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Beamspace Adaptive Channel Compensation for Sensor Arrays with Faulty Elements

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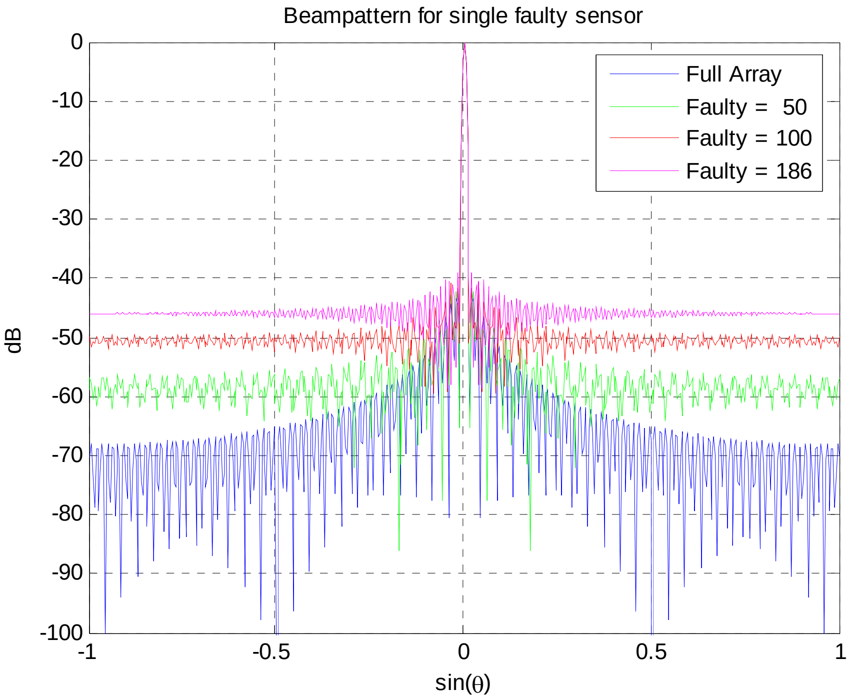
Outline

- Missing sensor problem
- Data model with missing sensors
- Covariance matrix estimation using single snapshot
- Previous methods
  - MMSE Interpolation
  - Elementspace ACC
- Proposed Algorithm
  - Beamspace ACC
- Results
- Summary and conclusion

Array Beam Pattern with Missing Sensors

- In practice, it is very common that some of the sensors in an array fail to operate. The conventional way of handling these faulty sensors is to null them prior to beamforming.
- Although zeroing the faulty sensors is a simple way of dealing with them, it results in higher sidelobes in the beampattern depending on their locations in the array.
- For an example scenario with one faulty sensor, the increase in the sidelobe level is related to the weight of the faulty sensor.
- Higher sidelobes if the faulty sensor is close to the middle of the array.



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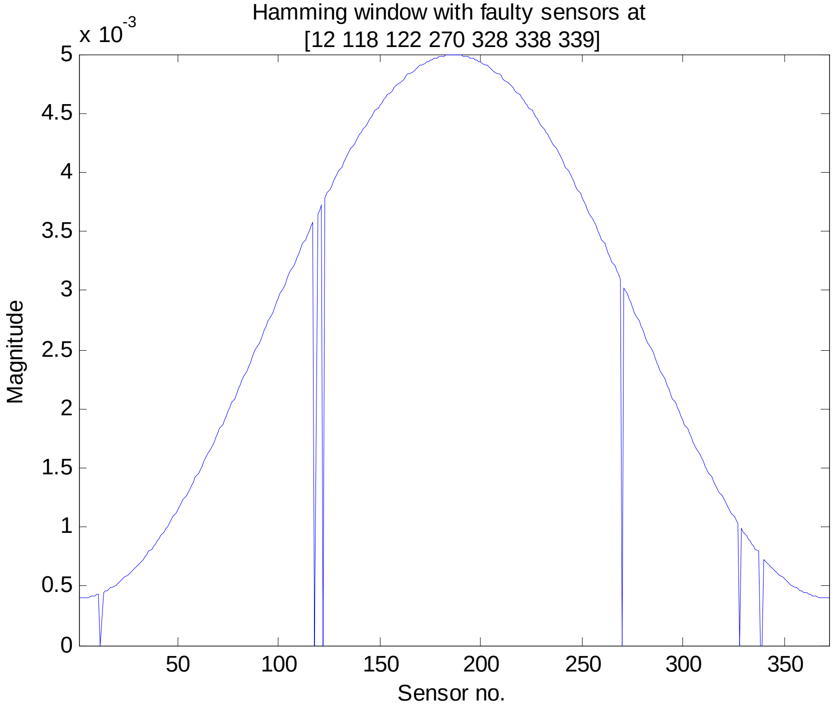
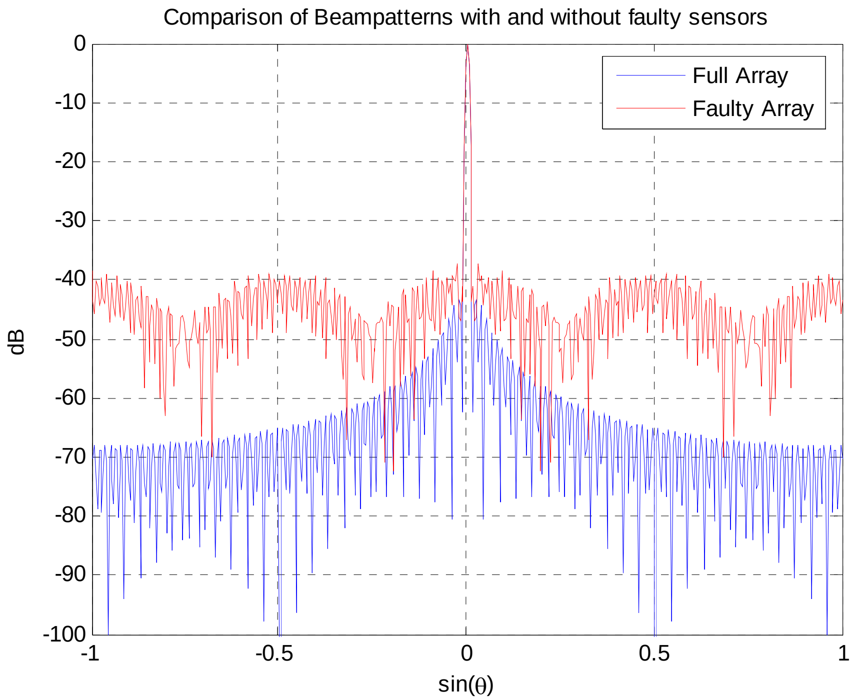
Array Beam Pattern with Missing Sensors

