

Partial focusing of radiation by a slab of indefinite media

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Negative refraction can occur at the interface between vacuum and an indefinite medium—an anisotropic medium for which not all elements of the permittivity and permeability tensors have the same sign. We show experimentally and via simulations that a metamaterial composed of split ring resonators, designed to provide a permeability equal to -1 along the longitudinal axis, will redirect s -polarized electromagnetic waves from a nearby source to a partial focus. The dispersion characteristics of indefinite media prohibit the possibility of true aplanatic points for a planar slab; however, by contouring the surfaces aplanatic points may be realized, as well as other geometrical optical behavior. © 2004 American Institute of Physics. [DOI: 10.1063/1.1690471]

An isotropic material whose permittivity ϵ and permeability μ are both simultaneously negative can be characterized by a negative index-of-refraction.^{1–3} Electromagnetic waves incident on a planar interface between two materials, one of which has positive refractive index and the other a negative refractive index, will undergo *negative refraction*; that is, the refracted wave will emerge on the same side of the surface normal as the incident wave. While a negative index-of-refraction has profound consequences for a broad range of electromagnetic phenomena, much of which was explored in an early paper by Veselago,¹ the impact of negative refraction on geometrical optics is perhaps the most immediately accessible phenomenon in terms of practicality. In 2001, an experiment demonstrating negative refraction was performed on a wedge-shaped sample composed of an artificial medium—or metamaterial, consisting of conducting wires and split ring resonators.⁴ The conclusion, that a metamaterial could be designed to have a negative index of refraction, has since been confirmed in two recent publications.^{5,6}

A particularly interesting consequence of negative refraction, pointed out by Veselago,¹ is that a planar slab with $\epsilon = -1$ and $\mu = -1$ possesses pairs of aplanatic points.⁷ That is, the rays from an electromagnetic source located on one side of the slab will be brought to a focus on the other side of the slab. An experimental confirmation of this focusing effect has recently been achieved within the context of a planar transmission line model.⁸ This refocusing theoretically extends to the near-fields associated with a source,⁹ although we do not discuss near-field refocusing or “perfect lens” effects here. The refocusing property of a negative index slab implies that very large refractive power is possible for only moderate magnitude of index. The focal length for a converging negative index lens with $n = -1$, for example, is identical to that for a positive index lens with $n = +3$, since $f = R/|n-1|$, where R is the radius of curvature of the lens.¹⁰ Because of their large refractive power and nearly matched surface impedance, negative index materials may be ideal candidates for lens applications.

While negative refraction is most easily visualized in an

isotropic material with negative refractive index, negative refraction can also be realized in anisotropic media for which not all elements of the diagonal permittivity and permeability tensors have the same sign.^{11,12} We term such media *indefinite* for brevity. Wave propagation in indefinite media has been previously explored in magnetized plasmas,¹³ and more recently exploited to design a photonic crystal slab that partially refocuses the rays from a nearby electromagnetic source.¹⁴ While a planar slab of indefinite media cannot yield pairs of aplanatic points, the partial refocusing available indicates that lenses composed of indefinite media, which are easier and less costly to fabricate, may be worthwhile pursuing.

In the present work, we focus attention on a particular type of indefinite medium, for which the permeability component (μ_z) along the propagation direction, or z axis, is negative, while all other permeability and permittivity tensor components are positive. To avoid a mathematical description of the refraction properties of indefinite media, we instead make use of the graphical representation of the dispersion relationships shown in Fig. 1. For anisotropic materials, these *isofrequency surfaces* provide a convenient means of understanding the refraction properties between complex media.¹⁵ In our case, the indefinite medium under consider-

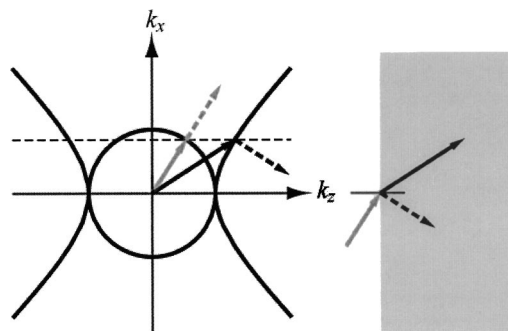


FIG. 1. Isofrequency curves for free space (circle) and indefinite media (hyperbolic sheets). The incident phase propagation direction is indicated by the lighter gray arrow, which is parallel to the incident group velocity (dashed gray arrow). The refracted wave vector and group velocities in the medium are indicated by the solid and dashed black arrows, respectively. Note that the group refraction is negative while the phase refraction is positive.

