Readings in Statistical Science Lecture

Statistical Signal Processing for Radar and Sonar in Complex Multipath Propagation Conditions

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Remote Sensing in Multipath Propagation Channels

**OVERALL GOAL:** Develop statistical signal and array processing techniques for electromagnetic and acoustic remote sensing which exploit complex multipath propagation to achieve enhanced performance.

**SUMMARY:**

- Radar and sonar signal processing methods have historically relied on plane-wave propagation models because of their analytic and computational simplicity.

- Methods for mitigating multipath propagation have been developed but typically exhibit performance which is upper bounded by their performance when multipath is absent.

- The idea of exploiting, rather than undoing, the effects of multipath propagation to achieve improved localization performance by use of a computational propagation model is the essence of matched-field processing (MFP).

- Our current projects involve multipath signal processing for passive and active sonar, over-the-horizon skywave radar, and tropospheric refractivity estimation using microwave clutter from the sea surface.
Sonar and Radar Multipath Phenomenology

- Interference of multipath components from a single point source at ~200 Hz. in a shallow-water acoustic channel results in highly variable field intensity.

- Refractivity changes above the sea surface can duct 3 Ghz. radar causing highly variable over-the-horizon clutter returns due to interfering multipaths.
Physics-Independent Multipath SSAP Strategies

• Traditional array processing methods use plane-wave models due to their: 1) analytic simplicity, and 2) elegant analogies between familiar time-domain filtering/spectral analysis and plane-wave beamforming/field-directionality mapping.

• Correlated multipath causing signal wavefront mismatch is mitigated by various incoherent sub-aperture methods (e.g. spatial smoothing, diversity combining, etc.).

• Post-detection gain against diffuse noise offered by incoherent sub-array combining methods typically $\sim 10\log(N/L) + 5\log(L)$, where $N$ is the total number of sensors and $L$ is the number of sub-arrays, which is much less than $10\log(N)$ for large $N$ and $L$.

• More recent physics-independent approaches involve “through-the-sensor” estimation of the multi-channel multipath impulse response. These include: 1) time-reversal methods (e.g. M. Fink, et.al., W. Kuperman, and W. Hodgkiss), 2) blind multichannel system identification methods (e.g. L. Tong, G. Xu, M. Nikias, G. Giannakis), and 3) direct training methods for space-time coding applications.

• Physics-independent methods for channel response estimation of limited applicability in radar/sonar settings because of requirements for cooperative targets, beacons, high signal-to-clutter-plus-noise ratios, and/or large observation time-bandwidth products.
Brief History of Physics-based Multipath SSAP

- Earliest physics-based multipath signal processing methods (before 1970) may be autocorrelation-based multipath ranging methods developed using raytrace models for deep-water channels (see e.g. Burdic and Urick texts).

- Full-field acoustic propagation modeling originally motivated by the need to coherently beamform larger sensor arrays to achieve array gains required to detect weaker targets.

- Matched-field processing first proposed by Tolstoy and Clay, Bucker, Hinich in 1970’s.

- Extensive deep-water experimental programs launched in the 1980’s (e.g. High Gain Initiative), shallow-water experimental programs in the 1990’s (e.g. SWELLEX series and Santa Barbara Channel), and new $40M ONR “Acoustic Observatory” program planned for 2001-2006.

- Availability of powerful, inexpensive computing has facilitated use of numerical propagation models such as raytracing, normal mode, or parabolic equation (PE) methods with SSAP techniques (see Schmidt, Kuperman, Porter text for models).

- More recently, physics-based multipath SSAP viewed as a means of providing new capabilities to existing sonars and radars (e.g. depth discrimination, altitude estimation, refractivity sensing) rather than just trying to achieve higher array gain.

- Difficult issues have been parameter estimation ambiguities, environmental mismatch, source motion, array calibration, and medium fluctuations.
Matched-field Beamforming for Passive Sonar

- **Objective:** To improve beamforming and source localization performance by exploiting full-field numerical propagation modeling of correlated multipath arrivals at a sensor array.