- If we have more than one faulty sensor, the faulty array sidelobes is no longer constant, but it takes the pattern of the faulty sensors.
- Peak sidelobe is approximately related to the ratio of the sum of the faulty sensor weights to that of the working sensors. [Ramsdale and Howerton 1980]
- Beam pattern with 8 missing sensors exhibits 7 dB higher peak sidelobes and up to 30 dB higher off-angle sidelobes compared to a full array.





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Array Data Modeling with Missing Sensors

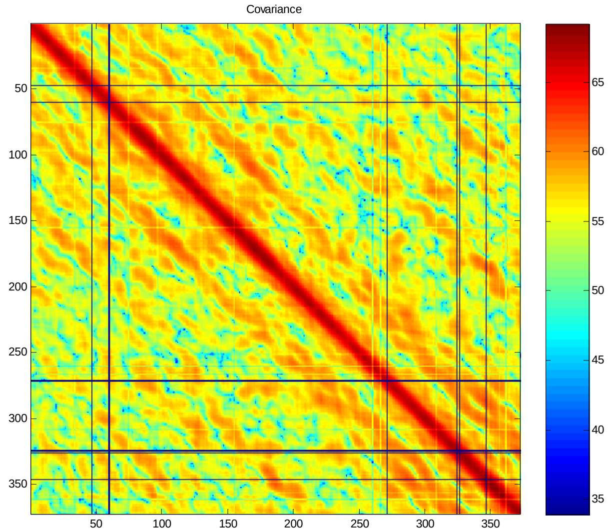
- Model the output from an *N*-element uniform linear array as zero-mean complex snapshots $\mathbf{x}_k$ with covariance matrix consisting of signal, interference, and noise:

$$\mathbf{R}_x = \sigma_s^2 d(\theta_s) d(\theta_s)^+ + \mathbf{APA}^+ + \sigma_n^2 \mathbf{I}$$

  where $d(\theta_s)$ and σ_s^2 are the signal wavefront and power, $\mathbf{APA}^+$ defines the unknown interference covariance, and $\sigma_n^2 \mathbf{I}$ represents the white noise component.

- Zeroing missing sensors corresponds to using a covariance matrix with correspondingly zeroed columns/rows e.g.

$$\begin{pmatrix} r_{1,1} & r_{1,2} & 0 & r_{1,4} & r_{1,5} \\ r_{2,1} & r_{2,2} & 0 & r_{2,4} & r_{2,5} \\ 0 & 0 & 0 & 0 & 0 \\ r_{4,1} & r_{4,2} & 0 & r_{4,4} & r_{4,5} \\ r_{5,1} & r_{5,2} & 0 & r_{5,4} & r_{5,5} \end{pmatrix}$$



- Classical interpolation methods can be thought of as "filling in" missing covariance matrix elements using Toeplitz approximation and then designing a linear filter for MMSE interpolation on a snapshot-by-snapshot basis.

Covariance Matrix Estimation with One Snapshot

- Covariance matrix estimation with as little as a single snapshot of data typically requires imposing a Toeplitz structure (i.e. spatially stationary field).
- A straightforward Toeplitz covariance matrix estimate can be computed by using the weighted projected covariance estimator [Barton and Smith (1997)] which uses biased average diagonals of the sample covariance matrix.
- For a fully populated array, this method is equivalent to computing a Toeplitz correlation matrix with the biased autocorrelation sequence of the array data.

$$r(k) = \frac{1}{N}\sum_{n=k}^{N-1} x(n)x^*(n-k) \qquad \hat{\mathbf{R}} = toeplitz(\mathbf{r})$$

- Toeplitz approximation models the interference subspace as a linear combination of uncorrelated plane wave returns corresponding to interference directions:

$$\mathbf{APA}^+ + \sigma_n^2\mathbf{I} \cong \hat{\mathbf{A}}\hat{\mathbf{P}}\hat{\mathbf{A}}^+ + \mathbf{\Phi}_{\mathbf{min}}\mathbf{\Lambda}_{\mathbf{min}}\mathbf{\Phi}_{\mathbf{min}}^+ \text{ where } \hat{\mathbf{A}}^+\mathbf{\Phi}_{\min} = \mathbf{0}$$

and $\hat{\mathbf{P}}$ and $\Lambda_{\min}$ are positive diagonal matrices.

