Robust Range-rate Estimation of Passive Narrowband Sources in Shallow Water

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Outline

- Introduction
- The Independent Mode Range-rate Estimator
- Comparison with Cramer-Rao Bound
- Application to SWellEx-96 data
- Conclusions.

Range-Rate Discrimination in Passive Sonar

Motivation:

- Range-rate is a more robust searching dimension with regard to wavenumber mismatch in Matched Field Processing (MFP) compared with absolute range.
- The discrimination capability in range-rate is helpful to detect targets in the presence of interferences with similar bearings but different relative range-rate.

Background:

- Normal mode theory for a narrowband moving source derived by Hawker (JASA, 1979). Projection of target Doppler onto different modes induces signal fluctuations which are target range-rate dependent.
- Direct application of MUSIC (Song, 1990) does not take advantage of available environmental information and requires a long observation time.
- Direct extension of Matched Field Processing (Zala, 1992) with range-rate is computational intensive due to parameter coupling.

Acoustic Normal Mode Theory For a Moving Source

The velocity potential emitted by a narrowband, horizontally uniform moving point source, in a range-independent stratified oceanic waveguide: (Hawker, 1979)

$$\psi(t) \approx C \sum_{m=1}^{M} \frac{U_m(z)U_m(z_s)}{\sqrt{k_m R_0}} \exp\left[j\omega_m t - jk_m R_0 \left(1 - \frac{v_r}{v_m^G}\right)\right] \tag{1}$$

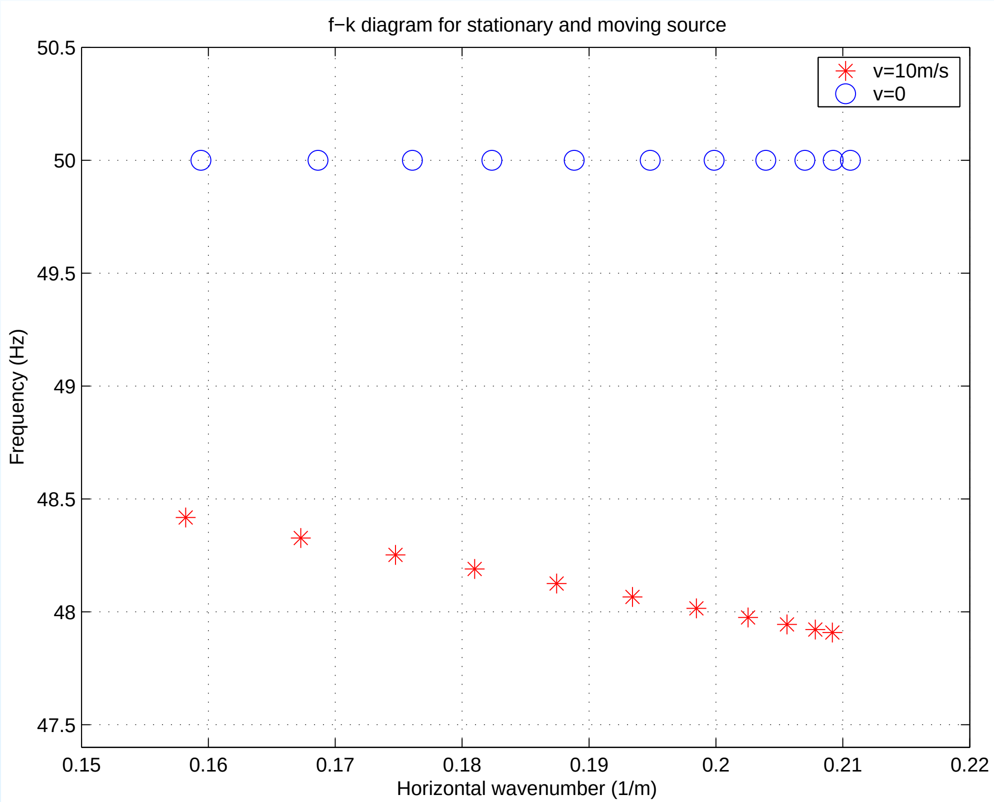
where $U_m(z)$ and $U_m(z_s)$ are mode eigenfunctions at receiver depth z and source depth z_s. k_m is the horizontal wavenumber. R_0 the initial range.

$$\omega_m = \omega_0 - k_m v_r (1 - \frac{v_r}{v_m^G}) \approx \omega_0 - k_m v_r$$

is the Doppler frequency, in which ω_0 is the intrinsic frequency, v_r the range-rate of the source and v_m^G the group velocity of mode m.

