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Electric-field-coupled resonators for negative permittivity metamaterials

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A lithographically patterned inductive-capacitive resonator is described that has a strong electric response. This resonator can be used to construct metamaterials with desired positive or negative permittivity. Such materials provide an alternative to wire media, and have the benefit of not requiring continuous current paths between unit cells. A planar medium composed of these resonators was simulated, fabricated, and measured in the microwave frequency range. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2166681]

The field of metamaterials and, in particular, negative index media, continues to grow in popularity.[1] Currently, the majority of implementations of negative index media are based on the split-ring resonators[2] (SRRs) and wire media,[3] originally pioneered by Pendry *et al.* As researchers have attempted to use these media in more diverse applications, a number of fundamental and not always desirable issues have arisen and been investigated.

One obvious property of wire media is that, unlike SRR media, it requires continuous electrical connections between unit cells. Breaks or terminations of the wires cause significant changes in the properties from the bulk values. When attempting to construct finite and contoured objects, such as a curved lens, one may be faced with both short and terminated wires in the critical areas of the device. Some researchers have found that finite wires of sufficient length will act like the bulk, but this length scale is much longer than a single unit cell.[4] (Another approach to obtaining electric response applies Babinet's principle to SRRs.[5] These resonators appear to also require intercell electrical continuity.)

For three-dimensional implementations of wire media, it has also been found that the three orthogonal arrays or wires must be electrically connected to each other.[6] While these connections are not a problem conceptually, fabrication requires cross-plane interconnectivity. This may be a barrier to mass production and is very tedious to implement even for single test samples.

Finally, recent research has pointed to the desirability of moving much deeper into the effective media regime than is commonly practiced. Obtaining a unit cell dimension a that is much smaller than the operating wavelength in the media λ has been demonstrated for "swiss rolls" as well as SRRs, but not for wire media. To achieve this with wire media would require extremely thin (and lossy) wires or some other means of increasing inductance.[3]

In this letter, we introduce an inductive-capacitive (LC) resonator with a fundamental mode that couples strongly to a uniform electric field, and negligibly to a uniform magnetic field. We will refer to such a resonator as an electric-LC (ELC) resonator. In this nomenclature, an SRR with two or four balanced splits would be a magnetic-LC resonator, since it couples only to magnetic field, and a single-split SRR would be an electromagnetic LC (EMLC) resonator, since it couples to both electric and magnetic fields. (Note that Pendry's original SRR with two coaxial, single-split rings is not balanced, and is thus an EMLC resonator.) The presented ELC resonator is suitable for implementing media with desired positive or negative permittivity in one to three dimensions, and does not require intercell or interplane electrical connectivity. Additionally, due to the fact that it is a local and self-contained oscillator, it should be more robust with regard to maintaining its bulk properties close to a boundary or interface.

The ELC resonator can be described qualitatively in terms of its equivalent circuit [Fig. 1(d)]. A capacitor-like structure couples to the electric field and is connected in parallel, to two loops, which provide inductance to the circuit. This allows the electric field to drive the LC resonance providing both positive and negative electric polarization at different frequencies along the resonance curve, where the phase of the resonator response is in phase and out of phase with the driving field, respectively. The two inductive loops are connected in parallel, so that the resonant frequency of the circuit model is $\omega_0=\sqrt{2/LC}$.

The ELC resonator structure possesses a fairly high level of symmetry, belonging to the C_{2h} group. It has been shown that structures belonging to this group do not possess magnetoelectric coupling.[7] The analysis required to use media composed of these resonators in devices is thus greatly simplified. Single-split SRR media, which do not possess this symmetry, have magnetoelectric coupling and cross-polarizing behavior.[8] The symmetry of the structure also dic-

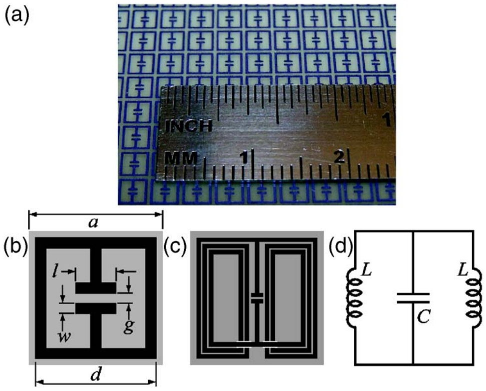

FIG. 1. (Color online) (a) The fabricated sample. (b) The geometric design: $a=3.333$ mm, $d=3$ mm, $l=1$ mm, $w=g=0.25$ mm. The copper thickness was 0.017 mm and the FR4 substrate thickness was 0.203 mm. (c) A proposed multilayer design with lower resonant frequency. (d) An equivalent circuit.