- More sophisticated methods for structured covariance matrix estimation have been developed by numerous researchers (e.g. Jansson and Ottersten 2000, Stoica and Li 1999, Abramovich et al 1998, Robey and Fuhrmann 1991, Cadzow 1988, Williams and Johnson 1988) but typically involve greater computational complexity.

Minimum MSE Interpolation of Faulty Sensors

- A natural solution for handling faulty channels is to interpolate them using a linear combination of the good sensor outputs.
- The linear filter, $\mathbf{T_b}$, which minimizes mean-square error (MSE) can be found as

$$\min_{\mathbf{T_b}} \left\| \mathbf{x_b} - \mathbf{T_b x_g} \right\|^2 \text{ is given by } \mathbf{T_b} = \mathbf{R_{bg} R_{gg}^{-1}}$$

where $\mathbf{x_g} = \mathbf{L}^+ \mathbf{x}$ are the *N*-*m* good channels and $\mathbf{x_b} = \mathbf{K}^+ \mathbf{x}$ are the *m* bad channels.

and $$E\left\{ \begin{bmatrix} \mathbf{x_b} \\ \mathbf{x_g} \end{bmatrix} \begin{bmatrix} \mathbf{x_b}^+ & \mathbf{x_g}^+ \end{bmatrix} \right\} = \begin{bmatrix} \mathbf{R_{bb}} & \mathbf{R_{bg}} \\ \mathbf{R_{gb}} & \mathbf{R_{gg}} \end{bmatrix}$$

- A single snapshot estimate of the cross-correlation between faulty and good channels is obtained from the Toeplitz covariance matrix estimate of the full array data.

- The Minimum MSE (MMSE) interpolated full array data snapshot can be written as a linear transformation matrix $\mathbf{T_m}$ operating on the original data snapshots:

$$\mathbf{q_m} = \mathbf{T_m x} \quad \text{where } \mathbf{T_m} = (\mathbf{L} + \mathbf{K T_b})\mathbf{L}^+$$

- Minimizing channel-wise MSE does not explicitly minimize sidelobe leakage of interferers into quiet directions and thus only indirectly mitigates impact of bad channels.

Element-Space Adaptive Channel Compensation (ACC)

Idea: Adaptively reconstruct full-array snapshot, **q,** such that interference leakage into nominally quiet directions is minimized.

- Adaptive channel compensation (ACC) minimizes interference leakage into quiet plane-wave directions subject to the constraint that the data distortion at the good elements is within some tolerance, ε i.e.

$$\min_{\mathbf{q}} \; (\mathbf{q}^{+}\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+}\mathbf{q}) \;\; \text{subject to the constraint} \;\; \left\| \mathbf{L}^{+}\mathbf{q} - \mathbf{x}_{\mathbf{g}} \right\|_2^2 < \varepsilon$$

$$\text{Solution:} \qquad \mathbf{q} = (\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+} + \mu\,\mathbf{L}\mathbf{L}^{+})^{-1}\,\mu\,\mathbf{L}\mathbf{x}_{\mathbf{g}}$$

where the columns of $\mathbf{\Phi}_{\min}$ are the sub-dominant eigenvectors

of the Toeplitz-approximated $\hat{\mathbf{R}}_{\mathbf{x}}$.

- Effect of ACC is to make interferers look more like plane-waves across the full-array which can then be effectively nulled by conventional beamformer shading.

ACC as the Solution to a Linear Inverse Problem

- Adaptive channel compensation can be interpreted as inversion for the complete data using the linear operator **T** operating on the corrupted data snapshot: $$\mathbf{T} = \left(\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+} + \mu\,\mathbf{L}\mathbf{L}^{+}\right)^{-1} \mu\,\mathbf{L}\mathbf{L}^{+}$$
- Decomposing **T** in terms of the dominant $\mathbf{\Phi}_{\max}\mathbf{\Phi}_{\max}^{+}$ and sub-dominant $\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+}$ subspaces of $\hat{\mathbf{R}}_{\mathbf{x}}$ gives:

$$\mathbf{T} = \left(\mathbf{\Phi}_{\max}\mathbf{\Phi}_{\max}^{+} + \left(\frac{\mu}{1+\mu}\right)\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+}\right)\left(\mathbf{I} - \mathbf{K}(\mathbf{K}^{+}\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+}\mathbf{K})^{-1}\mathbf{K}^{+}\mathbf{\Phi}_{\min}\mathbf{\Phi}_{\min}^{+}\right)\mathbf{L}\mathbf{L}^{+}$$

where $\mathbf{K}^{+}$ and $\mathbf{L}^{+}$ select the faulty and good elements and $\mathbf{L}\mathbf{L}^{+} + \mathbf{K}\mathbf{K}^{+} = \mathbf{I}$

- Signals in the subdominant subspace are not interpolated but suppressed by a factor of $\mu/(1+\mu)$
- Choice of $\mu = 10$ empirically found to provide good compensation of faulty channels with negligible signal distortion.

Beamspace Adaptive Channel Compensation

- Motivated by observation that target detection primarily affected by high receiver sidelobes of transmit *mainlobe* clutter and strong directional interference.