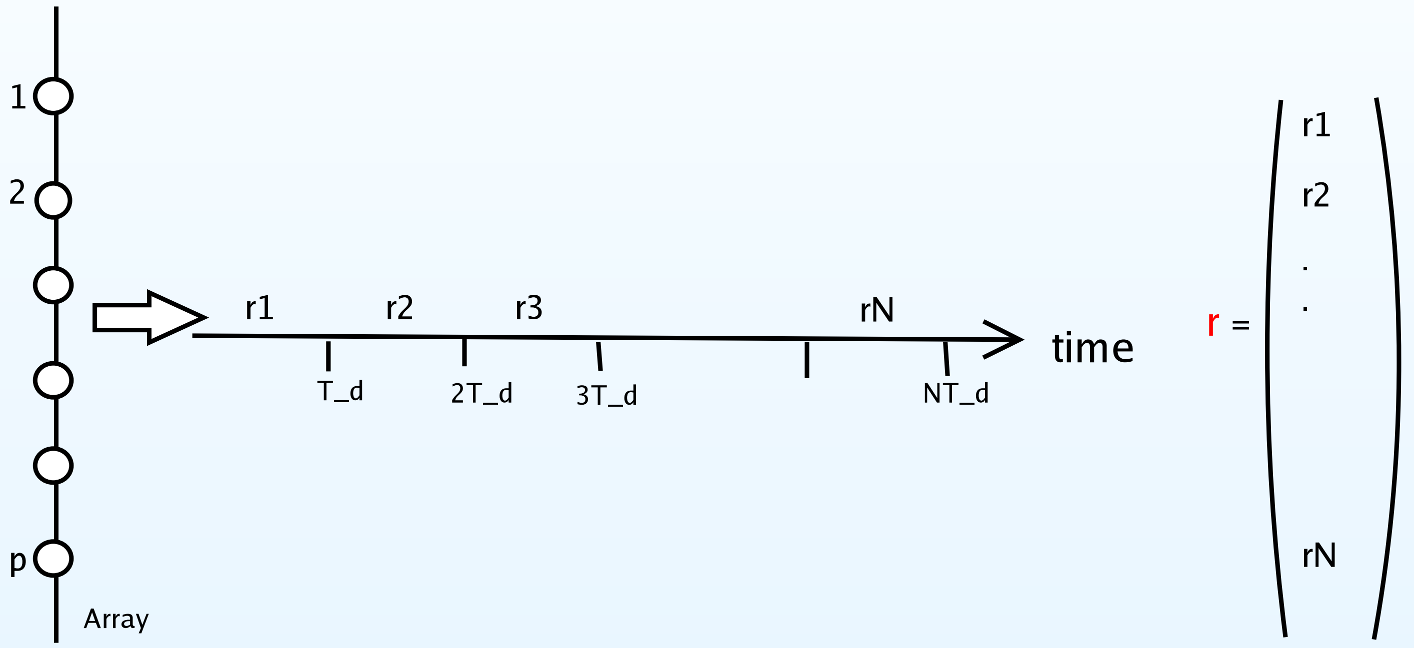
Differential Doppler for a Moving Source

Differential Doppler for moving vs. stationary source evident from slope of frequency distribution as a function of modal wavenumber for a single tonal source. Note that range-rate dependence does not require relative phase between modal components.



The Time-Frequency Data Snapshot Vector

Consider snapshot of narrowband data which consists of a concatenation of N conventional frequency domain snapshots over time.



Note range-rate processing can be performed before or after conventional beamforming.

The Signal Model (1)

Model the $(pN \times 1)$ space-time snapshot $\boldsymbol{r}$ as:

$$\boldsymbol{r} = s \sum_m a_m (\boldsymbol{d}_m \otimes \boldsymbol{u}_m) + \boldsymbol{n} \tag{2}$$

Where s is the signal amplitude, unknown nonrandom. a_m is a zero-mean complex random variable representing mode m's amplitude. $\boldsymbol{d}_m$ is a time harmonic vector for mode m:

$$\boldsymbol{d}_m = [1 \; e^{-jk_m v_r T_d} \; e^{-jk_m v_r 2T_d} \; \ldots \; e^{-jk_m v_r (N-1)T_d}]^T$$

$\otimes$ is the Kronecker product. $\boldsymbol{u}_m$ represents mth mode eigenfunctions evaluated at the depths of each array element:

$$\boldsymbol{u}_m = [U_m(z_1) \; U_m(z_2) \; \ldots \; U_m(z_p)]^T$$

and $\boldsymbol{n} \sim CN(0, \sigma_n^2 \boldsymbol{I})$ represents complex white Gaussian noise.

The Signal Model (2)

In matrix form

$$\boldsymbol{r} = s\boldsymbol{D}\boldsymbol{a} + \boldsymbol{n} \tag{3}$$

where

$$\boldsymbol{D} = [\boldsymbol{d}_1 \otimes \boldsymbol{u}_1 \quad \boldsymbol{d}_2 \otimes \boldsymbol{u}_2 \quad \ldots \quad \boldsymbol{d}_M \otimes \boldsymbol{u}_M]$$

and

$$\boldsymbol{a} = [a_1 \quad a_2 \quad \ldots \quad a_M]^T$$

The variance of mode amplitude is

$$\sigma_m^2 = E(a_m a_m^H) = U_m(z_s)^2 / k_m \tag{4}$$

In this model:

- Assume a_m and a_n are uncorrelated for $m \neq n$.
- The model is independent of absolute range.

The Independent Mode Range-rate Estimator (IMRE)

The covariance matrix is:

$$\boldsymbol{R} = E(\boldsymbol{r}\boldsymbol{r}^H) = P_s \boldsymbol{D}\boldsymbol{S}\boldsymbol{D}^H + \sigma_n^2 \boldsymbol{I} \tag{5}$$

