Performance Analysis of Cellular Networks with Generally Distributed Handoff Interarrival Times

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Abstract

Wireless cellular networks experience the handoff phenomenon, in which a call already in progress in a cell due to user mobility is "handed-over" (switched) to another cell. A common assumption in most of these dimensioning models is that call cell arrival processes (both for new and handoff) are Poisson. Call service time distributions and cell residence times are typically allowed to follow arbitrary distributions due to the well-known insensitivity property of loss queuing systems. Recently, studies that question the validity of the assumption of handoff arrivals being Poissonian have appeared in the literature. The above mentioned fact mandates the need to consider more general performance models that allow for arbitrarily distributed interarrival times. In this paper we provide numerical solutions for new and handoff call blocking and for arbitrary handoff traffic distribution. For this purpose, we first prove that the underlying stochastic process that captures the number of occupied cell channels is a Markov regenerative process and subsequently we use their mathematical theory to develop numerical techniques for important Quality of Service (QoS) measures.

1 Introduction

Wireless cellular networks experience the handoff phenomenon, in which a call already in progress in a cell due to user mobility is "handed-over" (switched) into another cell. Handoff traffic into a cell consists of calls already in progress in neighboring cells that attempt to enter the cell under study and compete for the same resources as with the new calls that may be initiated in the same cell. handoff traffic, in terms of volume, becomes important in micro-cellular structures and/or high mobility environments since in these environments the handoff probability increases. Due to the widespread deployment of cellular networks and services in recent years and bandwidth scarcity over the air, cellular network dimensioning is a topic of paramount importance to wireless service providers and has seen significant attention in the literature. A common assumption in most of these dimensioning models is that call cell arrival processes (both for new and handoff) are Poisson. Call service time distributions and cell residence times are typically allowed to follow arbitrary distributions due to the well-known insensitivity property of loss queuing systems. Another important dimension of cellular system performance models is the handoff call treatment strategy, or how are the new and handoff calls are treated in terms of channel assignment. Since the forced termination of an ongoing call is considered less desirable than the blocking of a new call, many priority schemes have been proposed in the literature [5, 6, 8, 16]. These schemes vary in terms of complexity from a very simple non-prioritized scheme where all calls are treated equally [14] to more complex schemes that assign priorities (dynamically) based on measurements [16]. One very popular scheme that is seen as a good compromise in terms of performance and complexity is the so called, "guard channel scheme" [4, 5, 6]. In this scheme, a fixed number of the channels that are available in a given cell are reserved for handoff calls. For a cell with (total) capacity of C cells, g channels are reserved for handoff calls. Therefore when a new call arrives and there are g or fewer channels available the call will be rejected. Note that this scheme provides a minimum for the number of handoff calls (g) that will exist in the cell at any given time. Other schemes based on call (handoff and/or new) queuing have also appeared in the literature [6, 8].

Two important Quality of Service (QoS) measures have been defined in cellular networks; the first one is the *new call blocking probability*, a measure similar to the one that we see in telephone trunk systems. The second one, is the *handoff call dropping probability*. Handoff call dropping is extremely important in cellular systems since it leads to the undesired phenomenon of a forced call termination.

In order to obtain the above mentioned call blocking (i.e., both new call blocking and handoff dropping) measures[1] in a cellular system, one could consider all cells (or a neighborhood of cells). An arrival stream (for new calls) being associated with each. The effects of user mobility are captured in the cell residence time parameter, after its expiration the user will move to a neighboring cell according to some probability distribution. This cellular network leads to a queuing network that unfortunately does not have a product-form solution [14]. A simpler approach, assuming of course that all cells are statistically identical, that was first suggested in [5] is to study only one cell, the typical cell, and the handoff effect originating from user movement to neighboring cells is captured in a (average) handoff rate parameter. The relationship between the (average) handoff rate and the new call arrival rate was first determined by Hong and Rappaport [5] and further assumed that handoffs follow a Poisson process. Unfortunately, the derived relationship depends on the blocking measures too, which cannot be obtained if the handoff rate is not known. The fixed point equation and the existence of a fixed point was proved in [4].

