Quantum Networking

Bell States and Bell Basis

- Maximally Entangled States
  - Correlation between two qubits are fully specified
  - Four Bell states form orthonormal basis for two-qubit composite system

$$\left|\psi_+\right\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \left|\psi_-\right\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \qquad |00\rangle = \frac{\left|\psi_+\right\rangle + \left|\psi_-\right\rangle}{\sqrt{2}}, |11\rangle = \frac{\left|\psi_+\right\rangle - \left|\psi_-\right\rangle}{\sqrt{2}}$$

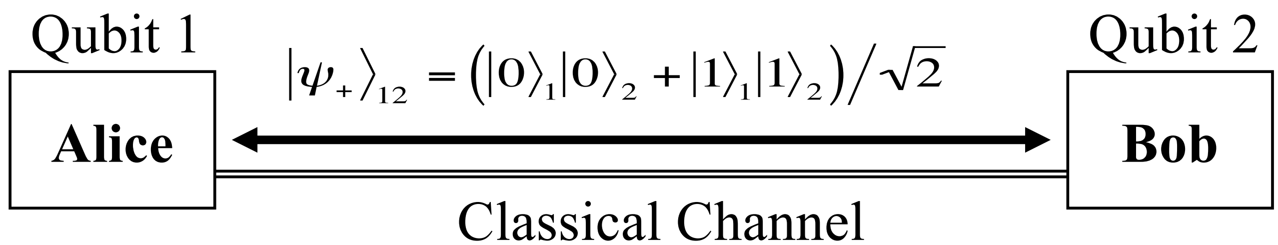
$$\left|\varphi_+\right\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, \left|\varphi_-\right\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \qquad |01\rangle = \frac{\left|\varphi_+\right\rangle + \left|\varphi_-\right\rangle}{\sqrt{2}}, |10\rangle = \frac{\left|\varphi_+\right\rangle - \left|\varphi_-\right\rangle}{\sqrt{2}}$$

  - Entangled states cannot be represented as a product of states from component state spaces

Quantum Teleportation

- Alice and Bob, in remote locations, share one qubit each from a Bell state
  - Alice has qubit 1 and Bob has qubit 2 $\quad \left|\psi_+\right\rangle_{12} = \left(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\right)/\sqrt{2}$
- Alice has a third qubit in state $\quad |\phi\rangle_3 = \alpha|0\rangle_3 + \beta|1\rangle_3$
  - Can Alice send the qubit to Bob using only classical channel?

$|\phi\rangle_3 = \alpha|0\rangle_3 + \beta|1\rangle_3$



- Total State $\quad \left|\psi_+\right\rangle_{12}|\phi\rangle_3 = \dfrac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\right)\left(\alpha|0\rangle_3 + \beta|1\rangle_3\right)$

$$= \frac{1}{\sqrt{2}}\left(\alpha|0\rangle_1|0\rangle_2|0\rangle_3 + \alpha|1\rangle_1|1\rangle_2|0\rangle_3 + \beta|0\rangle_1|0\rangle_2|1\rangle_3 + \beta|1\rangle_1|1\rangle_2|1\rangle_3\right)$$

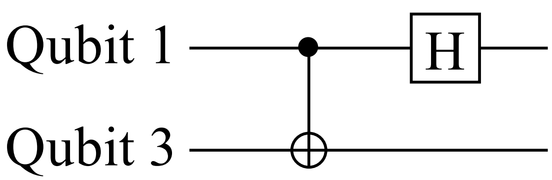
$$= \frac{1}{2}\left[\alpha|0\rangle_2\left(|\psi_+\rangle_{13} + |\psi_-\rangle_{13}\right) + \alpha|1\rangle_2\left(|\varphi_+\rangle_{13} - |\varphi_-\rangle_{13}\right) + \beta|0\rangle_2\left(|\varphi_+\rangle_{13} + |\varphi_-\rangle_{13}\right) + \beta|1\rangle_2\left(|\psi_+\rangle_{13} - |\psi_-\rangle_{13}\right)\right]$$

$$= \frac{1}{2}\left[\left(\alpha|0\rangle_2 + \beta|1\rangle_2\right)|\psi_+\rangle_{13} + \left(\alpha|0\rangle_2 - \beta|1\rangle_2\right)|\psi_-\rangle_{13} + \left(\alpha|1\rangle_2 + \beta|0\rangle_2\right)|\varphi_+\rangle_{13} + \left(\alpha|1\rangle_2 - \beta|0\rangle_2\right)|\varphi_-\rangle_{13}\right]$$