  ![Diagram](image)

- Conventional matched-field processing (MFP) is essentially beamforming using signal wavefront predictions from a full-wave computational model for sound propagation in an ocean waveguide.

- Two of the first difficulties faced by MFP are high sidelobe levels and sensitivities to the errors in the environmental model used to drive numerical predictions of the field.
Conventional MFP with a Normal Mode Model

- The frequency-domain linear sensor array output due to a random source at range, $r_s$, depth, $z_s$, and bearing, $\theta_s$, in additive noise can modeled as a sum of normal modes:

$$x_n = s_n d(\theta_s, r_s, z_s) + \eta_n = s_n U(\theta_s) a + \eta_n$$

where $[U(\theta_s)]_{ml} = \phi_l(z_m) e^{-jk_m \gamma \sin \theta_s}, [a(r_s, z_s)] = \phi_l(z_s) e^{-jk_r \gamma}$, $\gamma$ is array tilt, and the modal eigenfunctions, $\phi_l(z)$, and horizontal wavenumbers, $k_l$, are determined by numerical solution of a Helmholtz equation including boundary conditions at the surface and bottom.

- Conventional MFP simply correlates predicted wavefront replica vectors for different hypothesized source locations with the received pressure field. Example with vertical array data collected in the Mediterranean for source at 5600 m. range and 80 m. depth.
Robust Minimum Variance MFP Processors

- Adaptive minimum variance (MV) beamformers are motivated by the need for ambiguity/sidelobe suppression but multiple point constraints and/or diagonal loading required to improve robustness to signal mismatch.

- MV with neighborhood location constraints (NLC) [Schmidt, 1990] and environmental perturbation constraints (EPC) [Krolik, 1992] used to avoid signal cancellation.

- Example of MV-NLC (left) and MV-EPC (right) with SACLANT Mediterranean data.
MFP in a Dynamic Littoral Environment

- Range-depth-bearing adaptive MFP requires accurate prediction of \((k_i - k_j)r_s\) which is difficult for large range. Further, interferer motion decorrelates its multipath components over the observation times required to estimate data covariance matrices.

- Typical range-time ambiguity surfaces at known target depth in the presence of moving interferer for SWELLEX data taken off San Diego coast. Limited ability of adaptive beamformers to suppress moving surface interferers commonly observed.
Matched-field Altitude Estimation for OTH Radar

OBJECTIVE: To estimate aircraft altitude by modeling dwell-to-dwell changes in the complex delay-Doppler over-the-horizon (OTH) radar return due to unresolved direct and surface-reflected multipath from a moving target.

BACKGROUND:

• Previous attempts at altitude estimation with OTH radar have required either excessive signal bandwidth (> 100 kHz) or a large number of revisits (> 30 min.) to resolve micro-multipaths in slant range or Doppler.

• Target motion in classical MFP techniques viewed as problematic since it decorrelates multipath components over the observation times used with stationary source models.

• In contrast, our approach exploits target dynamics to estimate altitude without the need to model the complex relative multipath amplitudes usually required for MFP.

• Matched-field altitude estimation (MFAE) depends primarily on horizontal wavenumber differences multiplied by the change in target range between dwells.

• Currently implemented on a real-time demonstration system attached to the Navy’s Relocatable OTH Radar (ROTHR) in Chesapeake, Virginia.
Counter-Drug Surveillance Using OTH Radar

Altitude estimation important for classification purposes as well as cueing searches for small aircraft done by visual spotting or with radars with a limited field of view.
Micro-multipath Returns in Delay-Doppler Space

- Overlapping micro-multipaths consist of a coherent sum of direct and surface-reflected returns which are unresolved in log-amplitude delay-Doppler space.