Analyzing cellular networks for call blocking performance measures using either the multi-cell structure or a single representative cell have both their merits and demerits. It can be safely said that multi-cell models are much more complex to handle while a key issue in single cell models is the appropriate characterization of relevant parameters. In this paper we adopt the single cell model. We do allow however for an arbitrary distribution for the handoff call interarrival times, thus shifting the problem to the accurate characterization of the handoff arrival process.

Chlebus and Ludwin [1] were probably the first ones to question the Poisson assumption for handoff traffic. In their paper, they show that given Poisson new arrivals, the blocking phenomena in neighboring cells makes the handoff traffic non-Poisson. Using a two moment characterization of the interarrival time distribution they show that the handoff traffic is smooth (i.e., the variance of the interarrival times is smaller than its mean). They finally employ the Erlang fixed-point approximation (a.k.a reduced load approximation) to capture the dependencies between adjacent cells. By comparing approximate analysis with simulation, they conclude that the Poisson approximation for handoff traffic (when considering blocking effects) is a reasonable approximation in particular for light to moderate loading conditions. The smoothness of the handoff traffic due to blocking was also demonstrated by Rajaratnam and Takawira [11] that showed that Poisson process may not be appropriate for the handoff traffic when the handoff calls have traversed a large number of cellular boundaries. They also employ a two-moment approximate analysis method that shows that the Poisson assumption overestimates handoff call blocking. In a later work of theirs [12], they extend their two-moment performance analysis to cellular networks with channel reservation and general arrival process of handoff using discrete event simulation.

Orlik and Rappaport [9] addressed the issue of the handoff arrival process by considering a neighborhood of cells where all but one cell, the cell under study, is assumed to generate handoff arrivals according either to a Poisson or a two-state Markov-Modulated Poisson Process. In contrast to previous authors they find small differences between Poisson handoff traffic single cell analysis and non-Poisson/multiple cell analysis especially for heavy loads.

Finally, Zeng and Chlamtac [20] have addressed the effect of cell residence time on the handoff interarrival time distribution in both blocking and non-blocking environments. They show that the cell residence time distribution influences the handoff traffic statistics. Using a Gamma distribution for the cell residence times, they show that, in a non-blocking environment, for a large cell residence time variance, handoff traffic cannot be characterized by a Poisson process.

From the above we can conclude that there is significant evidence that the handoff traffic cannot be always modeled as a Poisson process. The distribution type of the interarrivals is still an open issue. Consequently, there is a great need to develop performance models that allow for general distributions in handoff interarrivals.

Performance analysis of wireless cellular systems with Poisson handoff arrivals that specifically adopt the guard channel scheme has been carried out by a number of authors [4, 8, 15, 17, 18, 19]. In particular, the "wireless Erlang-B formulae" are derived and their important properties are discussed in [4]. As we saw earlier, these results may not be suitable for the performance analysis of emerging third generation cellular networks or other cellular networks where the handoff traffic is known to be non-Poissonian. To obtain

[1]Call blocking measures are also known as Grade of Service (GoS)

realistic performance measures for wireless networks, one should consider the appropriate distribution for the handoff interarrival times. Thus, in this paper we extend the previous work of [4] and our objective is to present corresponding performance measures for a cellular system with non-Poissonian handoff traffic. More specifically, we considered r-stage Erlang and hyperexponential distributions, all covered under the general framework of Markov regenerative theory. We develop a solution method to obtain the performance measures for both new and handoff calls.

The paper is organized as follows. The model description and assumptions that capture the cellular system behavior are discussed in Section 2. In Section 3, the MRGP model for the underlying system is developed. The solution method for our performance measures is shown in Section 4. Comparative study of performance measures for the wireless systems with non-exponentially distributed handoff interarrival times to that with exponentially distributed case is carried out in Section 5. Pointers to further research and conclusions are given in Section 6.