Idea: The proposed BACC method adaptively reconstructs the receive beams, **q,** of the full array in the transmit mainlobe region so that strong directional components have minimal leakage into adjacent beams, subject to the constraint that data distortion at the good elements $\mathbf{x_g}$ is within some tolerance, ε.

$$\min_{\underline{\mathbf{q}}} \; (\underline{\mathbf{q}}^{+}\tilde{\mathbf{\Phi}}_{\min}\tilde{\mathbf{\Phi}}^{+}_{\min}\underline{\mathbf{q}}) \;\; \text{subject to the constraint} \;\; \left\| \mathbf{L}^{+}\mathbf{U}\underline{\mathbf{q}} - \underline{\mathbf{x}}_{\mathbf{g}} \right\|_2^2 < \varepsilon$$

Solution: $$\underline{\mathbf{q}} = (\tilde{\mathbf{\Phi}}_{\min}\tilde{\mathbf{\Phi}}^{+}_{\min} + \mu\, \mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}\mathbf{U})^{-1}\mu\, \mathbf{U}^{+}\mathbf{L}\underline{\mathbf{x}}_{\mathbf{g}}$$

The sensor data, $\hat{\underline{\mathbf{x}}}_{acc}$, can be formed by projecting **q** back to the element space

$$\hat{\underline{\mathbf{x}}}_{acc} = \mathbf{U}\underline{\mathbf{q}} = \mathbf{U}(\tilde{\mathbf{\Phi}}_{\min}\tilde{\mathbf{\Phi}}^{+}_{\min} + \mu\, \mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}\mathbf{U})^{-1}\mu\, \mathbf{U}^{+}\mathbf{L}\underline{\mathbf{x}}_{\mathbf{g}}$$

where $\mathbf{U}^{+}$ is the beamforming matrix, the columns of $\tilde{\mathbf{\Phi}}_{\min}$ are sub-dominant eigenvectors of the beamspace covariance matrix $\mathbf{R}_u = \mathbf{U}^{+}\hat{\mathbf{R}}_x\mathbf{U}$, and **L** is the selection matrix for the good sensors, $\underline{\mathbf{x}}_{\mathbf{g}}$.

- $\hat{\mathbf{R}}_{\mathbf{x}}$ is the Toeplitz approximated element space covariance matrix of full array data.

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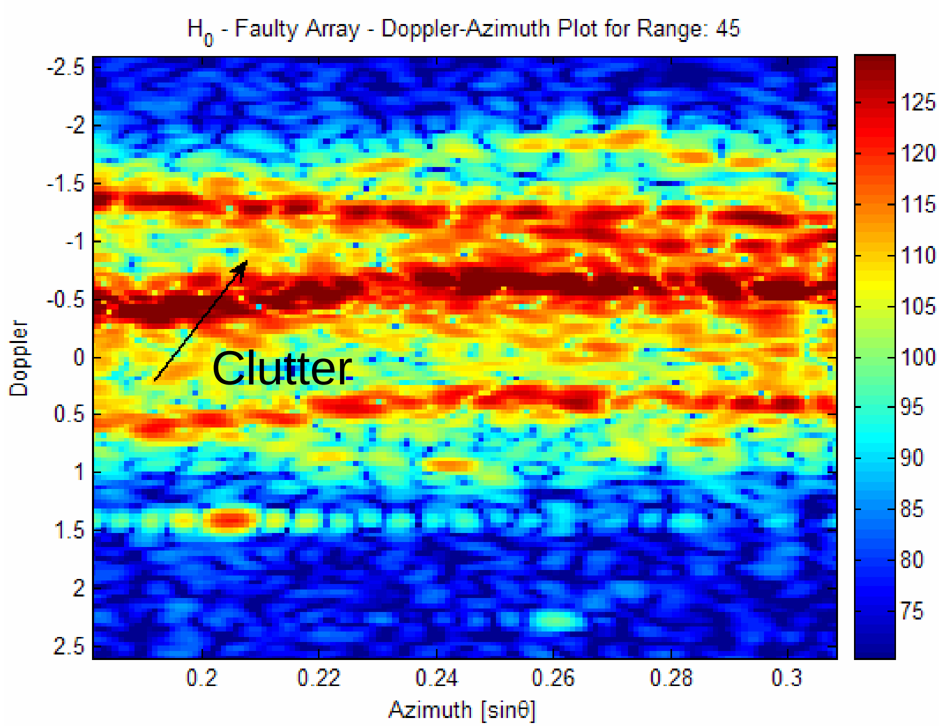
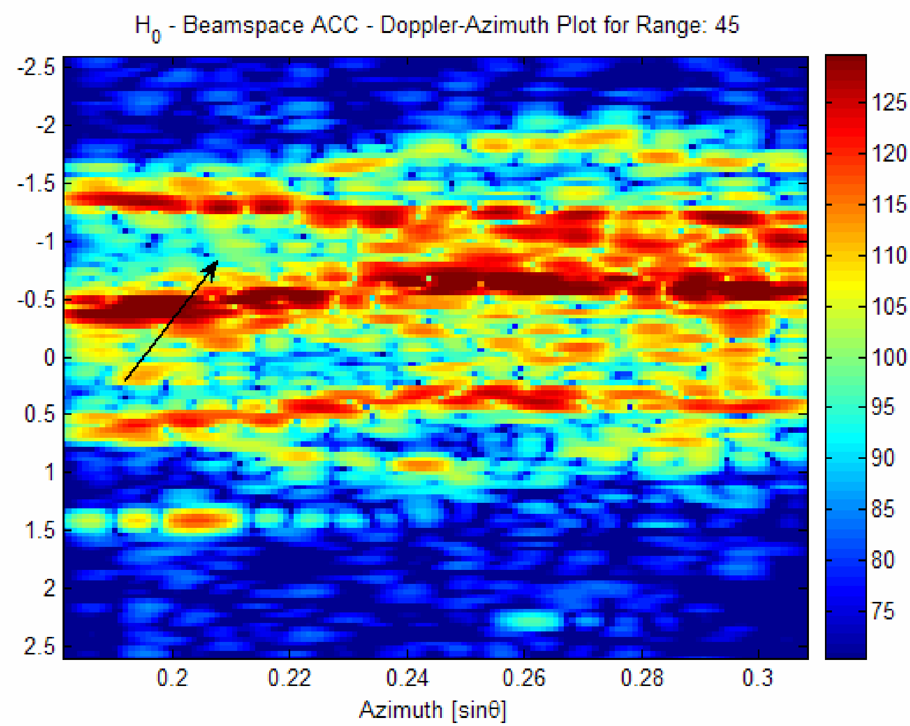
BACC Features