- Within and across revisits, delay and Doppler differences between micro-multipaths result in complex target peak shape changes and fading which is altitude dependent.
Delay-Doppler Modeling of Multipath OTHR Returns

- Altitude-dependent dwell-to-dwell changes in the complex micro-multipath amplitudes, $x_k$, can be handled by a Markov state equation given by:

$$x_k = \rho A_k(z, \dot{r}) x_{k-1} + \sqrt{1 - \rho^2} \nu_k$$

where $\rho$ is the dwell-to-dwell correlation coefficient and where the additive process noise accounts for elevation angle errors, altitude rate uncertainties and Doppler jitter.

- The fluctuating radar return, $y_k$, at dwell $k$, is then obtained by taking a weighted sum of micro-multipath replica vectors multiplied by a complex unknown scalar, $\alpha_k$:

$$y_k = \alpha_k H_k(z) x_k + \eta_k$$

- Coherent, partially coherent, and incoherent maximum likelihood estimates of altitude, $z$, are obtained by assuming, respectively, $\rho = 1$, $0 < \rho < 1$, $\rho = 0$.

- Early MFAE versions essentially assumed $\rho = 1$ which made them less robust to non-altitude-dependent inter-dwell variability.
The Geometry of Micro-Multipath Phase Changes

- Between consecutive dwells at time $t_k$, the phase change undergone by the $l^{th}$ micro-multipath return is $d\theta_{l,k} = \frac{\partial}{\partial r_k} d r_k + \frac{\partial}{\partial \omega_k} d \omega_k$ where $dr_k$ is the change in ground range and $d\omega_k$ is the change in carrier frequency between dwells.

- The ray elevation angles of the micro-multipaths lead to a simple geometric expression for $\frac{\partial}{\partial r_k} = \frac{\omega_k}{c} (\cos \beta_1 + \cos \beta_2)$ and $\frac{\partial}{\partial r_k} d r_k = \frac{\omega_k}{c} (\cos \beta_1 + \cos \beta_2) v_r (t_k - t_{k-1}) = \omega_{l,k} (t_k - t_{k-1})$ where $v_r$ is the radial velocity of the target.
Recursive Bayesian Matched-field Altitude Estimation

- MFAE recursion on each dwell for each hypothesized altitude involves:
  1) Prediction of micro-multipath coefficient vector, $\hat{x}_{n|n-1;1}$, from $y_{n-1;1}$.
  2) Prediction of $y_n$ using $\hat{x}_{n|n-1;1}$ and estimation of signal amplitude, $\alpha_n$.
  3) Accumulation of posterior probability using $\log p(y_n \mid y_{n-1;1}, \hat{\alpha}_{n;1}, z)$.
  4) Update of $\hat{x}_{n|n-1;1}$ using current dwell data, $y_n$.

- Assuming known signal amplitudes and a Gaussian model for the micro-multipath coefficients and observation noise, a Kalman filter could be used to recursively estimate, $x_k$.

- The ML estimate of altitude, $z$, is then obtained by computing:

$$\log p(y_{n;1} \mid \hat{\alpha}_{n;1}, z) = \sum_{i=1}^{n} \log p(y_i \mid y_{i-1;1}, \hat{\alpha}_{i;1}, z)$$

where $y_{n;1} = [y_n, ..., y_1]$ and $\hat{\alpha}_{i;1} = [\hat{\alpha}_i, ..., \hat{\alpha}_i]$ are signal amplitude estimates.
Recursive Bayesian Matched-field Altitude Estimation

- Markov modeling of the micro-multipath coefficients permits recursive Bayesian estimation of complex shape changes across an entire sequence of dwells.
- Separation of the *dynamical model* and *observation model* permits adaptive control of both signal level and predicted correlation between dwells.
- Estimation of signal amplitude and dynamical process predictability made on each dwell to better match modeled multipath return with measured data.