2 Cellular Model Description

We assume that the cellular network under study is homogeneous, i.e., all cells are identical and experience the same traffic patterns. This allows us to consider only one cell for our performance study and capture all interactions with neighboring cells through a handoff call arrival process. We further assume that the number of channels, C, that are allocated to the "cell under study" are fixed over time (i.e, the system employs the fixed channel allocation scheme). A call is accepted only when the cell can find a channel not in use. Otherwise, the call is rejected. Call arrivals in cellular system can be classified as new calls and handoff calls. New calls are generated by mobiles originating calls in the chosen cell, whereas handoff calls are ongoing calls transferring from other cells. A handoff call could be blocked due to insufficient bandwidth available in the new cell, and in such case, the handoff call is said to be dropped. The dropping of a handoff call is considered more severe than the blocking of a new call. We adopt the previously mentioned "guard channel scheme" [5, 8] for giving priority to handoff calls against the new calls that may attempt to join the system. We set aside $g \leq C$ out of C as guard channels and therefore the maximum number of channels that new calls can gain access to is $C - g$. We assume that ongoing call (new or handoff) completion times are exponentially distributed with parameter μ_d and the time at which the mobile station engaged in the call departs the cell is also exponentially distributed with parameter μ_h. We also assume that the interarrival times of new calls are exponentially distributed with parameter λ_n and of handoff call interarrivals are generally distributed with (cumulative) distribution function $P(T \leq t) = G(t)$ and density function $g(t)$ having finite mean $1/\lambda_h$ which is independent of new call arrival times. Note that new calls that find all $C - g$ channels busy will leave the system as also handoff calls which find all C channels busy will leave the system. The state transition diagram for this model is shown in Figure 1. In this figure, a dotted arc denotes a system transition triggered by the arrival of a handoff call and $\mu = \mu_d + \mu_h$. Since the two types of calls, new and

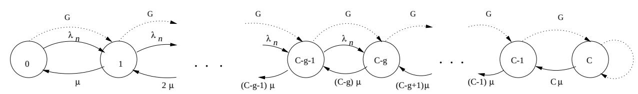

Figure 1: State transition diagram for the performance model of the cellular system

handoff, are treated differently, performance measures of interest are the steady state *new call blocking probability* and *handoff call dropping probability*. The new call blocking probability is simply the probability that an incoming new call finds at least $C-g$ channels busy in the system, while the handoff dropping probability is the probability that an incoming handoff call finds all the channels busy in the system. In the particular case where handoff call interarrival times follow the exponential distribution with parameter λ_h, Haring et al. in [4] provided the formulae for the above mentioned performance indices. In Section 5, we compare the results of [4] with our results for the special case of exponentially distributed handoff call interarrival times.

3 The Markov Regenerative Processes Model

Consider the cellular system state process $\{Z(t), t \geq 0\}$, where $Z(t)$ is the number of busy channels (or ongoing calls) in the system at time t. Clearly, this is a continuous-time discrete state process. The process $\{Z(t)\}$ is not a continuous time Markov chain (CTMC) since sojourn times are not exponentially distributed. Furthermore, $Z(t)$ is not even a semi-Markov process since whenever the channels are busy and a call completes, we have to keep track of the remaining handoff interarrival time in order to predict the future behavior. From the stochastic behavior of handoff arrivals, we observe that the underlying process satisfies the Markovian property at handoff arrival instants only. Define S_{n+1} as the time instant of the $(n+1)$-st handoff call arrival, where $n \geq 0$. Let $S_0 = 0$ and let $(Z(S_n^-) = i)$, $i \in \Omega' = \{1, 2, \ldots, C\}$. Here, the elements of Ω' are called regeneration points and they occur only at handoff call arrival instants. Note that entry into state 0 of the process is not a regeneration instant. Consider the sequence of epochs $\{S_n, n \geq 0\}$ at which the process $\{Z(t), t \geq 0\}$ is observed. Note that the state of the process $Z(t)$ can change between S_n and S_{n+1}, because of new call arrivals and ongoing call completions. We observe that the sequence $(Z(S_n^-), S_n)$ is a Markov renewal sequence and $\{Z(t), t \geq 0\}$ is a Markov regenerative process. See [7, 3] for both definitions. Two matrices define the behavior of the MRGP; the local kernel $E(t)$ that describes the behavior of the MRGP between two regeneration epochs and the global kernel $K(t)$ that describes the behavior of an embedded Discrete Time Markov Chain (DTMC) at regeneration epochs [7]. We now proceed to determine the local kernel $E(t)$ and the global kernel $K(t)$ matrices. Since the cardinality of Ω' is C, the matrix $K(t)$ has entries $K_{i,j}(t)$ that denote the actual state labels according to the state transition diagram. The interpretation for the elements of matrix $K(t)$ is as follows: $K_{i,j}(t)$ denotes the probability that the system will be in state $j \in \{1, 2, \ldots, C\}$ at the time of next handoff arrival (i.e., regeneration instant) which occurs on or before time t given that the system was in state $i \in \{1, 2, \ldots, C\}$ just after the instant of previous handoff arrival. Since the new calls are not accepted when the system reaches $C - g$, $K_{i,j}(t)$ for $i \in \{1, 2, \ldots, C-g\}$, $j \in \{C-g+2, C-g+3, \ldots, C\}$ and for $i \in \{C-g+1, C-g+2, \ldots, C-2\}$, $j \in \{i+2, i+3, \ldots, C\}$ are zero. Also, if we let $\{Y_n, n \geq 0\}$, $Y_n \in \Omega'$ be the number of ongoing calls in the system at the time of n-th handoff arrival, then we can claim that, $\{Y_n, n \geq 0\}$ is a DTMC with one-step transition probability matrix $K(\infty)$. The matrix $E(t)$ describes the dynamics of the process during the time between two consecutive regeneration points starting from a state at regeneration. The explanation for the elements of matrix $E(t)$ is as follows: $E_{i,j}(t)$ denotes the probability that the system will be in state $j \in \{0, 1, \ldots, C\}$ at time t and the next handoff arrival occurs after t given that the system was in state $i \in \{1, 2, \ldots, C\}$ just after the instant of previous handoff arrival. Note that, $E_{i,j}(t) = 0$ for $i \in \{1, 2, \ldots, C-g\}$ and $j \in \{C-g+1, C-g+2, \ldots, C\}$. Similarly, $E_{i,j}(t) = 0$ for $i \in \{C-g+1, C-g+2, \ldots, C\}$ and $j > i$.

