)=&\mu_2\sum_{n=1}^{N}np_n(x)\\
=&\mu_2p_0(x)\left[\sum_{n=1}^{N-g-1}n\frac{A_x^n}{n!}+\sum_{n=N-g}^{N}n\frac{A_x^n}{n!}\alpha_x^{n-(N-g)}\right]\\
=&\mu_2p_0(x)A_x\\
&\times\left[\sum_{n=0}^{N-g-2}\frac{A_x^n}{n!}+\sum_{n=N-g}^{N}\frac{A_x^{n-1}}{(n-1)!}\alpha_x^{n-(N-g)}\right]\\
=&\mu_2p_0(x)A_x\\
&\times\left[\sum_{n=0}^{N-g-1}\frac{A_x^n}{n!}\right.\\
&\qquad\left.+\alpha_x\sum_{n=N-g+1}^{N}\frac{A_x^{n-1}}{(n-1)!}\alpha_x^{n-1-(N-g)}\right]
\end{aligned}$$

$$
\begin{aligned}
&= \mu_2 p_0(x) A_x \\
&\quad \times \left[Y_1(N,g,x) + \alpha_x \sum_{k=N-g}^{N-1} \frac{A_x^k}{k!} \alpha_x^{k-(N-g)} \right] \\
&= \mu_2 p_0(x) A_x \\
&\quad \times \left[Y_1(N,g,x) + \alpha_x Y_2(N,g,x) - \alpha_x \frac{A_x^N}{N!} \alpha_x^g \right] \\
&= \frac{\mu_2}{\mu} p_0(x) \lambda_1 Y_1(N,g,x) + x \frac{\mu_2}{\mu} p_0(x) Y_1(N,g,x) \\
&\quad + \mu_2 p_0(x) A_x \frac{x}{\lambda_1 + x} \left(Y_2(N,g,x) - \frac{A_x^N}{N!} \alpha_x^g \right) \\
&= \frac{\mu_2}{\mu} p_0(x) \lambda_1 Y_1(N,g,x) \\
&\quad + x \frac{\mu_2}{\mu} p_0(x) \\
&\quad \times \left(Y_1(N,g,x) + Y_2(N,g,x) - \frac{A_x^N}{N!} \alpha_x^g \right) \\
&= \frac{\mu_2}{\mu} \lambda_1 (1 - P_b(x)) + x \frac{\mu_2}{\mu} (1 - P_d(x)).
\end{aligned}
$$

I. Proof that $P_d(x)$ is Increasing in x

$$
\begin{aligned}
P_d^{-1}(x) &= \frac{N!}{A_x^N \alpha_x^g} [Y_1(N,g,x) + Y_2(N,g,x)] \\
&= \frac{N!}{(\lambda_1 + x)^{N-g} x^g} \\
&\quad \times \left[\sum_{n=1}^{N-g-1} \frac{1}{n!} (\lambda_1 + x)^n \right. \\
&\quad \left. + \sum_{n=N-g}^{N} \frac{1}{n!} x^n \left(\frac{\lambda_1 + x}{x} \right)^{N-g} \right] \\
&= \sum_{n=1}^{N-g-1} \frac{N!}{n!} \frac{1}{(\lambda_1 + x)^{N-g-n} x^g} \\
&\quad + \sum_{n=N-g}^{N} \frac{N!}{n!} \frac{1}{x^{N-g}} \\
&= \sum_{k=1}^{N-g} \frac{N!}{(N-g-k)!} \frac{1}{(\lambda_1 + x)^k x^g} \\
&\quad + \sum_{k=0}^{g} \frac{N!}{(N-k)!} \frac{1}{x^k}
\end{aligned}
$$

which is a sum of decreasing functions of x. Therefore, $P_d(x)$ is increasing with respect to x.

J. Proof that $P_b(x)$ is Increasing in x

$$
\begin{aligned}
P_b^{-1}(x) &= 1 + \frac{Y_1(N,g,x)}{Y_2(N,g,x)} \\
&= 1 + \frac{\sum_{n=1}^{N-g-1} \frac{1}{n!} (\lambda_1 + x)^n}{\sum_{n=N-g}^{N} \frac{1}{n!} x^n (\frac{\lambda_1 + x}{x})^{N-g}} \\
&= 1 + \frac{\sum_{n=0}^{N-g-1} \frac{1}{n!} \frac{1}{(\lambda_1 + x)^{N-g-n}}}{\sum_{k=0}^{g} \frac{x^k}{(k+N-g)!}}.
\end{aligned}
$$

So $P_b^{-1}(x)$ can be written as $1 + (H(x)/G(x))$ where $H(x)$ is decreasing wrt x and $G(x)$ is increasing with respect to x. Therefore, $P_b(x)$ is increasing with respect to x.

ACKNOWLEDGMENT

The authors would like to thank the Universitat de les Illes Balears, Palma, Spain, for providing the sunny and floral atmosphere conducive to this research. They would also like to thank M. Curiel for her assistance with drawing a figure and Y. Cao for carrying out some of the computations in this paper.