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**Sounder Data**

1. **RTRT Ionosphere and 2-D Ray Tracing**
2. **Micro-Multipath Propagation Model**
3. **Delay-Doppler Surface Replica Generation**

- **Ground Track and Hypothesized Altitude**
- **Align Peak Data with Slant Track**
- **Accumulate Multi-dwell Altitude Log-Likelihood**
- **Prediction**
- **Aircraft Altitude and Confidence Interval**
Simulation of Non-parametric Model Mismatch

- Log-likelihood surfaces for simulation example of original "coherent" MFAE versus robust MFAE assuming incoherent and partially coherent inter-dwell predictability, respectively. True inter-dwell correlation coefficient = 0.6
Incoherent vs. Partially Coherent MFAE for Real Data Example

- Real data result using the robust MFAE method for a high altitude aircraft on 6/22/2000 with standard 30 second revisit, 16 kHz bandwidth, fixed-frequency DIR.
- For this 30 dB high SNR track, partially coherent MFAE converges within 5 dwells while incoherent MFAE converges within 20 dwells. Note correspondence between suppression of partially coherent MFAE ambiguity versus width of incoherent MFAE likelihood surface.
- The final altitude estimate is 35,300 feet and the true altitude is 35,000 feet.
Performance Bounds for Partially Coherent MFAE

- The two-dwell Cramer-Rao lower bound (CRLB) on altitude estimation performance illustrates the potential gain from modeling the dwell-to-dwell evolution of the micro-multipath reflection coefficients.

- Comparison of MFAE simulation results to the two-dwell CRLB as a function of dwell-to-dwell coherence indicates the small-error performance bound is not tight for partially-coherent data.

- Performance of robust MFAE (in red) better than original MFAE (in green) but both algorithms are dominated by large errors.
MFAE Performance versus Track Length

- Monte Carlo simulation results versus number of dwells used for MFAE indicate accuracy improves more quickly given higher dwell-to-dwell predictability.
- Left plot indicates robust MFAE RMSE when dwell-to-dwell correlation is known a priori.
- Right plot indicates robust MFAE RMS when the assumed dwell-to-dwell correlation of 0.6 is mismatched from the true value which varies from 0 to 1.
Microwave Remote Sensing of Surface Based Ducts

**OBJECTIVE:** Develop robust efficient techniques for estimating the refractivity parameters required to predicting surface-based ducted microwave propagation.

**BACKGROUND:**

- The tropospheric refractivity profile in coastal regions, which is primarily a function of water-vapor concentration and temperature, determines to a large extent the performance of shipboard radar and communication systems.

- Direct measurement of atmospheric conditions at sea is difficult, so estimation of refractivity from clutter (RFC) is being developed to provide synoptic characterization of surface-based ducts over large spatial extents using sea surface radar backscatter.
Phenomenology of Clutter in Tropospheric Ducts

- Surface-based ducted conditions result in clutter returns from well beyond the horizon. Clutter power versus azimuth display from 2.8 Ghz SPANDAR data taken off Wallops Island on April 2, 1998.

- Temporal variability of ducted SPANDAR returns over a 30 minute interval on May 4, 2000 suggests that tracking refractivity conditions requires sub-hourly environmental measurements (Movie courtesy of NSWC).
Markov Models for Solving the Wave Equation

• The parabolic approximation for solving the wave equation in inhomogeneous media is a marching solution which lends itself to Markov-model based estimation methods:

\[ H(\alpha) = \exp(-j\alpha^2 \Delta r / 2k) \]

• Each Fourier “split-step” of the parabolic equation can be expressed in linear system terms:

\[ \exp(jk(n_{k-1}^2 - 1)\Delta r / 2) \]
RFC as Range-Recursive State Estimation

- The split-step operation, $f(u_k, g_k)$, on the field, $u_k$ at range step $k$ is combined with a Markov model for refractivity parameters, $g_k$, to obtain the non-linear state equation:

$$
\begin{bmatrix}
    x_{k+1} \\
g_{k+1}
\end{bmatrix} = \begin{bmatrix}
u_{k+1} \\
g_k
\end{bmatrix} = f(u_k, g_k) + \begin{bmatrix} 0 \\
g_k
\end{bmatrix}
\epsilon_{g_k}
$$

where $\epsilon_{g_k}$ controls the smoothness of the refractivity parameters across range.