- BACC transformation matrix is $\mathbf{T} = \mathbf{U}(\tilde{\mathbf{\Phi}}_{\min}\tilde{\mathbf{\Phi}}_{\min}^{+} + \mu\ \mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}\mathbf{U})^{-1}\mu\ \mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}$
- μ controls the amount of suppression of the sidelobe leakage into other beams. Similar to elementspace ACC, the noise subspace component is suppressed approximately by a factor $\mu/(1+\mu)$
- $\tilde{\mathbf{\Phi}}_{\min}$ represents the noise subspace eigenvectors.
- The larger the size of $\tilde{\mathbf{\Phi}}_{\min}$, the more quiet directions to suppress the leakage.
- In clutter dominated regions, the quiet directions are limited. However in interference dominated regions, the size of $\tilde{\mathbf{\Phi}}_{\min}$ is larger. In our experiments, 10 out of 18 beams are used to represent the noise subspace.
- Note that if the size of $\tilde{\mathbf{\Phi}}_{\min}$ is zero, $\mathbf{T} = \mathbf{U}(\mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}\mathbf{U})^{-1}\mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}$ which means no suppression, just interpolation in the mainlobe directions similar to Fourier interpolation where $\mathbf{T} = \mathbf{U}\mathbf{U}^{+}\mathbf{L}\mathbf{L}^{+}$
- Because matrices to invert and decompose are of dimension equal to the number of beams (e.g. 18) vs. the number of elements (e.g. 372), beamspace ACC is much faster for large receive arrays.
- For our real data experiments, beamspace ACC is ~ **40 times faster** than element-space ACC and **~ 5 times faster** than MMSE.

Beam-space ACC Doppler-Azimuth Spectra

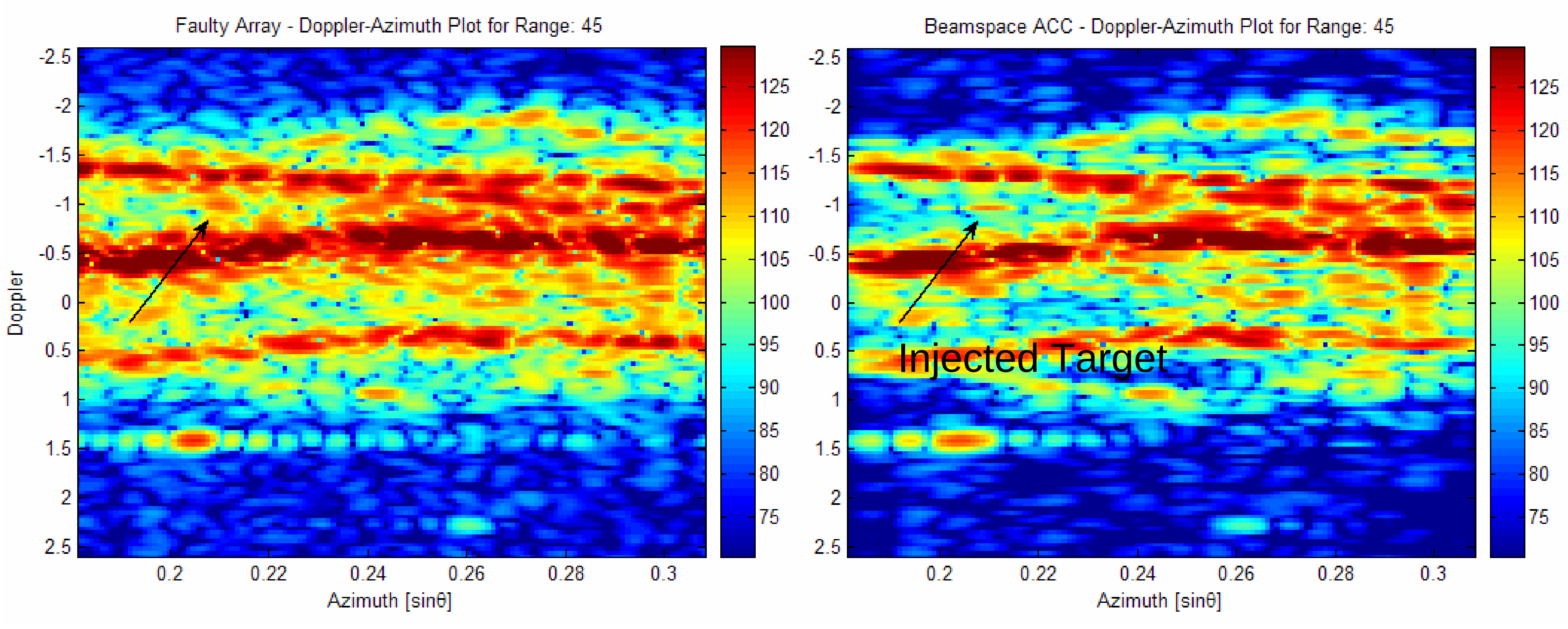
- The Doppler-Azimuth spectra comparing the Conventional (left) vs. Beamspace ACC (right) in transmitter mainlobe.
- Note the higher clutter level for the faulty sensor case because of the high sidelobes of the directional interference leaking to the other beams. (Arrow points to potential clutter peak).