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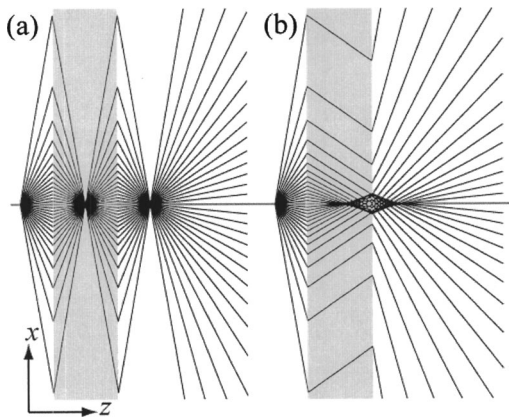


FIG. 2. A ray-tracing diagram showing the manner in which the trajectories of rays emanating from a point source are refocused by (a) a slab with isotropic negative index ($n = -1$), and (b) the indefinite medium slab.

ation is characterized by a hyperbolic isofrequency surface rather than the common elliptical surface that characterizes positive or negative definite anisotropic media.^{11,12}

For an isotropic negative index material, the group (or energy) and phase velocities are antiparallel. The directions of group and phase velocities in an anisotropic medium, however, are not fixed, but rather vary with the direction of propagation in the material with respect to the principal axes. The condition that the wave vector parallel to the interface be conserved implies two possible solutions in the medium, which can be distinguished by requiring the group velocity point away from the interface. Figure 1 indicates the correct choice of solution. As can be seen from the figure, for the medium used here the phase velocity will undergo positive refraction, but the group velocity will undergo negative refraction. Clearly, for shallow angles of incidence, the indefinite planar slab will provide some degree of refocusing in the same manner as an isotropic negative index material. The ray-tracing diagram of Fig. 2 compares the trajectories of rays from a point source passing through an isotropic ($\epsilon = \mu = -1$) and an indefinite medium slab.

To confirm the ray-tracing picture of Fig. 2, a full-wave simulation of the refocusing was carried out using HFSS (Ansoft), a finite-element based frequency-domain electromagnetic solver. Materials with negative isotropic permittivity and/or permeability can be simulated using HFSS,^{10,16} as well as anisotropic materials with either positive or negative values for the permittivity and permeability tensor elements. The geometry simulated was chosen to match the experimental geometry. A current line source, positioned 2 cm away from one face of a planar slab of indefinite material, was used to launch a cylindrical *s*-polarized wave (electric field perpendicular to the plane of propagation). The indefinite medium slab was 4 cm in width, 16 cm in length, and was modeled with an indefinite permeability tensor for which $\mu_z = -1$ (the *z* axis is in the propagation direction). The permittivity was isotropic and equal to unity, and all other principal elements of the permeability tensor were also unity. Perfect electric conductors spaced 1 cm apart terminated the computational domain parallel to the plane of propagation, essentially forming a two-dimensional waveguide. Radiation boundary conditions terminated all other surfaces.

The simulation results are presented in Fig. 3 (left),

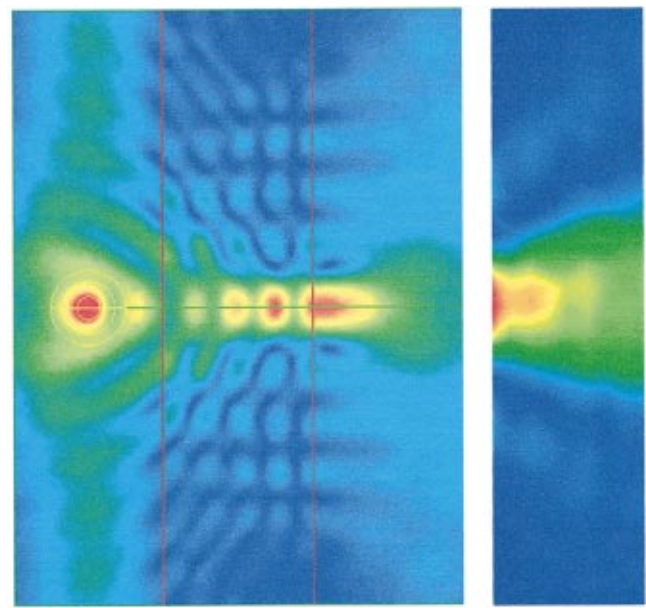


FIG. 3. (Color) A spatial map of the magnitude of the electric field. (Left panel) The simulated slab, indicated by the solid lines, has $\epsilon = 1$ and a diagonal permeability tensor for which the longitudinal component $\mu_z = -1$ and $\mu_x = \mu_y = +1$. The slab is 16 cm long, with a line source placed 2 cm from the. The slab thickness was 4 cm. (Right panel) Experimentally obtained spatial map of the electric field in the 4 cm region to the right of the slab.

which shows a spatial map of the magnitude of the computed electric field. The source is on the left-hand side of the figure. Unlike the matched isotropic $n = -1$ slab, there is a substantial mismatch between the indefinite medium slab and free space, so that the reflected field is considerable. Nevertheless, the map shows an intense partial focus on the far side of the slab relative to the line source. Similar results have also been found recently in a numerical study of focusing by indefinite media slabs.¹⁷

To experimentally confirm the focusing effect, we utilized a metamaterial composed of split ring resonators (SRRs). The SRRs are identical in all respects to those utilized in Ref. 4, including substrate material, physical dimensions, and overall size. The unit cell of the SRR lattice is 3.33 mm in the vertical direction and 5.0 mm in the transverse (propagation) directions.