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tates its coupling to a magnetic field. Because the two inductive loops are equivalent but oppositely wound with respect to the capacitor, they act as a magnetic field gradiometer that does not couple to a uniform magnetic field; a uniform magnetic field cannot drive the fundamental LC resonance.

Like an SRR, the ELC resonator has both inductive and capacitive elements, only one type of which is involved in coupling the fundamental mode to the desired external field. For the SRR, the inductive element (the ring) couples to the magnetic field, and the capacitive element (the split) does not. This gives useful independent control over coupling strength and resonant frequency. Analogously, in the ELC resonator, the capacitive element couples strongly to the electric field and the inductive loops do not. Wire media do not offer this independent control.

Our simulations have shown that increasing the capacitance of the coupling element—by decreasing the gap g, or increasing the length l—decreases the coupling strength. Weak coupling has the undesirable effect of narrowing the resonance and increasing the dispersion at the operating frequency. Thus, to achieve better effective media with a lower resonant frequency, increasing the loop inductance is preferred over increasing the coupling element capacitance. A proposed structure for doing this is shown in Fig. 1(c).

We have designed, built, and measured an ELC resonator medium for use in the microwave frequency range. We restricted ourselves to inexpensive (but lossy) copper clad FR4 material, in a single-layer design, and with a very modest lithographic line width of 250 μm. With these constraints we were able to achieve a permittivity of $-1+0.2i$ at an operating frequency of 15.7 GHz and an effective media ratio $\lambda/a=5.7$. The design parameters are given in Fig. 1(b).

We simulated our ELC resonator medium using a finite difference time domain (FDTD) Maxwell's equations solver, Microwave Studio by CST. Our simulations used electric and magnetic boundary conditions on the transverse boundaries and two open ports to simulate the S-parameter response of a single infinite layer medium to a normally incident plane wave. Solving the inverse problem we extracted the material properties (ε and μ) from the S-parameters.[9] We ran these simulations for various orientations of the ELC resonators relative to the incident field, and in all cases we assumed a cubic unit cell for de-embedding and extraction, even though for some orientations this specification was arbitrary.

In Fig. 2 we show the real part of the material properties for two specific orientations. The darker colored curves show the properties for the orientation shown in the top left of the figure, where the electric field is normal to the "capacitor plates" and thus couples to the capacitive element. Here we see a strong electric resonance, with the real part of the permittivity ranging from nearly 30 down to -7. As expected, with our moderate λ/a, we see a "magnetic antiresonance" associated with the electric resonance. This artifact results from attempting to extract spatially local material properties in a frequency range where the medium is highly spatially dispersive.[10] We also see a higher-order electric resonance which we did not investigate.

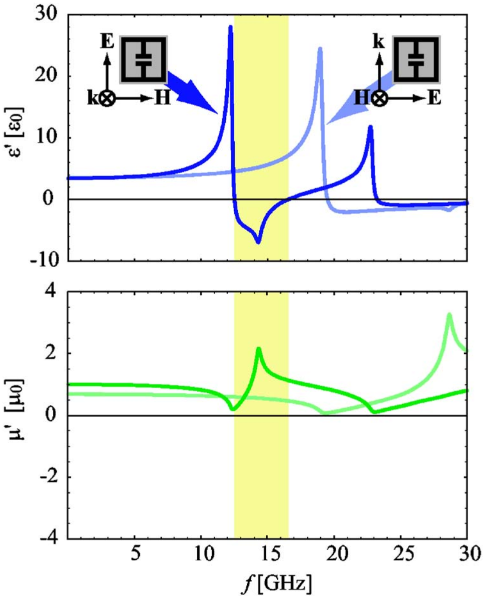

FIG. 2. (Color online) The real part of the permittivity (blue) and permeability (green) of a single layer of media extracted from FDTD simulation of the S-parameters. The darker color shows the properties for the configuration (left inset) for which the electric field is coupled to capacitive element. The frequency range where the real part of the permittivity is negative is highlighted in yellow. The lighter color shows the lack of magnetic response when the applied magnetic field threads the loops (right inset configuration).