4 MRGP Model Solution

In this section, we present a solution method for the blocking measures described above for a cellular unit with g guard channels and C as the total number of available channels. To obtain these performance measures, we need to know (closed form) expressions for $K(t)$ and $E(t)$. First, we find $K(t)$ in the next subsection.

4.1 Closed form expression for $K(t)$

We classify the elements of matrix $K(t)$ into two sets based on their row indices, viz., $\{1, 2, \ldots, C-g\}$ and $\{C-g+1, C-g+2, \ldots, C\}$. In order to find the elements of matrix $K(t)$, we need the following definition. Let $\Omega(i)$ for $i = 1, 2, \ldots, C-g$, be the set of all states reachable from state i in which the *subordinated* CTMC can spend a non-zero time before the next EMC transition occurs and is given by $\Omega(i) = \{0, 1, \ldots, C-g\}$. Thus for instance j new calls arrive and k ongoing calls depart during $(0, S_1)$, the new state will be $i + j - k$. Here, the evolution of the Markov regenerative process between the Markov regeneration epochs can be described by a CTMC infinitesimal generator Q, as the only state transitions that take place during this time are due to exponentially distributed events. The matrix Q of the corresponding subordinated CTMC with initial state i is given as

$$\begin{pmatrix} -\lambda & \lambda & 0 & \cdots & 0 & 0 \\ \mu & -(\lambda+\mu) & \lambda & \cdots & 0 & 0 \\ 0 & 2\mu & -(\lambda+2\mu) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -(\lambda+(C-g+1)\mu) & \lambda \\ 0 & 0 & 0 & \cdots & (C-g)\mu & -(C-g)\mu \end{pmatrix}.$$

Let $p_{i,j}(t)$ be the probability that the subordinated CTMC will be in state j at time t given that it was in state i initially. Define $\mathbf{p}(t) = [p_{i,0}(t), p_{i,1}(t), \ldots, p_{i,C-g}(t)]$. Then

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{p}(t)Q, \;\; p_{i,i}(0) = 1, \;\; i \in \{1, 2, \ldots, C-g\}. \quad (1)$$