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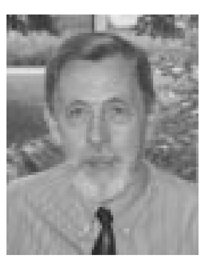
Guenter Haring (M'86–SM'96) has been a Full Professor of applied computer science at the University of Vienna, Austria, since 1985.

From 1989 to 1993, he was President of the Austrian Computer Society. Since January 2000, he has been Dean of the School of Business, Economics and Computer Science, University of Vienna. He was one of the three founding members of the Austrian Center for Parallel Computation (ACPC). He was also founding member of the Computer Measurement Group Central Europe (CMG-CE). He has published over 140 scientific papers and is coeditor of four books. His research interests include performance evaluation of computer systems, distributed and communication systems, mobile communication, cooperative work, and user interface design. He was and is a Leader of several national and international projects in these areas, covering both applied and basic research.

Mr. Haring is a member of the IEEE Computer Society, the Association for Computing Machinery (ACM), the Austrian Computer Society (OCG) and the German Gesellschaft fir Informatik (GI).



Raymond Marie received the Doctorat d'Ing. and the Doctorat d'Etat es Sciences Mathématiques degrees from the University of Rennes, France, in 1973 and 1978, respectively.

From 1977 to 1999, he was a Research Manager of an INRIA group in modeling. He spend the 1981–1982 academic year as a Visiting Associate Professor at North Carolina State University, Raleigh, NC. Since 1983, he has been a Professor at the Computer Science Department, University of Rennes. His active research interests include performance evaluation of computer systems, high-speed networks, and reliability computation of complex systems.

Ramon Puigjaner (M'69) received the Industrial Engineer degree from the Universitat Polithcnica de Catalunya, Barcelona, Spain, in 1964, the Masters degree in aeronautical sciences from the Ecole Nationale Supirieure de l'Aironautique de Paris,France, the Ph.D. degree from the Universitat Polithcnica de Catalunya, Barcelona, Spain, in 1972, and the degree of License in Informatics from the Universidad Politicnica de Madrid, Spain, in 1972.

From 1966 to 1987, he shared his time between the Universitat Polithcnica de Catalunya, where he taught and researched automatic control, computer architecture and computer performance evaluation. He held several positions in the industry, mainly from 1970 to 1987, in UNIVAC (after SPERRY and finally UNISYS), where he was in charge of computer performance measuring and modeling for tuning and sizing in Spain. In 1987, he joined (full time) the Department of Computer Science, Universitat de les Illes Balears, Palma de Mallorca, Spain, where he is currently a Professor of computer architecture and technology and Director of the Polytechnic School of the same university. He is the Spanish Representative at the IFIP TC 6 Communications. He has been involved and is still involved in several ESPRIT projects as well as in several projects funded by the Spanish Comisisn Interministerial de Ciencia y Tecnologma and has acted as Project Reviewer and Evaluator for the Commission of the European Union. He is on the editorial board of the *Journal on Computer Networks* and is the author of a book on computer performance evaluation and of more than 90 reviewed papers in international journals and conferences. His current research interests are the performance evaluation of computer systems and computer networks and the diffusion of these techniques in the industrial milieu mainly in the field of real-time and distributed systems.

Dr. Puigjaner is a member of the ACM, the IFIP WG 6.3 Performance of Computer Networks, the IFIP WG 6.4 High-Performance Networks, and the IFIP WG 10.3 Distributed Systems. He was awarded the IFIP Silver Core.

Kishor Trivedi (M'86–SM'87–F'92) received the B.Tech. degree from the Indian Institute of Technology, Bombay, India, and the M.S. and Ph.D. degrees in computer science from the University of Illinois, Urbana-Champaign.

He holds the Hudson Chair in the Department of Electrical and Computer Engineering, Duke University, Durham, NC. He also holds a joint appointment in the Department of Computer Science at the same university. He is the Duke Site Director of an NSF Industry–University Cooperative Research Center between North Carolina State University and Duke University for carrying out applied research in computing and communications. He has been on the Duke University faculty since 1975. He has served as a Principal Investigator on various AFOSR, ARO, Burroughs, DARPA, Draper Lab, IBM, DEC, Alcatel, Telcordia, Motorola, NASA, NIH, ONR, NSWC, Boeing, Union Switch and Signals, NSF, and SPC funded projects and as a consultant to industry and research laboratories. He is a codesigner of HARP, SAVE, SHARPE, SPNP, and SREPT modeling packages. These packages have been widely circulated. He has supervised 34 Ph.D. dissertations. He has published over 300 articles and lectured extensively. He is the author of the well-known text *Probability and Statistics with Reliability, Queuing and Computer Science Applications* (Englewood Cliffs, NJ: Prentice-Hall). He has recently had two books published: *Performance and Reliability Analysis of Computer Systems* (Norwell, MA: Kluwer) and *Queueing Networks and Markov Chains* (New York: Wiley). His research interests are in reliability and performance assessment of computer and communication systems.

Dr. Trivedi was an Editor of the IEEE TRANSACTIONS ON COMPUTERS from 1983 to 1987. He is a Golden Core member of the IEEE Computer Society.