The experiments were performed over a frequency range from 9 to 14 GHz, in a two-dimensional scattering chamber consisting of two aluminum plates (3 ft \times 3 ft \times 1/4 in.) separated by a distance of 1 cm. Toothed absorber was arranged in a circular pattern around the sample area to minimize reflection from the plate edges back into the scattering region. Strips of SRRs three unit cells high (1 cm) and 32 unit cells long (16 cm) were cut and assembled into an anisotropic sample using spacer layers cut from the same blank circuit board material. The resulting metamaterial slab was 4 cm in length (the propagation direction) and 16 cm in width (transverse to propagation), with the axes of the rings oriented along the propagation direction.

The SRR lattice used to construct the indefinite medium slab has been previously studied and shown to have a frequency region where the effective permeability is negative.¹⁸ A band structure calculation (not shown) predicts a band gap of ~ 0.7 GHz, starting from just under 11 GHz. The gap

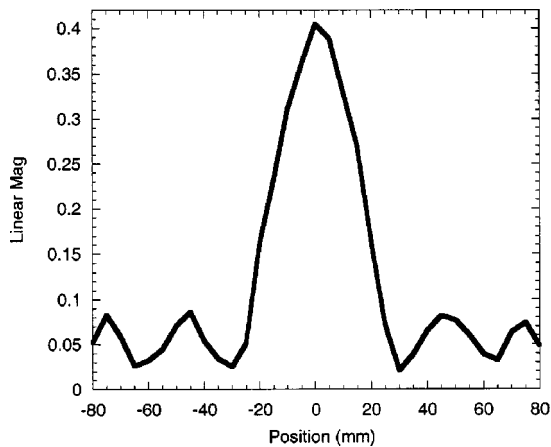


FIG. 4. A cross section of the field magnitude taken perpendicular to the direction of propagation (parallel to the slab).

occurs where the permeability is negative, with μ tending to zero at the upper band edge and tending to infinity at the lower band edge where the SRR is resonant. Transmission experiments on the SRR structure, with the axes of the rings perpendicular to the direction of propagation (but still in the plane of propagation), confirm the metamaterial is opaque over roughly the range of frequencies corresponding to the gap in the band structure. The simulations and transmission experiments on the SRR lattice indicate the component of the permeability perpendicular to the SRR axes should be near the value of -1 near 11.3 GHz.

To perform an experimental mapping of the electric field near the focus of the slab, a slot (80 mm \times 2.1 mm) was machined into the upper aluminum plate of the scattering chamber perpendicular to the plane of propagation, and a probe (PRD 250A) inserted into this slot was used to detect the local electric field. An Agilent 8722ES vector network analyzer was used to excite a coaxial source antenna inserted into the center of the lower plate, and detect the power (amplified by an HP8449B 30dB gain amplifier) from the probe. The probe was moved along the length of the slot, a total distance of 16 cm, in steps of 0.5 cm. After each linear scan, the upper plate was moved a distance of 0.5 cm in the propagation direction so that a new region could be scanned. The total distance in the direction of propagation mapped was 4 cm, starting at a distance of 0.5 cm away from the exit face of the slab. Each data point was sampled ten times, as a function of frequency from 9 through 14 GHz, thus allowing spatial field maps of 4 cm \times 16 cm containing 288 data points to be produced at any frequency within the range.

The result of the spatial mapping is shown in Fig. 3 (right), with a cut across the fields shown in Fig. 4. The field

map shown was taken at a frequency of 11.3 GHz, very near where the effective μ_z is expected to be near -1 . A scan through the spatial maps at all other frequencies did not reveal any other frequency where a similar local field enhancement occurred. The range where the pseudofocus was detectable covered a frequency span of less than two tenths of a GHz. As can be seen from the figure, the enhancement was well-resolved and consistent with the simulation results.

Our results confirm another of the predicted behavior for negative refractive media:¹ focusing by a planar structure. Because the material used is formed from indefinite rather than isotropic media, the focusing is imperfect and aplanatic points are not achieved. However, substantial field enhancement can readily be observed, as is consistent with both ray tracing arguments as well as full wave numerical simulations. Because the partial focusing implies nontrivial refractive power, these results indicate the potential for indefinite materials as lens structures. Aplanatic points, for example, should be achievable with the present indefinite medium by contouring the front and back surfaces of the slab. While simulations based on continuous material suggest this to be the case, the implementation in an actual metamaterial is complicated by the finite unit cell size of the structure.

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