From the definition of the kernel $K(t)$ of an MRGP (see [7]), we write the first set of elements of kernel $K(t)$ as follows:

$$K_{i,j}(t) \;\; = \;\; \begin{cases} \int_0^t p_{i,j-1}(x) dG(x), & j = 1, \ldots, C-g+1 \\ 0, & j = C-g+2, \ldots, C \end{cases}$$

where $p_{i,j}(t)$ are obtained by solving equation (1). This finite birth and death process is the $M/M/C-g/C-g$ loss system and the transient solution [13] is given by:

$$p_{i,j}(t) \;\; = \;\; \frac{\rho^j/j!}{\sum_k \rho^k/k!} + \frac{(C-g)!}{j!}\rho^{C-g-i} \times \sum_{r=1}^{C-g} \frac{D_i(x_r)D_j(x_r)}{rD_{C-g}(x_r)D'_{C-g}(x_r+1)} e^{x_r \mu t} \quad (2)$$

where

$$\rho = \lambda/\mu,$$

$$D_n(x) = \begin{cases} 1, & n=0 \\ x+\rho, & n=1 \\ (x+\rho+n-1)D_{n-1}(x) & \\ -(n-1)\rho D_{n-2}(x), & n=2,\ldots,C-g \end{cases}$$

and $x_r, r = 1, 2, \ldots, C-g$ are the roots of $D_{C-g}(x+1) = 0$. For the second set of rows of $K(t)$ with $i \in \{C-g+1, C-g+2, \ldots, C\}$, we have $\Omega(i) = \{0, 1, \ldots, i\}$. Thus for instance j new calls arrive and k ongoing calls depart during $(0, S_1)$, the new state will be $i+j-k$ and there is no arrival of new calls when more than $C-g$ calls are in the system. The infinitesimal generator matrix of the corresponding subordinated CTMC with initial state i is

$$Q^*(i) = \begin{pmatrix} -\lambda & \lambda & 0 & \cdots & 0 & 0 \\ \mu & -(\lambda+\mu) & \lambda & \cdots & 0 & 0 \\ 0 & 2\mu & -(\lambda+2\mu) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -(i-1)\mu & 0 \\ 0 & 0 & 0 & \cdots & i\mu & -i\mu \end{pmatrix}.$$

Define $\mathbf{q}(t) = [q_{i,0}(t), q_{i,1}(t), \ldots, q_{i,i}(t)]$. Then

$$\frac{d\mathbf{q}(t)}{dt} = \mathbf{q}(t)Q^*(i), \quad q_{i,i}(0) = 1,$$
$$i \in \{C-g+1, C-g+2, \ldots, C\}.$$

Now we write the second set of elements of kernel $K(t)$ as follows: For $i \in \{C-g+1, C-g+2, \ldots, C-1\}$,

$$K_{i,j}(t) = \begin{cases} \int_0^t q_{i,j-1}(x)dG(x), & j=1,2,\ldots,i+1 \\ 0, & j=i+2,\ldots,C \end{cases} \quad (3)$$

and for $i = C$,

$$K_{C,j}(t) = \begin{cases} \int_0^t q_{C,j-1}(x)dG(x), \\ j=1,2,\ldots,C-1 \\ \int_0^t [q_{C,C-1}(x) + q_{C,C}(x)]\, dG(x), \\ j=C \end{cases} \quad (4)$$

Note that, the last element $K_{C,C}(t)$ is different from the other elements because, any one of the ongoing calls could be completed before the arrival of the next handoff call or no calls might completed before the next handoff call arrival. The transient solution $q_{i,j}(t)$ can be written as

$$q_{i,j}(t) = \left[e^{Q^*(i)t}\right]_{i,j} \quad (5)$$

where $[.]_{i,j}$ denotes the i-th row and j-th column of matrix $e^{Q^*(i)t}$. $q_{i,j}(t)$ can also be expressed as a weighted sum of exponential terms by using the transient solution of the (finite) birth death process. In particular, it is observed from the transient solution of pure death process that, for $j = C-g+1, C-g+2, \ldots, i$, equation (5) reduces to

$$q_{i,j}(t) = \begin{pmatrix} i \\ j \end{pmatrix} e^{-j\mu t}(1-e^{-\mu t})^{i-j}. \quad (6)$$

Equations (2), (3) and (4) complete the elements of global kernel $K(t)$.