- The averaged log-amplitude radar return, $y_k$, is computed using the field near the surface:

$$
y_k = \frac{10}{\ln(10)} \ln(C^H x_k x_k^H C S_k + \sigma_n^2) - \frac{10 \gamma}{\ln(10)} + \nu_k
$$

where $C = [1, \ldots, 0]^T$, $s_k$ is the sea-surface backscatter strength, $\sigma_n^2$ is the receiver noise level, and $\nu_k$ is a zero-mean Gaussian with variance dependent on the pulse averaging.

- Range-dependent RFC goal is to compute the maximum a posteriori estimate of each, $g_k$, or the joint estimate of the sequence, $g_1, \ldots, g_k$, given the radar returns, $y_1, \ldots, y_k$. 

Recursive MAP RFC Estimation by Particle Filtering

- The non-linear RFC state estimation problem can be solved by representing the posterior density function of the state by a set of random particles, rather than a continuous function over some high dimensional state space [e.g. Gordon, Salmond, Smith, 1993]

- For example, suppose at step $k$, random particles, $\mathbf{x}_{k-1}(i), i = 1, ..., N$, are available from $p(\mathbf{x}_{k-1} \mid y_1, ..., y_{k-1})$. Then particles, $\mathbf{x}_k^*(i)$ from $p(\mathbf{x}_k \mid y_1, ..., y_{k-1})$ computed using these as input to the state equation with samples, $\varepsilon_{g_i}$, drawn from its known distribution.

- The updated posterior density is approximated at each particle, $\mathbf{x}_k^*(i)$, by forming:

$$q_i = \frac{p(y_k \mid \mathbf{x}_k^*(i))}{\sum_{j=1}^{N} p(y_k \mid \mathbf{x}_k^*(i))}$$

- Particles, $\mathbf{x}_k(i), i = 1, ..., N$, can be computed by bootstrap resampling $N$ times from the discrete distribution defined such that for any $j$, $\Pr\{\mathbf{x}_k(j) = \mathbf{x}_k^*(i)\} = q_i$. The forward MAP estimate of $g_k$ is the particle corresponding to the maximum $q_i$.

- These steps are repeated for each range step to obtain a recursive estimate.
RFC Sequence Estimation via a Monte Carlo Viterbi Method

- Joint MAP estimation of the range-dependent refractivity sequence, $g_1, ..., g_k$, facilitates revision of previous refractivity estimates when, for example, new clutter rings appear.

- A Monte Carlo Viterbi approach [Godsill, Doucet, West, 2000] uses the same particle trajectories as filtering methods.

- Instead of choosing the estimate based on the marginal posterior distribution of the state at range step, $k$, the set of maximum \textit{a posteriori} refractivity particles is computed by:

$$W_k(i) = \log p(y_k \mid x_k(i)) + \max_j [W_{k-1}(j) + \log P(x_k(i) \mid x_{k-1}(j))]$$

where $W_{k-1}(j) = \max_{x_1, ..., x_{k-1}} (\log p(x_1, ..., x_{k-1} \mid y_1, ..., y_{k-1})$.

- The MAP estimate of the refractivity parameter sequence at each range step is obtained by finding argmax of $W_k(i)$ and then tracing back through the trellis.

- In the RFC problem, the transition probability distribution, $p(x_k(i) \mid x_{k-1}(j))$, is truncated Gaussian with variances and a threshold designed to avoid discontinuities in the state trajectories.
Simulated and Real Clutter Analysis from SPANDAR

- The Space and Range Radar (SPANDAR) at NASA’s Wallops Island facility was used to collect clutter data between 3/4/98 – 04/06/98 at 2.8 Ghz @ 500 Hz. PRF with 448 600 m. wide range bins, 1 MW power, and 100 ft. antenna elevation with 0.5 degree beamwidth.