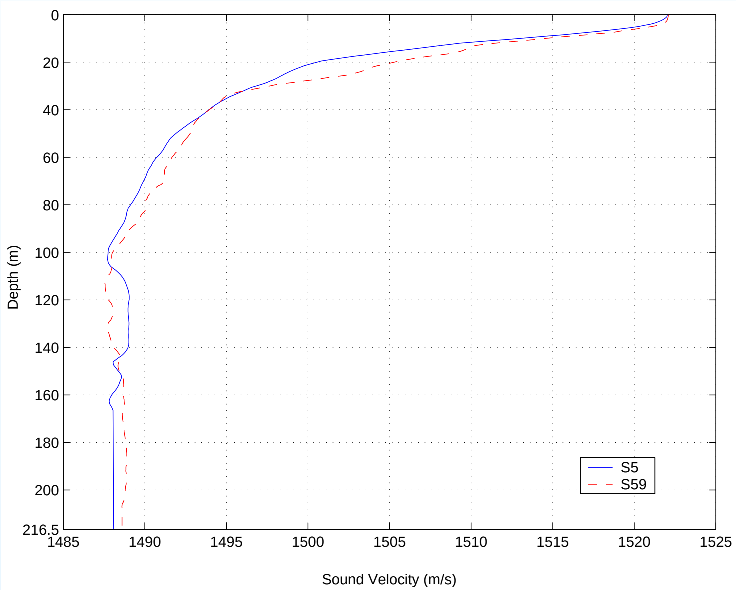
where $P_s = |s|^2$ and $\boldsymbol{S} = E(\boldsymbol{a}\boldsymbol{a}^H) = diag[\sigma_1^2 \quad \sigma_2^2 \quad \ldots \quad \sigma_M^2]$.
Given the environment and target depth z_s, the signal covariance matrix $\boldsymbol{R}_s = \boldsymbol{D}\boldsymbol{S}\boldsymbol{D}^H$ could be computed for each hypothesized range-rate v_r.
Suppose the dominant eigenvector of $\boldsymbol{R}_s$ is $\boldsymbol{h}_1$ (normalized), then a Bartlett type estimator can be expressed as:

$$P(v_r) = \frac{1}{\lambda_1} \boldsymbol{h}_1^H \hat{\boldsymbol{R}} \boldsymbol{h}_1 \tag{6}$$

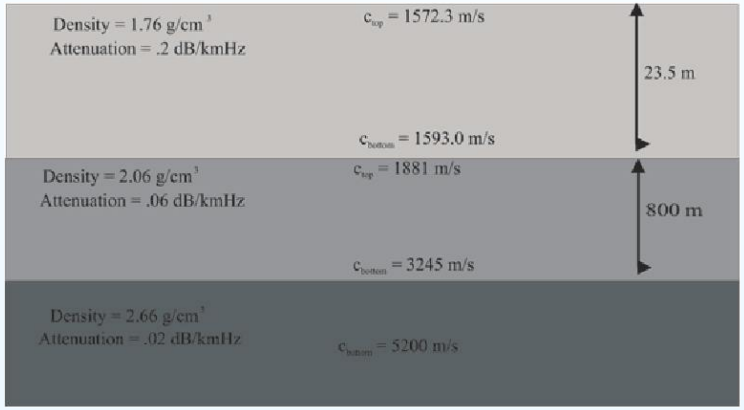
which maximizes output SNR in white noise. $\hat{\boldsymbol{R}}$ is the sample covariance matrix and λ_1 is the maximum eigenvalue of $\hat{\boldsymbol{R}}$.

The SWellEx-96 Environment

Typical shallow water environmental profiles of SWellEx-96 are used in simulations:

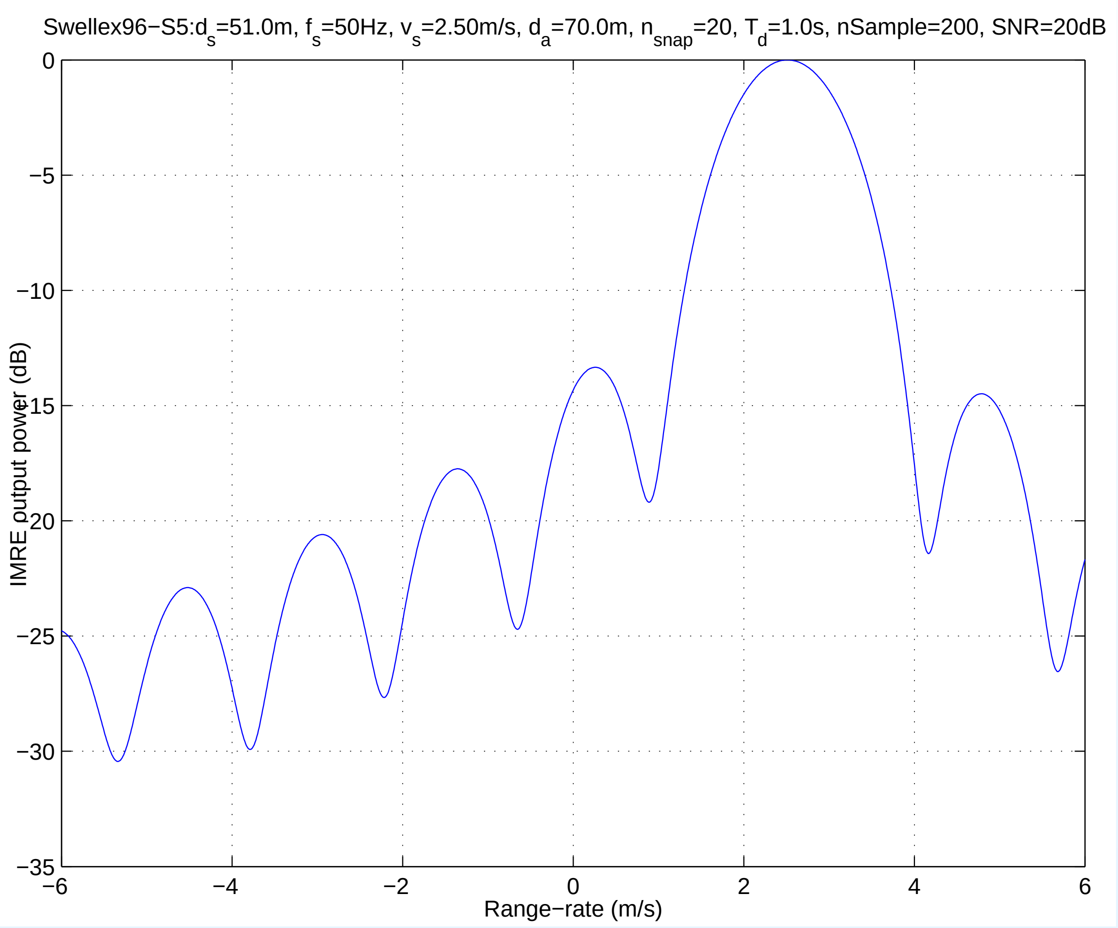


SWellEx-96 Water Column Sound Velocity Profile



SWellEx-96 Sea Bottom Properties

Typical IMRE Output



The sidelobe is 13dB down from the mainlobe.

Typical Eigenspectrum of Covariance Matrix R

Same settings as previous slide without noise. Source is moving at 2.5m/s.

Eigenvalue number	Eigenvalues (dB)	Normalized eigenvalues
1	-18.3	0.9702
2	-33.4	0.0297
3	-59.6	0.7231×10^{-4}
4	-85.2	0.1998×10^{-6}
5	-111.7	0.4465×10^{-9}
6	-140.8	0.5507×10^{-12}
7	-	0
8	-	0
$\vdots$	$\vdots$	$\vdots$
20	-	0

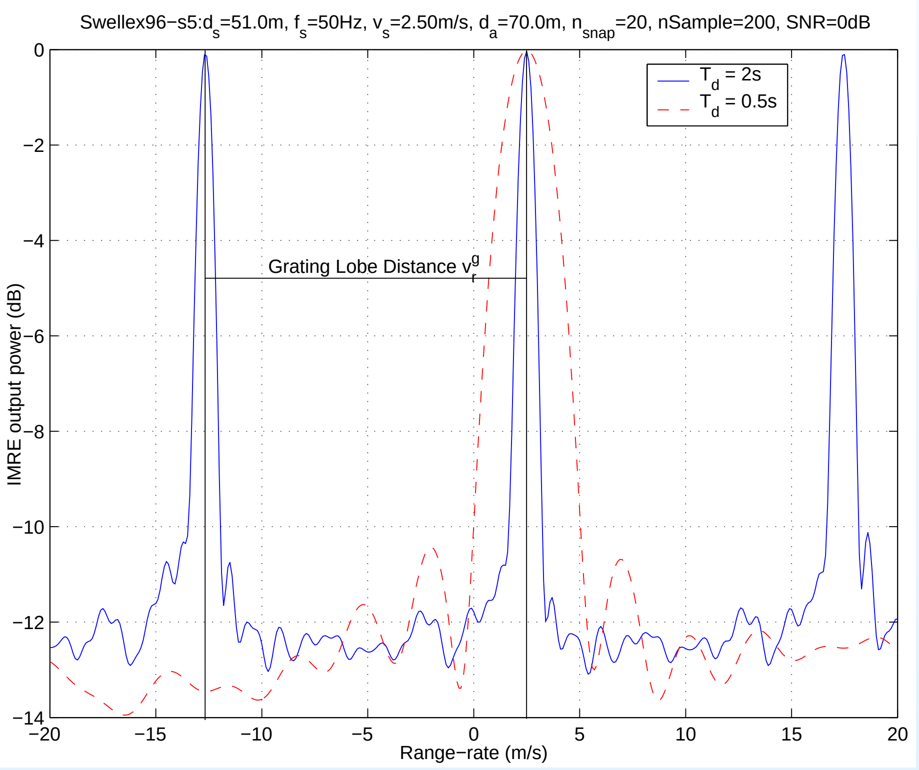
The dB difference between 1st and 2nd eigenvalues corresponds to mainlobe/sidelobe difference in previous slide.