4.2 Computing $\mathbf{K}(\infty)$

We now proceed to obtain expression for $\mathbf{K}(\infty)$. It is not difficult to show that for $i = 1, 2, \ldots, C-g$:

$$K_{i,j}(\infty) = \frac{\rho^{(j-1)}/(j-1)!}{\sum_k \rho^k/k!} + \frac{(C-g)!}{(j-1)!}\rho^{C-g-i} + \sum_{r=1}^{C-g} \frac{D_i(x_r)D_{(j-1)}(x_r)}{rD_{C-g}(x_r)D'_{C-g}(x_r+1)}\hat{g}(-x_r\mu).$$

Here $\hat{g}(.)$ denotes the Laplace transform of $g(.)$. Furthermore, for $i \in \{C-g+1, C-g+2, \ldots, C-1\}$,

$$K_{i,j}(\infty) = \begin{cases} \int_0^\infty q_{i,j-1}(x)dG(x), \\ j=1,2,\ldots,i+1 \\ 0, \\ j=i+2,i+3,\ldots,C \end{cases}$$
$$= \begin{cases} \sum_{k=0}^{i} \frac{A'_{i,(j-1)}(\rho_k)}{\prod_{l=0,l\neq k}^{i}(\rho_k-\rho_l)}\hat{g}(-\rho_k), \\ j=1,2,\ldots,i+1 \\ 0, \\ j=i+2,i+3,\ldots,C \end{cases}$$

where $A'_{ij}(\rho_k)$ is the $(j,i)^{th}$ cofactor of the characteristic matrix of $Q^*(i)$ and the ρ_k's are eigenvalues of $Q^*(i)$ and are given by

$$\rho_k = \begin{cases} 0, & k=0 \\ x_k, & k=1,2,\ldots,C-g \\ -k\mu, & k=C-g+1, C-g+2, \ldots, i. \end{cases}$$

Similarly based on equation (4), we obtain for $i = C$,

$$K_{C,j}(\infty) = \begin{cases} \sum_{k=0}^{C} \frac{A'_{C,(j-1)}(\rho_k)}{\prod_{l=0,l\neq k}^{C}(\rho_k-\rho_l)}\hat{g}(-\rho_k), & j \neq C \\ \sum_{k=0}^{C}\left(\frac{A'_{C,(C-1)}(\rho_k)}{\prod_{l=0,l\neq k}^{C}(\rho_k-\rho_l)} + \frac{A'_{C,C}(\rho_k)}{\prod_{l=0,l\neq k}^{C}(\rho_k-\rho_l)}\right)\hat{g}(-\rho_k), & j = C. \end{cases}$$

4.3 Closed form expression for $E(t)$

The matrix $E(t)$ describes the behavior of the process during the time between two consecutive handoff call arrivals starting from a state of the system just after the last handoff call arrival. Hence it is a $(C) \times (C+1)$ matrix and is given by:

$$E_{i,j}(t) = \begin{cases} p_{i,j}(t)(1-G(t)), & i \in \{1,2,\ldots,C-g\}; \\ & j \in \{0,1,\ldots,C-g\} \\ q_{i,j}(t)(1-G(t)), & i \in \{C-g+1,\ldots,C\}; \\ & j \in \{0,1,\ldots,i\} \\ 0, & \text{otherwise} \end{cases}$$

where $p_{i,j}(t)$ are defined in equation (2) and $q_{i,j}(t)$ are defined in equations (5) and (6).

For our performance measures that we will discuss in the next section we will need

$$\alpha_{i,j} = \int_0^\infty E_{i,j}(t)dt.$$

It is not difficult to show that for $i = 1, 2, \ldots, C-g$ and $j = 0, 1, \ldots, C-g$

$$\alpha_{i,j} = \left(\int_0^\infty \left[\int_t^\infty g(x)dx\right] dt\right) \frac{\rho^j/j!}{\sum_k \rho^k/k!} + \frac{(C-g)!}{j!} \rho^{C-g-i} \sum_{r=1}^{C-g} \frac{D_i(x_r)D_j(x_r)}{rD_{C-g}(x_r)D'_{C-g}(x_r+1)} \hat{G}_c(-x_r\mu) \quad (7)$$

where $\hat{G}_c(.)$ means the Laplace transform of complementary distribution function $G(.)$ and for $i = C-g+1, C-g+2, \ldots, C$ and $j = 0, 1, \ldots, i$,

$$\alpha_{i,j} = \sum_{k=0}^{i} \frac{A'_{ij}(\rho_k)}{\prod_{l=0,l\neq k}^{i}(\rho_k - \rho_l)} \hat{G}_c(-\rho_k). \quad (8)$$