Quantum Teleportation: continued

$$= \frac{1}{2}\Big[\big(\alpha|0\rangle_2 + \beta|1\rangle_2\big)|\psi_+\rangle_{13} + \big(\alpha|0\rangle_2 - \beta|1\rangle_2\big)|\psi_-\rangle_{13} + \big(\alpha|1\rangle_2 + \beta|0\rangle_2\big)|\varphi_+\rangle_{13} + \big(\alpha|1\rangle_2 - \beta|0\rangle_2\big)|\varphi_-\rangle_{13}\Big]$$

- Alice can make a Bell-basis measurement .
  - Answer is 2-bit classical information describing Bell basis she has in qubit 1 & 3
  - For example, she needs a quantum circuit



Input	Output
$\lvert\psi_+\rangle = \big(\lvert 00\rangle_{13} + \lvert 11\rangle_{13}\big)/\sqrt{2}$	$\lvert 00\rangle_{13}$
$\lvert\psi_-\rangle = \big(\lvert 00\rangle_{13} - \lvert 11\rangle_{13}\big)/\sqrt{2}$	$\lvert 10\rangle_{13}$
$\lvert\varphi_+\rangle = \big(\lvert 01\rangle_{13} + \lvert 10\rangle_{13}\big)/\sqrt{2}$	$\lvert 01\rangle_{13}$
$\lvert\varphi_-\rangle = \big(\lvert 01\rangle_{13} - \lvert 10\rangle_{13}\big)/\sqrt{2}$	$\lvert 11\rangle_{13}$

  - Alice sends the two classical bits to Bob over classical channel
  - Based on Alice's information, Bob applies following operation to his qubit

$$|00\rangle_{13} \longrightarrow I\big(\alpha|0\rangle_2 + \beta|1\rangle_2\big) = \alpha|0\rangle_2 + \beta|1\rangle_2$$

$$|10\rangle_{13} \longrightarrow Z\big(\alpha|0\rangle_2 - \beta|1\rangle_2\big) = \alpha|0\rangle_2 + \beta|1\rangle_2$$

$$|01\rangle_{13} \longrightarrow X\big(\alpha|1\rangle_2 + \beta|0\rangle_2\big) = \alpha|0\rangle_2 + \beta|1\rangle_2$$

$$|11\rangle_{13} \longrightarrow ZX\big(\alpha|1\rangle_2 - \beta|0\rangle_2\big) = \alpha|0\rangle_2 + \beta|1\rangle_2$$

Entanglement Swapping

- If Alice's qubit #3 was originally entangled with another qubit #4,
  - After teleportation, qubit #3 and #1 are "destroyed" by the measurement in teleportation process
  - The state of qubit #3 is teleported to qubit #2
  - Since qubit #3 and qubit #4 were originally entangled, now qubit #4 and qubit #2 are entangled
  - Qubit #4 and qubit #2 never directly interacted, yet ends up entangled