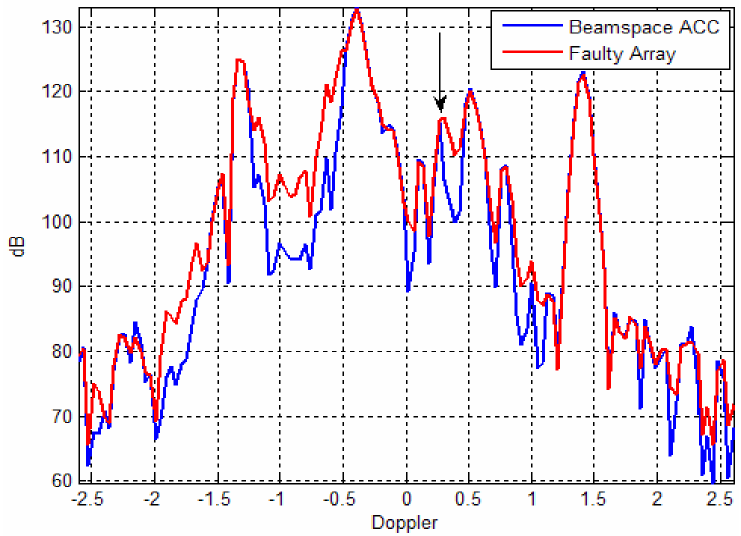
Beam-space ACC with an Injected Target

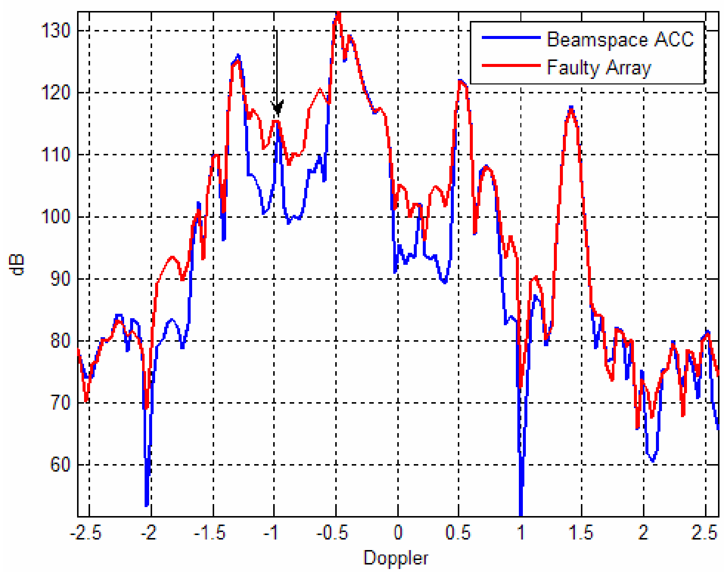
- The Doppler-Azimuth Spectra with the injected target comparing the Conventional (left) vs. Beamspace ACC (right) in transmitter mainlobe.
- Target injected in region of clutter peak for conventional processing (arrows).
- Note that the injected target can be seen clearly with beam-space ACC.

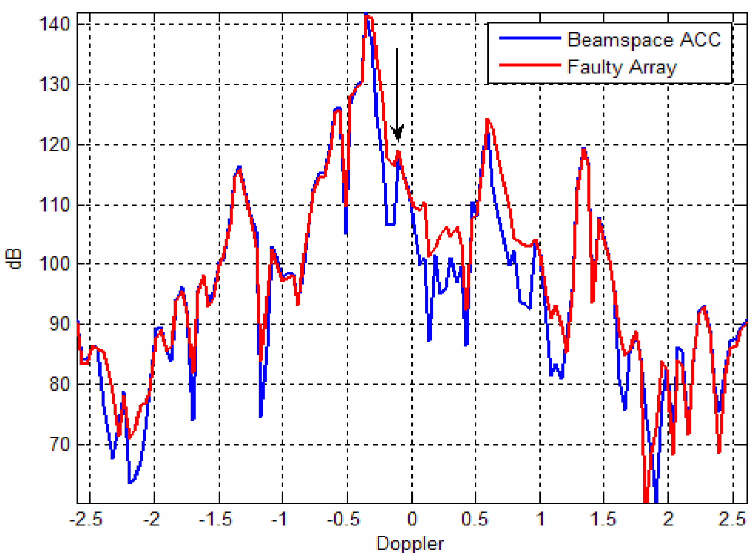


Beam-Space ACC Doppler Spectra in Clutter

- Conventional (red) vs. Beam-space ACC (blue) with injected targets (arrows) illustrates unmasking.
- The injected target is shown with the arrow.
- Note that the target can be seen clearly using BACC.