Range-rate Aliasing

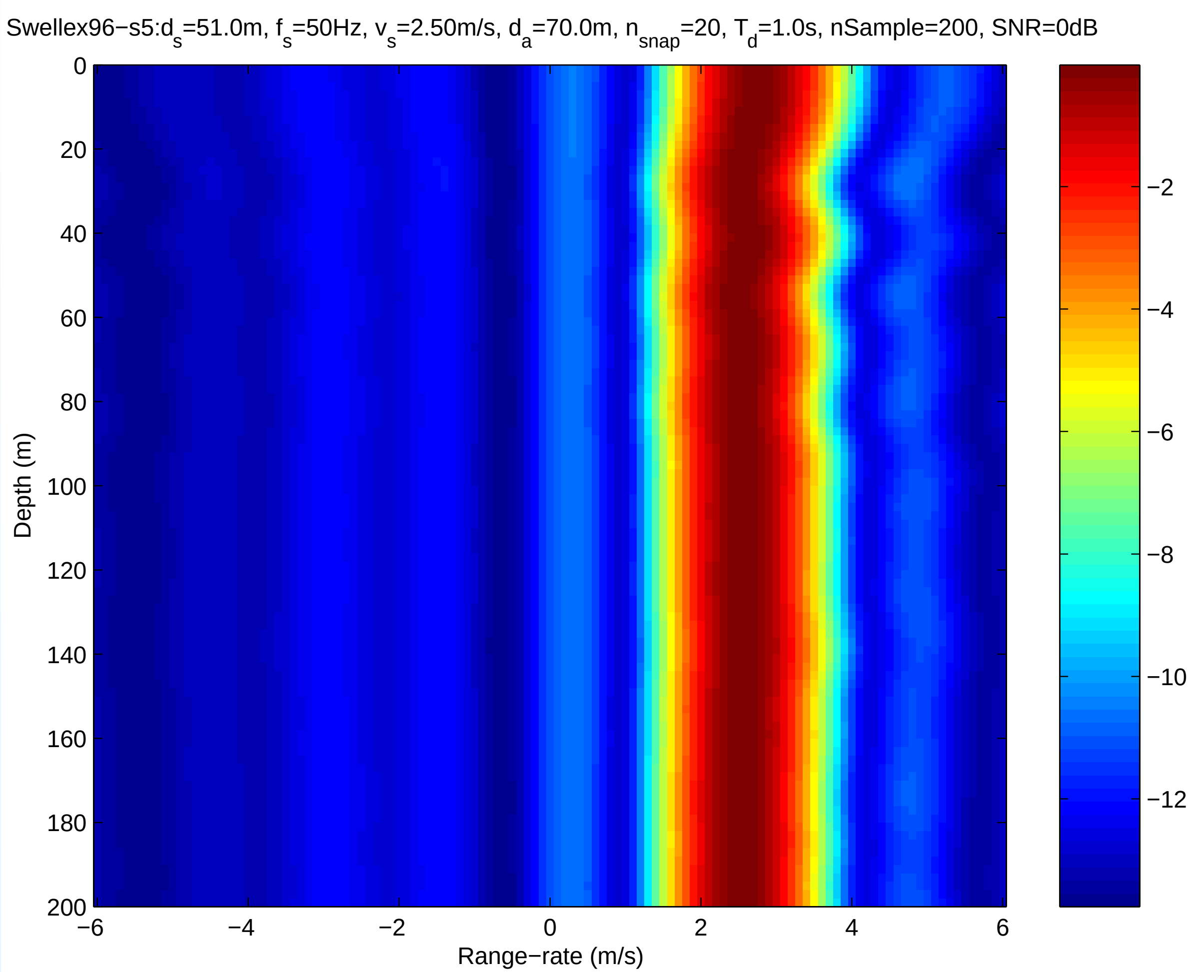
Range-rate difference between grating lobes:

$$v_r^g \approx \frac{2\pi}{\bar{k}T_d} \tag{7}$$

where $\bar{k}$ is the mean value of k_m. T_d is the snapshot delay.



Insensitivity to Target Depth



IMRE search in both depth and range-rate with one sensor

Cramer-Rao Low Bound (1)

- The unknown parameters are: $\boldsymbol{\Theta} = (v_r, P_s, \sigma_n^2)^T$.
- each element of Fisher Information Matrix (FIM) could be represented by:

$$J_{ij} = N_s tr \left[\boldsymbol{R}^{-1} \frac{\partial \boldsymbol{R}}{\partial \theta_i} \boldsymbol{R}^{-1} \frac{\partial \boldsymbol{R}}{\partial \theta_j} \right] \quad (8)$$

where N_s is the number of sampled $\boldsymbol{r}$.
To compute J_{ij}, recall

$$\boldsymbol{R} = P_s \boldsymbol{D} \boldsymbol{S} \boldsymbol{D}^H + \sigma_n^2 \boldsymbol{I}$$

So

$$\frac{\partial \boldsymbol{R}}{\partial P_s} = \boldsymbol{D} \boldsymbol{S} \boldsymbol{D}^H \quad (9)$$

$$\frac{\partial \boldsymbol{R}}{\partial \sigma_n^2} = \boldsymbol{I} \quad (10)$$

Cramer-Rao Low Bound (2)

Let $\boldsymbol{c} = [0 \quad 1 \quad 2 \quad \ldots \quad (N-1)]^T$ and $\boldsymbol{k} = [k_1 \quad k_2 \quad \ldots \quad k_M]^T$.
Define matrix $\boldsymbol{G}$:

$$\boldsymbol{G} = -jT_d \boldsymbol{c}\boldsymbol{k}^T$$

and

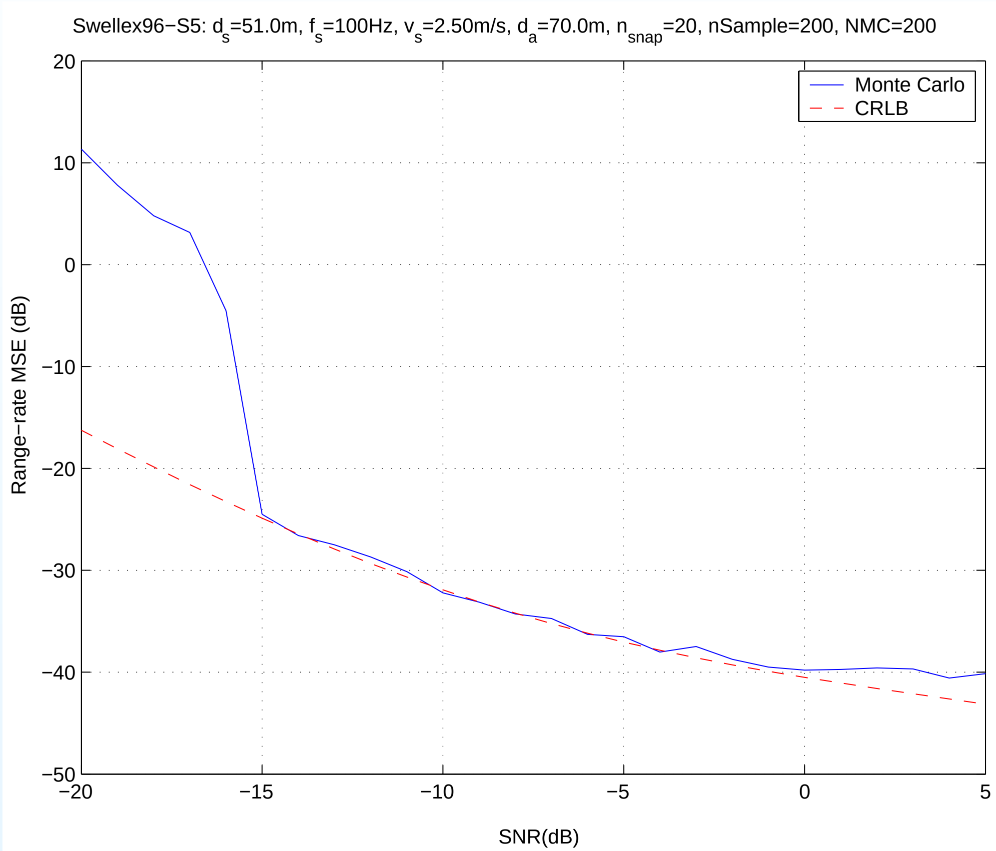
$$\boldsymbol{H} = \boldsymbol{G} \otimes \boldsymbol{h}$$

where vector $\boldsymbol{h}$ is a p (array length) by 1 vector with all its elements being 1.
Thus

$$\begin{aligned} \frac{\partial \boldsymbol{R}}{\partial v_r} &= P_s\left(\frac{\partial \boldsymbol{D}}{\partial v_r}\boldsymbol{S}\boldsymbol{D}^H + \boldsymbol{D}\boldsymbol{S}\frac{\partial \boldsymbol{D}^H}{\partial v_r}\right) \\ &= P_s((\boldsymbol{H} \odot \boldsymbol{D})\boldsymbol{S}\boldsymbol{D}^H + \boldsymbol{D}\boldsymbol{S}(\boldsymbol{H} \odot \boldsymbol{D})^H) \end{aligned} \tag{11}$$

where $\odot$ is element-by-element Hadamard product.

Comparison of IMRE with CRB



Between SNR $-15 \sim 0$dB, the mean square error of IMRE achieves CRLB. The threshold happens at -16dB.

High SNR Behaviour Analysis

Express the dominant eigenvector of the sample covariance matrix as:

$$\hat{b}_1 = b_1 + \eta \tag{12}$$

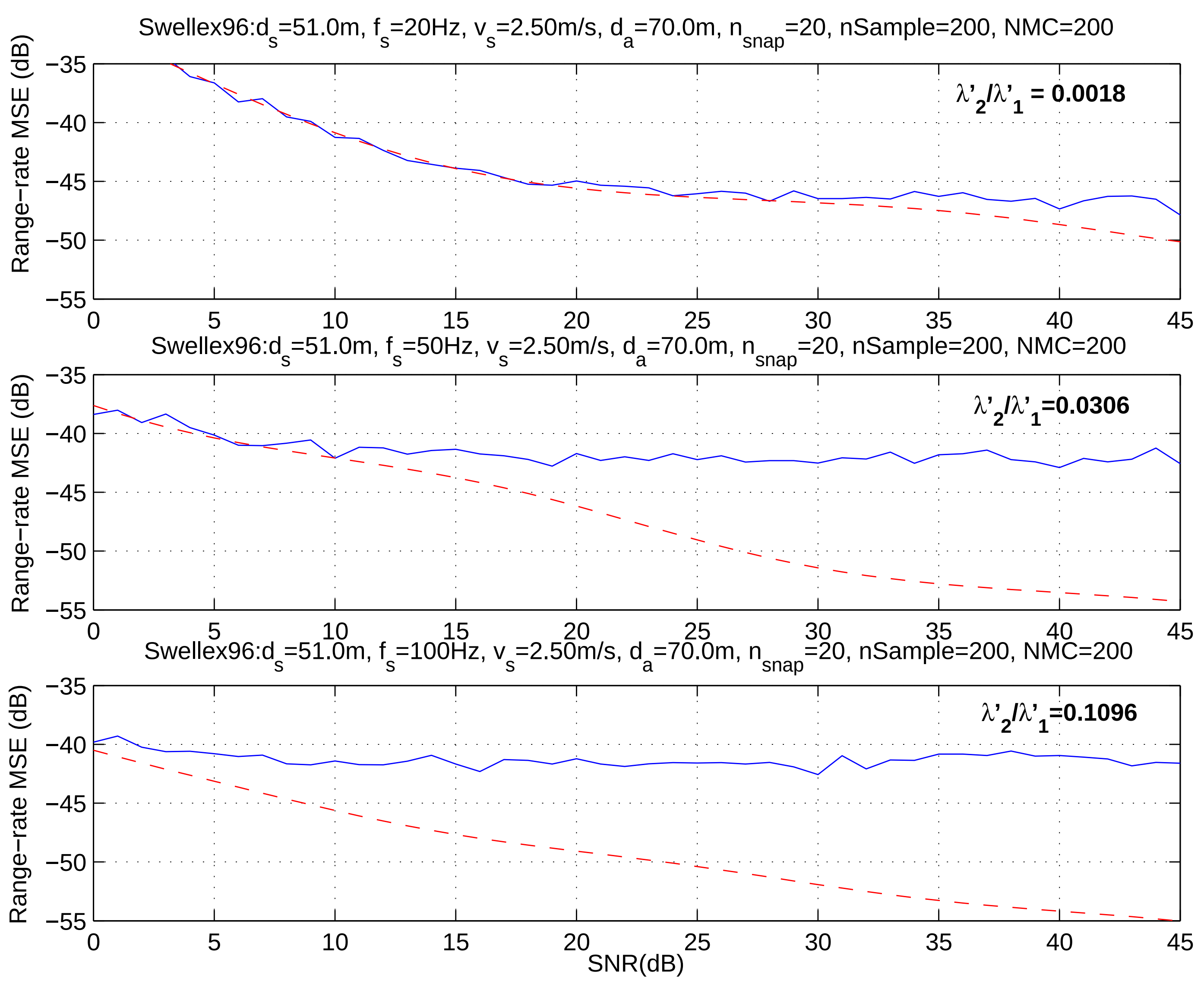
The estimator error vector η has the following asymptotic correlation function:

$$E[\eta\eta^H] = \frac{\lambda_1}{N_s} \sum_{k=2}^{N} \frac{\lambda_k}{(\lambda_1' - \lambda_k')^2} b_k b_k^H$$

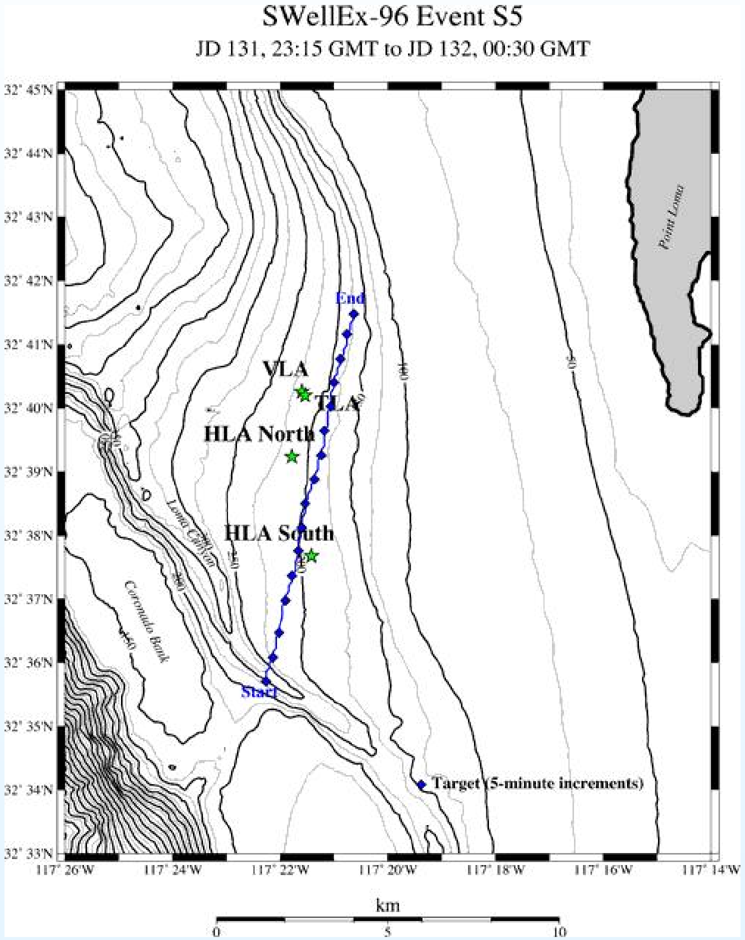
where λ's are the eigenvalues of $P_s DSD^H$. Looking at $\frac{\lambda_1 \lambda_k}{(\lambda_1' - \lambda_k')^2}$, we have:

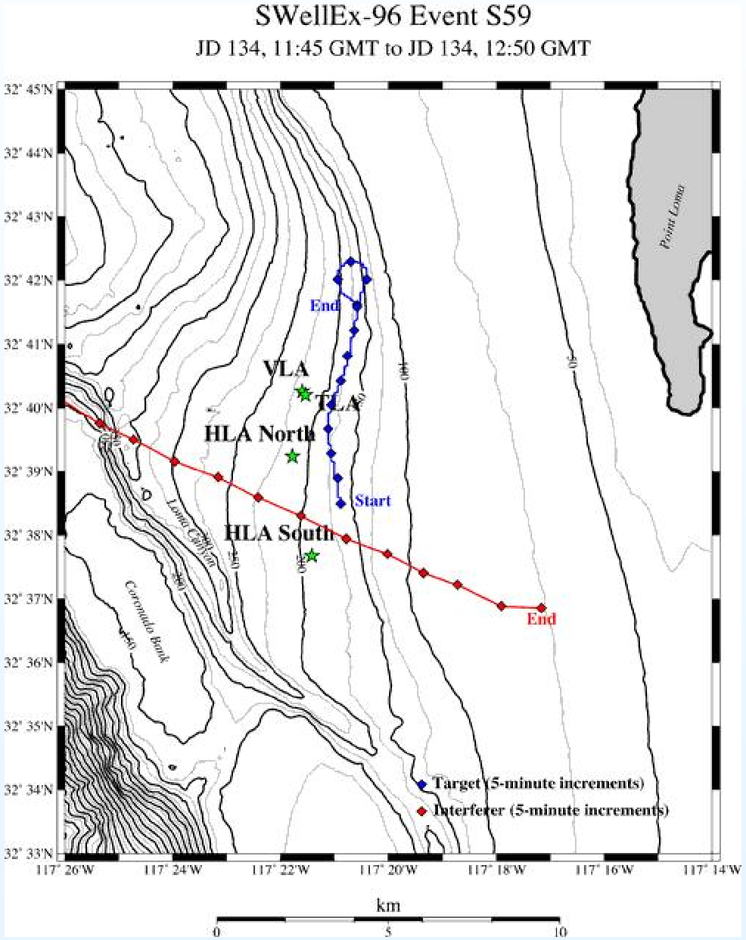
- When the noise variance σ_n^2 is far smaller than the second eigenvalue λ_2' of $P_s DSD^H$, the performance of the algorithm will not improve with higher SNR, i.e., not stick to the CRB.
- The smallest MSE achieveable is determined by $\frac{\lambda_2'}{\lambda_1'}$.