Alice has qubits 1, 3, 4

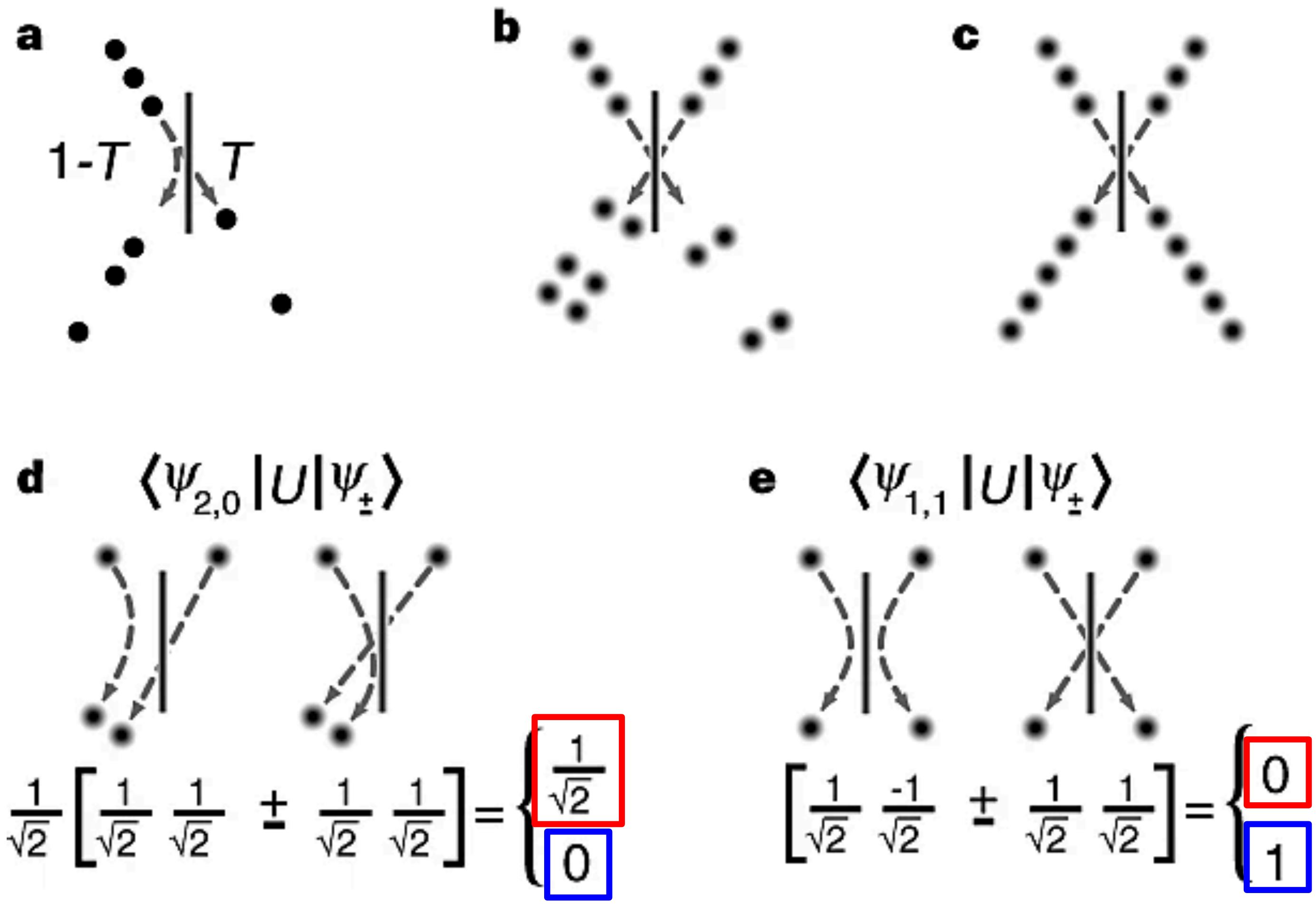
Bob has qubit 2

$$|\psi_+\rangle_{34} = (\cancel{|0\rangle_3}|0\rangle_4 + \cancel{|1\rangle_3}|1\rangle_4)/\sqrt{2} \qquad |\psi_+\rangle_{12} = (\cancel{|0\rangle_1}|0\rangle_2 + \cancel{|1\rangle_1}|1\rangle_2)/\sqrt{2}$$

$$|\psi_+\rangle_{24} = (|0\rangle_2|0\rangle_4 + |1\rangle_2|1\rangle_4)/\sqrt{2}$$