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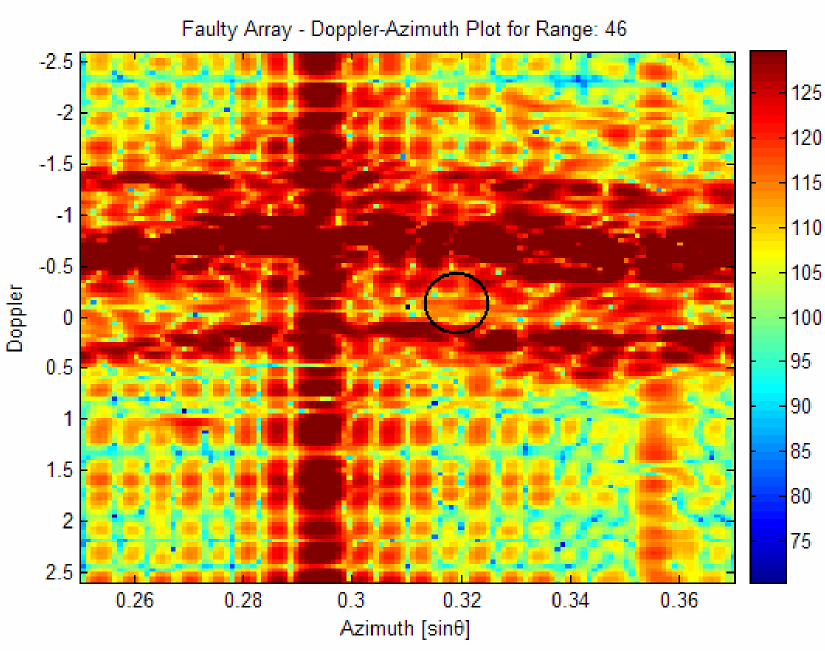
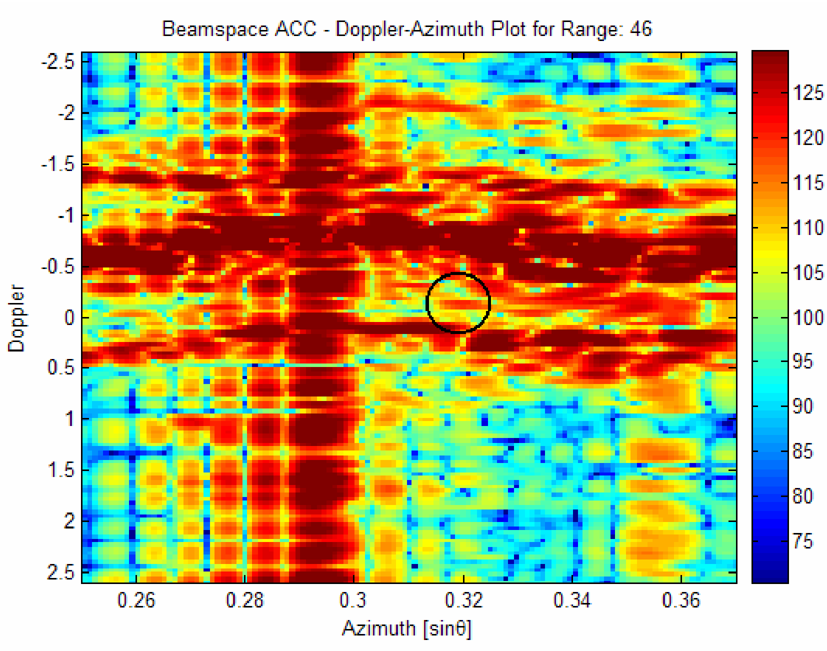
Beamspace ACC Performance Against RF Interference

- The Doppler-Azimuth plots comparing the conventional (left) vs. Beam-space ACC.
- Note the strong interference, assumed to be RFI, in the transmit mainlobe and the high sidelobes leaking into adjacent beam. An injected target is shown with a circle.
- The sidelobes are suppressed and injected target can be seen clearly with ACC.





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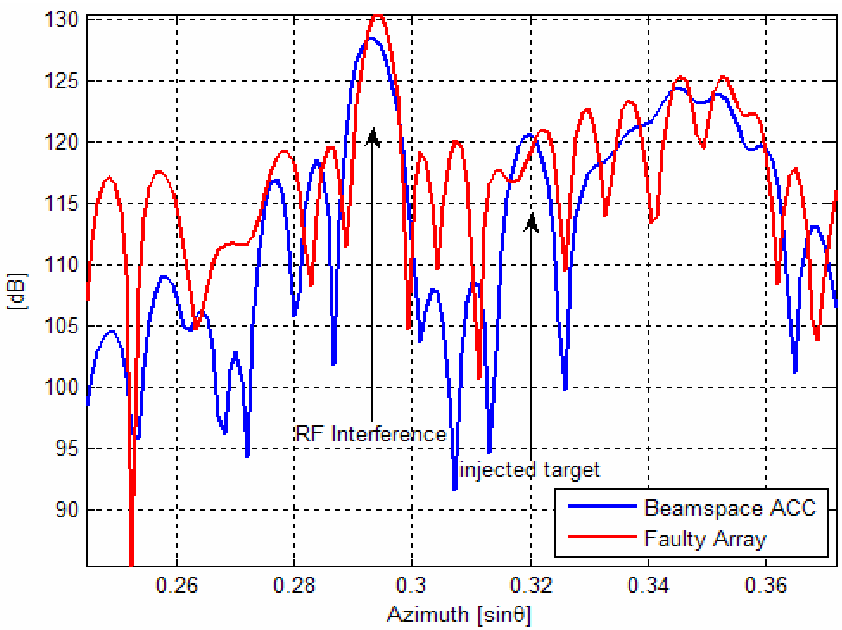
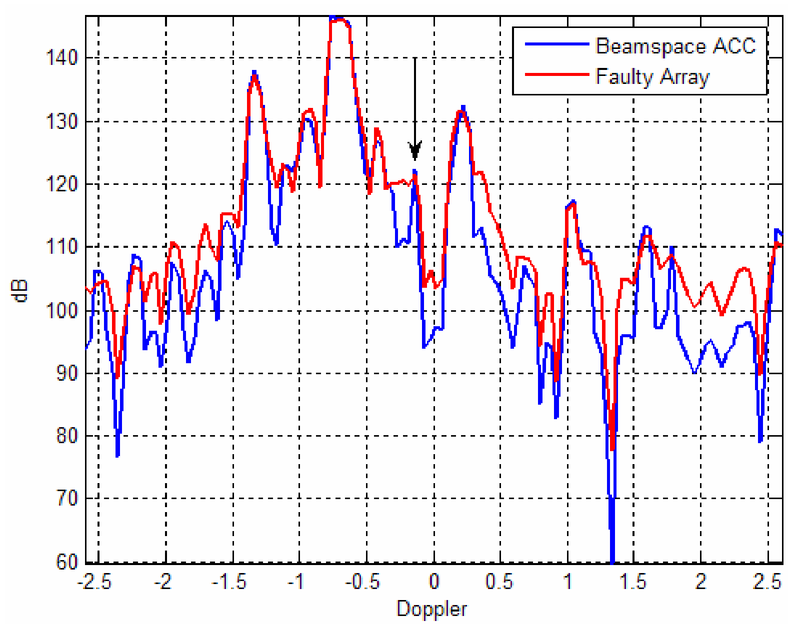
Beam-Space ACC with RFI