High SNR Performance at Different Frequencies



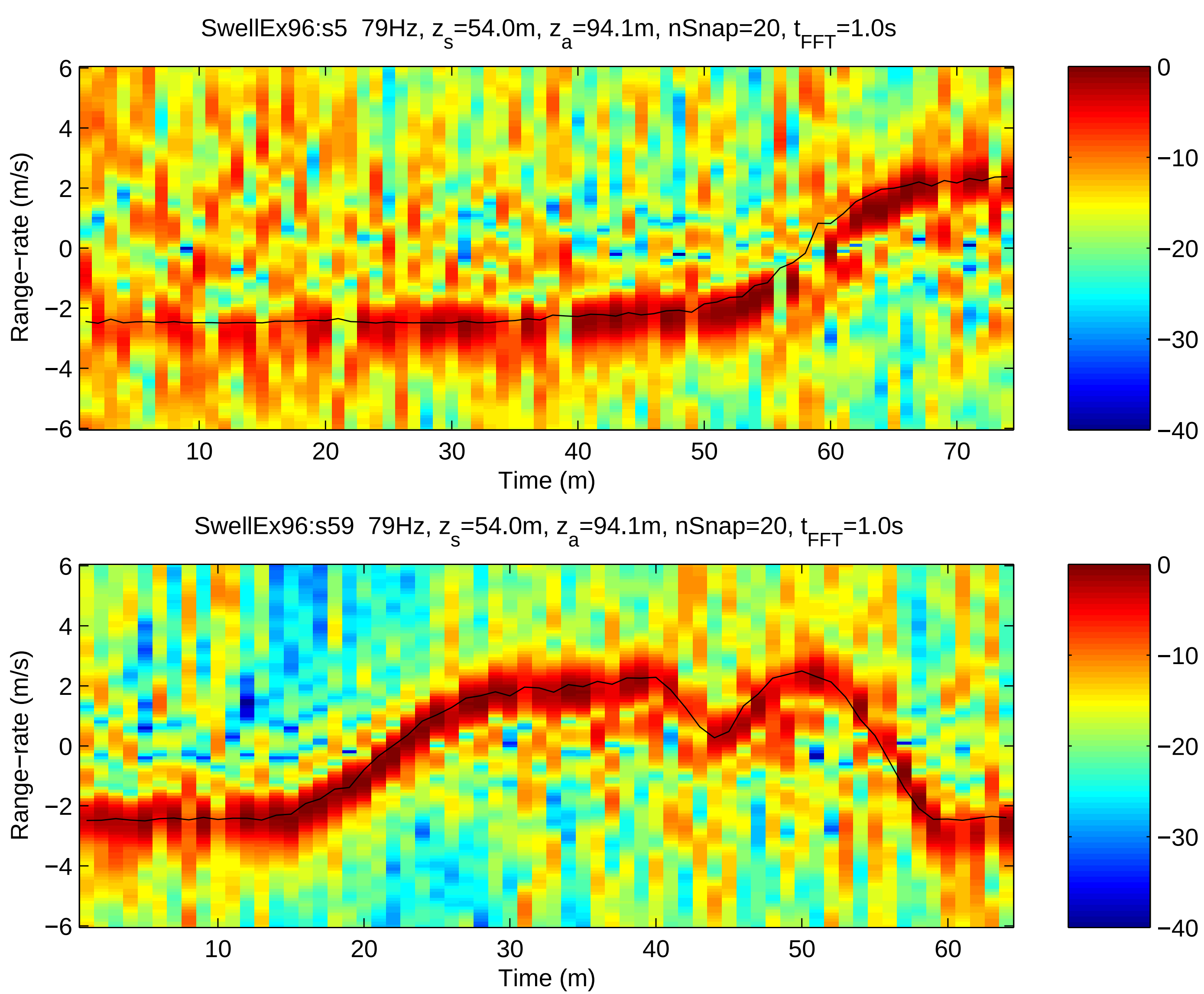
Source Tracks in the SWellEx-96 Experiment





Source: UCSD Marine Physical Laboratory SWellEx-96 Website.

Range-rate Track of S5 and S59 Events in SWellEx96



Summary and Comments

- A narrowband range-rate estimator, IMRE, is proposed. The method exploits existing environmental information to obtain a more robust and accurate estimation of range-rate.
- The aliasing and robustness of IMRE are discussed.
- The performance of IMRE is compared favorably with Cramer-Rao Lower Bound. The high SNR behavior is analysized.
- Application of IMRE to the SWellEx-96 data set illustrates the practical usage of the algorithm. Note in application of IMRE to real data, the demodulation of FFT will introduce a bias, which will be addressed in future research.

Thank You!