4.4 Performance measures

In order to compute call blocking measures for our cellular system we are interested in obtaining the system stationary probabilities at call arrival time instants. The system state probabilities (at an arbitrary time point) are defined as follows [7]:

$$P(C, g, j) = \frac{\sum_{k\in\Omega'} \nu_k \alpha_{k,j}}{\sum_{k\in\Omega'} \nu_k \beta_k} = \lambda_h \sum_{k=1}^{C} \nu_k \alpha_{k,j}$$

where ν_k's are the solution of

$$\nu = \nu K(\infty), \sum_{k=1}^{C} \nu_k = 1,$$

$\alpha_{i,j}$ are given in equations (7) and (8) and $\beta_k = \sum_{l=0}^{C} \alpha_{k,l} = \frac{1}{\lambda_h}$. Now since the new calls arrive according to a Poisson process, the system state distribution as seen by (new call) arrivals is the same as the system state distribution observed at an arbitrary time instant [2]. This not true, however, for the handoff call arrivals and the distribution as seen by handoff call arrivals would be the solution of the EMC, i.e., $\nu_j, \; j = 1, 2, \cdots, C$. Using these facts we find the performance measures of interest as follows:

$$P_b(C, g) = \sum_{j=C-g}^{C} P(C, g, j) = \sum_{j=C-g}^{C} \lambda_h \sum_{k=1}^{C} \nu_k \alpha_{k,j}, \quad (9)$$

$$P_d(C, g) = Pr\{\text{all channels occupied} \mid \text{handoff arrived}\} = \nu_C. \quad (10)$$

Solution method outline: We now outline the procedure to compute the performance measures of interest.

1. Compute $K(\infty)_{i,j}$ and $\alpha_{i,j}, \forall, i, j$.
2. Solve the linear system $\nu = \nu K(\infty)$ with the constraint $\sum_i \nu_i = 1$.
3. Obtain $P_d(C, g)$ simply as ν_C (Equation (10))
4. Substitute ν_i and $\alpha_{i,j}$ in Equation (9) to obtain values of $P_b(C, g)$.

5 Comparative Study

In this section we illustrate the numerical solution of equations (9) and (10) under different non-exponentially distributed handoff interarrival times. We set $\lambda_n = 0.5$ new calls per minute, $\mu_d = 0.05$, $\mu_h = 1/3$ calls per minute. For simplicity we choose the number of guard channels $g = 3$. For the purpose of comparison we consider different types of distributions for handoff interarrival times with fixed mean 2.5 minutes per call. We plot the blocking probability and dropping probability versus number of channels from $C = 6$ to $C = 10$ in Figures 2 and 3, respectively. We

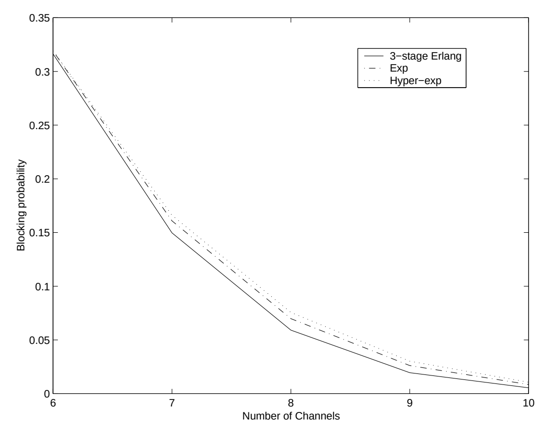

Figure 2: Blocking probability for fixed mean handoff interarrival time

choose the parameters $r = 3$ and each stage arrival rate is 1.2 per minute for 3-stage Erlang distribution and $\alpha_1 = 0.4, \alpha_2 = 0.6, \lambda_1 = 1, \lambda_2 = 0.2857$ for hyper-exponential distribution. We observe from Figure 2 that, there is a difference in the values between exponential and non-exponential distributed cases. Note that, 3-stage Erlang distribution gives the lowest blocking probability whereas hyper-exponential distributed handoff interarrivals gives the highest value. Also, we observe that, with exponentially distributed handoff interarrivals, blocking probability lies in between that of hyper-exponential and 3-stage Erlang cases. Similarly, in Figure 3 we observe that the 3-stage Erlang distribution gives the lowest probability, hyper-exponential distribution gives the highest probability and exponential distribution stays in between 3-stage Erlang and hyper-exponential distributions. This is explained from the fact that the handoff call blocking depends on the variance (or equivalently the coefficient of variation) of interarrival times and distributions with higher variances, for the same mean, give rise to larger blocking measures. Also, note that, when the number of channels increases, the dropping probabilities tend to approach each other for various distributions as we expect. In Figures 4 and 5, we have blocking measures