- Range cut (left) and Doppler cut (right) with target injected into the clutter in the presence of a strong directional RF interference.
- The interferer and the injected target are shown with arrows.
- Note that suppression of RFI by ACC results in target unmasking.

Beam-power vs. Bearing plot at target Doppler



Beam-power vs. Doppler at target azimuth



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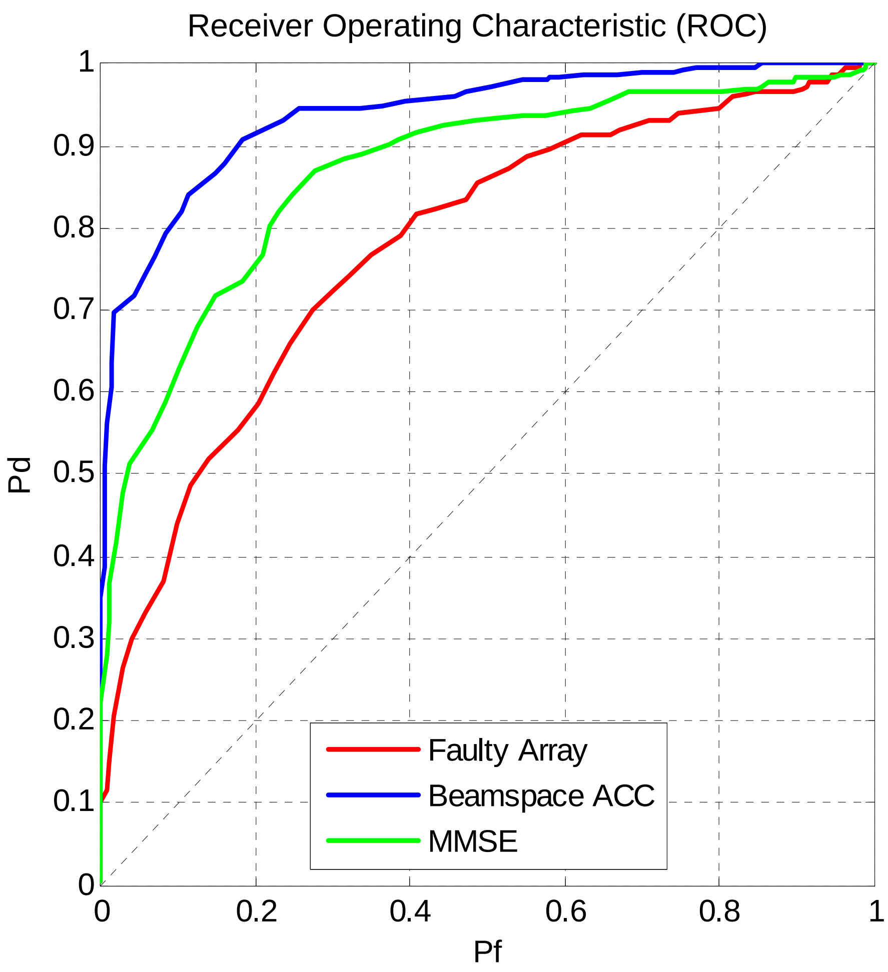
ROC Detection Performance for Beamspace ACC in RFI

- Receiver operating characteristic (PD vs. PFA) for conventional, MMSE, and Beam-space ACC.
- Based on 300 Monte Carlo trials using synthetic data generated from the real data covariance matrix.
- Beamspace ACC outperforms MMSE and conventional faulty array processing.
- Beamspace ACC is **5 times faster** than MMSE.
- Comparison with adaptive beamforming of interest, although azimuthally-spread interference and extremely limited snapshot support makes this a challenge.



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Summary and Conclusion

- Adaptive channel compensation (ACC) of faulty sensors important when directional clutter and/or interference present.
- MMSE interpolation methods not explicitly aimed at reducing leakage of strong interference into quiet look directions.
- ACC performed on a single snapshot of post-Doppler processed data which avoids the stationarity requirements of spatially adaptive beamforming solutions.
- ACC proposed as a means of compensating the entire array so that strong interferers made to look more like uncorrelated plane-waves which can then be suppressed by conventional beamformer shading.
- Beamspace ACC is 40 times faster than the element-space ACC.
- Unlike Element-space ACC, which suppresses the sidelobe leakage from all directions, Beam-space ACC is focused on the transmitter mainlobe where the detection is performed.

Questions?

Thank You!