- PPI data consists of 128 averaged log-amplitude returns per azimuth. Clutter power versus range display (right) indicates strong surface-based ducting weakly correlated over time.
Simulated and Real Clutter Analysis Summary

- RFC performance of both the MAP particle filter and Viterbi estimators was performed.
- The particle filter was implemented using a 3 tri-linear parameters corresponding to M-deficit, base-height, and trapping-layer thickness.
- The Viterbi MAP sequence estimator used 4 tri-linear parameters corresponding to the heights and M-unit values of the two critical points of a tri-linear profile.
- Range variability was assumed limited to 1 m/km for base height and thickness and 1 M-unit/km for M-deficit.
- The prior distribution of refractivity parameters was assumed uniform on base-height from –20 m to 180m, thickness from 10m to 100m, and M-deficit from 5 M-units to 65 M-units.
- The backscatter cross-section of the sea surface was assumed constant and estimated by using the median clutter value between 0 and 15 km.
- 50 Monte Carlo trials were performed for both matched and mismatched refractivity models. Comparison between helo-measured and RFC-estimated coverage diagrams were performed using 12 SPANDAR clutter versus range measurements.
Good MAP Refractivity Sequence Estimate for SPANDAR

- Helo vs. RFC refractivity (top), Helo and RFC coverage diagrams (middle), and clutter fit (bottom) with average absolute prop loss error of 4.54 < range-independent helo data.
Poor MAP Refractivity Sequence Estimate for SPANDAR

- Helo vs. RFC refractivity (top), Helo and RFC coverage diagrams (middle), and clutter fit (bottom) with average absolute prop loss error of 10.6 > range-independent helo data.
Propagation Differences for RFC Sequence Estimate vs. Helo

- Propagation loss differences for all 12 SPANDAR cases with helicopter-measured refractivity profiles taken as ground-truth extended to 100 km in range.
- RFC sequence estimation via Monte Carlo Viterbi method using 400 particles gives mismatch typically less than 5 dB in the duct with larger errors above.
Propagation Differences for RFC Particle Filter vs. Helo

- Propagation loss differences for all 12 SPANDAR cases with helicopter-measured refractivity profiles taken as ground-truth extended to 100 km in range.
- RFC particle filter estimate using 400 particles degrades at further ranges due to its inability to revise refractivity estimates at previous ranges as clutter peaks encountered.
Refractivity Profile Fits for RFC Sequence Estimate vs. Helo

- Refractivity profile comparisons with helicopter-measured refractivity profiles for all SPANDAR cases out to 60 km in range.
- RFC captures the essential features of the duct in all cases but sometimes overestimates the M-deficit and duct height.
Summary of MAP RFC Sequence Estimate SPANDAR Results

- Average absolute propagation loss prediction errors (computed 0-100 km in range and 0-200 m. in height) for SPANDAR data set using MAP Monte Carlo Viterbi method.

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Conclusions

• Physics-based multipath SSAP has the potential to significantly enhance sonar and radar performance given sufficiently accurate modeling of the propagation environment.

• Matched-field processing using full-field numerical models of complex underwater acoustic channels to improve the array gain of sonar systems has yet to be achieved in the fleet, but many interesting experiments at sea have been performed.

• “Differential” matched-field methods which model changes in complex multipath propagation caused by target motion are less sensitive to environmental mismatch and thus hold promise in providing new capabilities to existing sonar and radar systems.

• Matched-field altitude estimation is an example of differential matched-field processing across OTH radar dwells in complex delay-Doppler space. Currently applying this approach in the space-time domain for sonar target depth-range-rate estimation.

• Tighter coupling of computational electromagnetics with SSAP methods, such as the link between the PE solver and Markov modeling, has the potential for fast solution of large remote sensing problems (e.g. refractivity estimation from radar sea clutter).