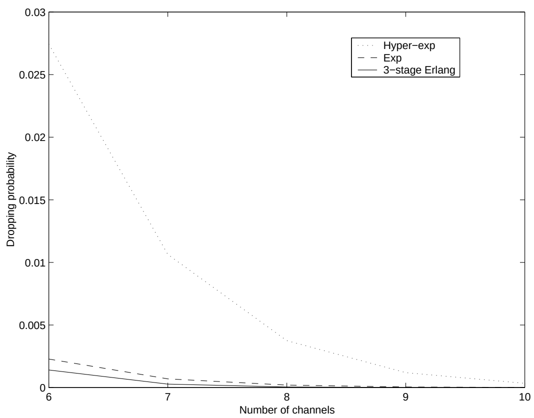

Figure 3: Dropping probability for fixed mean handoff interarrival time

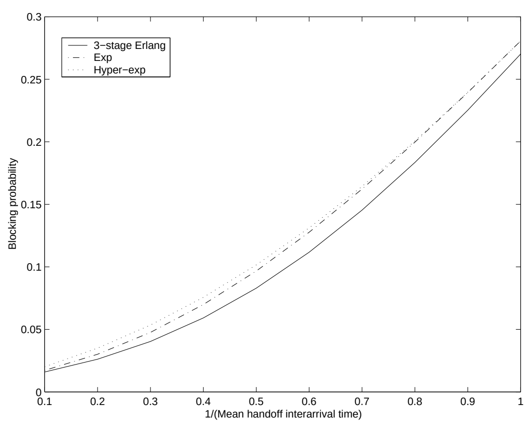

Figure 4: Blocking probability for fixed number of channels

for fixed the number of channels $C = 8$ and for various values of inverse of mean handoff interarrival times from 0.1 to 1.0. From Figure 4 we observe that there is a difference in the results for different distributions as in Figure 2 and also 3-stage Erlang distribution gives

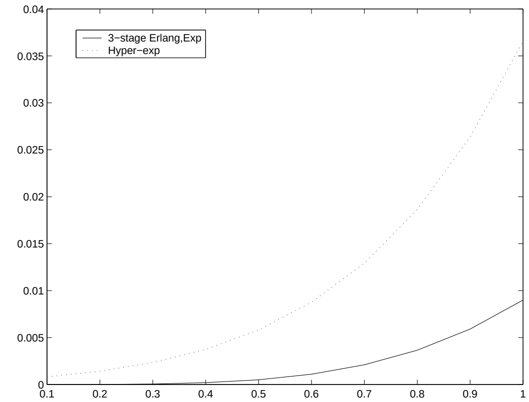

Figure 5: Dropping probability for fixed number of channels

the lowest blocking probability. We observe that handoff interarrival time distribution does not affect the new calls blocking probability significantly. But the distribution of handoff interarrival times causes noticeable variations in dropping probability.

6 Conclusion and Future Work

In this paper, we have presented an analytical performance model to study (new and handoff) call blocking for non-exponential interarrival times. For this reason, we first identified the underlying stochastic process as a Markov regenerative one and subsequently provided a general solution method for an arbitrary handoff interarrival time distribution function. It is our belief that the proposed model may be of great interest in the design and operation of second generation as well as third generation cellular communication networks. In our exact performance model, the "new call" arrival process is Poisson but the "handoff call" arrival process is a renewal process. We obtained numerical results for various types of (interarrival time) distributions, namely, exponential, Erlang and hyper-exponential. Our results validate the well-known fact that the variance (or equivalently coefficient of variation) in addition to the mean is important in determining the handoff call blocking measures. We observe that the larger the variance (or coefficient of variation), for the same mean, the larger the call blocking value. This makes our analysis more valuable since the Poissonian handoff arrival assumption may underestimate/overestimate the blocking probabilities and this may result in erroneous network dimensioning. We are currently working on efficient algorithms to compute call blocking measures as well as the solution of optimization problems related to this work.

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