Hong-Ou-Mandel Interference

- Interference of identical particles at a 50/50 beamsplitter



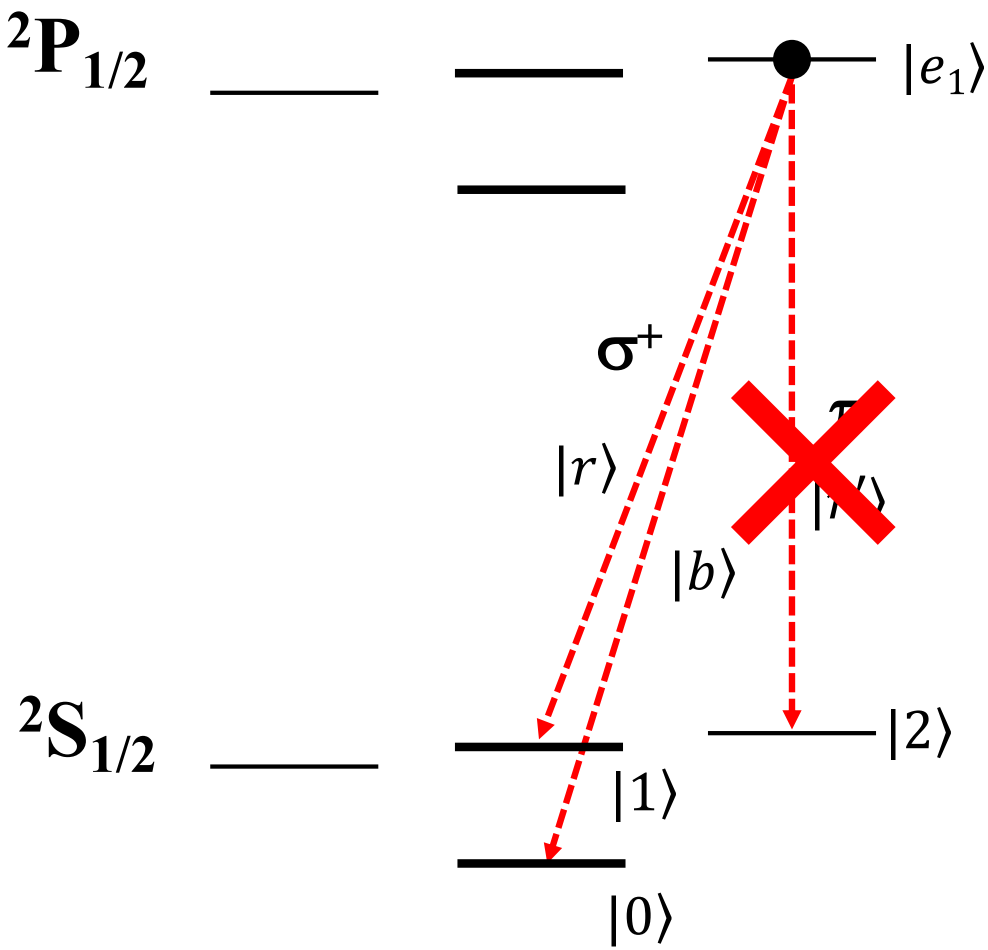
Bosons

Fermions

Liu, Odom, Yamamoto & Tarucha, Nature 391, 263 (1998)

Atom-Photon Entanglement

- Consider hydrogen-like atom
  - One electron, nuclear spin $I = ½$

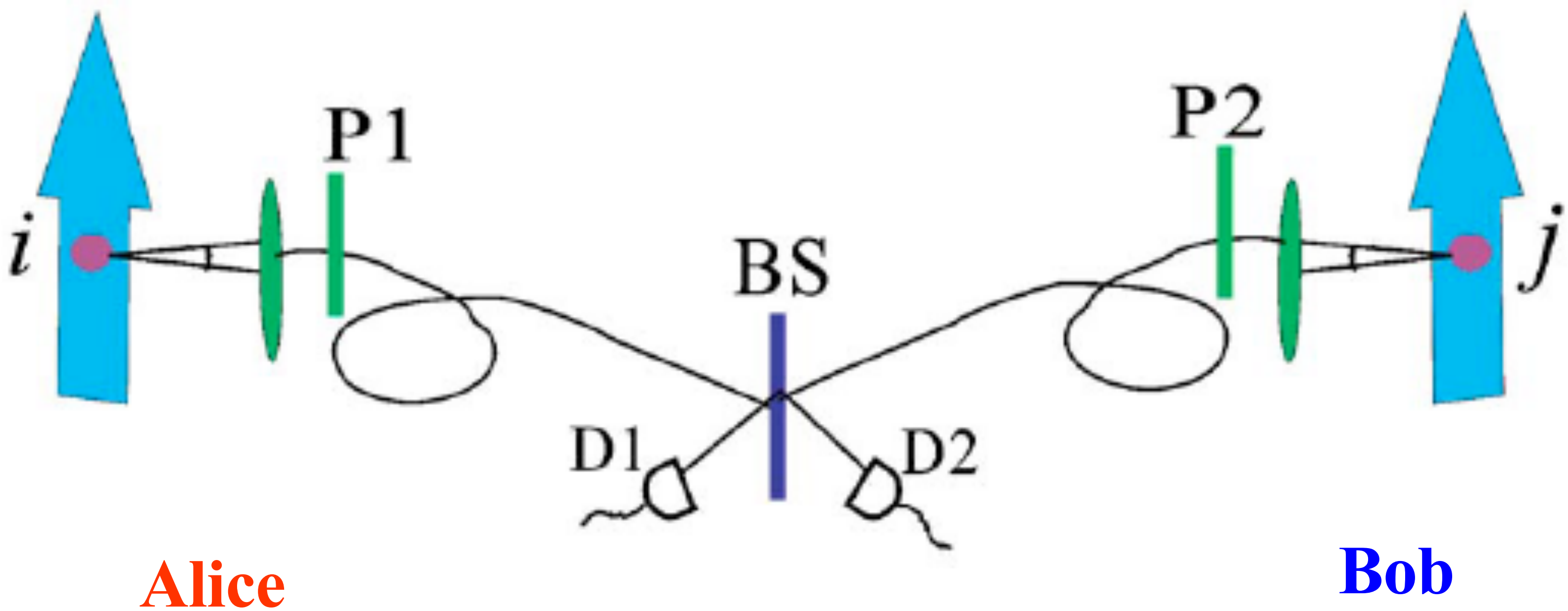


$$|\boldsymbol{\psi}_+\rangle_{ap} = \left(|\mathbf{0}\rangle_a|\boldsymbol{b}\rangle_p + |\mathbf{1}\rangle_a|\boldsymbol{r}\rangle_p\right)/\sqrt{\mathbf{2}}$$

L.-M. Duan et al., Phys. Rev. A73, 062324 (2006)

Atom-Atom Entanglement via Photons

- Two identical setups at Alice and Bob
  - Coincidence detection in both D1 and D2 heralds successful entanglement of the two atoms *i* at Alice and *j* at Bob



$$|\psi_+\rangle_{AB} = (|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)/\sqrt{2}$$

L.-M. Duan et al., Phys. Rev. A73, 062324 (2006)