The lighter colored curves in Fig. 2, are for the orientation where the magnetic field is normal to the plane of the substrate and threads the loops of the ELC resonators, and the electric field is parallel to the "capacitor plates" and thus does not couple to the capacitive element. As expected, there is no significant resonant magnetic response, we only see the "magnetic antiresonance" associated with an electric resonance.

We performed S-parameter measurements on a single layer of the medium, as shown in Fig. 1(a). Unlike SRR media, we can excite the desired, fundamental mode with normally incident radiation for in-plane structures, since this mode couples to in-plane fields. We used a focused beam system comprised of a pair of Ku-band lens horn antennas (Rozendal and Associates) connected to a network analyzer (Agilent N5230A). Phase referencing and normalization were performed in transmission by removing the sample from the signal path, and in reflection, by replacing the sample with an aluminum plate. Similar measurements have been described elsewhere.[11]

The phase referencing required some further postprocessing adjustments, or de-embedding, to match the standard conventions for S-parameters. For transmission, the phase change associated with traversing one lattice parameter in free space had to be added to S_{21}. For reflection, the relevant length for adjusting the phase of S_{11}, is twice the distance between the front of the aluminum plate and the front of the sample unit cell. Due to imperfect flatness of the sample and lack of rigidity of the sample mount, this latter value was difficult to estimate, and was left as an adjustable parameter in fitting the simulation and experimental S-parameters to each other. However, the value of the parameter obtained from the fit was reasonable.

The agreement between simulation and experimental S-parameters and the extracted material properties is excellent (Fig. 3). One real and one complex adjustable fitting parameter were used in fitting the data. The real parameter was the reflection de-embedding length mentioned earlier. The complex parameter was the complex permittivity of the FR4 substrate material. The use of this latter adjustable parameter was necessitated by the variability of the microwave

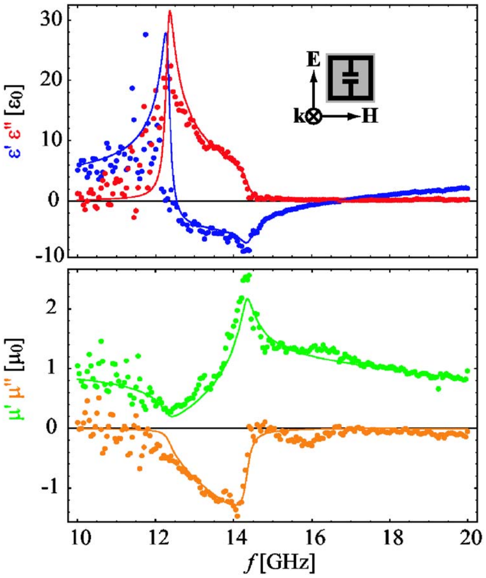

FIG. 3. (Color online) Real and imaginary parts of the permittivity ε and the permeability μ extracted from simulation (lines) and experimental data (points) for a single infinite layer of media with structure oriented as shown.

dielectric properties of commercial FR4. The value obtained from the fit ($\varepsilon = 3.75 + 0.084i$) is consistent with the range of values observed in this material. We note that the experimental data is substantially noisy in the 10 to 12 GHz range. In this frequency range the signal thoughput of the Ku-band horns rolls off significantly, leading to a poor signal-to-noise ratio.

We have demonstrated both by simulation and experiment a viable alternative to wire media, but a number of improvements are possible. The basic concept embodied in this structure could be extended to multidimensional media. Further geometric optimization and use of better substrate materials could lead to lower media loss, and the effective media limit ratio λ/a could be improved by increasing the inductance with multiple windings, as in Fig. 1(c).

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