Limitations on subdiffraction imaging with a negative refractive index slab

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A planar slab of material, for which both the permittivity and permeability have the values of −1, can bring not only the propagating fields associated with a source to a focus, but can also refocus the nonpropagating near fields, thereby achieving resolution beyond the diffraction limit. We study the sensitivity of this subwavelength focus to the slab material properties and periodicity, and note the connection to slab surface plasmon modes. We conclude that significant subwavelength resolution is achievable with a single negative index slab, but only over a restrictive range of parameters. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1554779]

A wave incident on the interface between two materials whose indices of refraction are of opposite sign will undergo *negative refraction*. This recently appreciated propagation phenomenon has been predicted to lead to numerous interesting optical phenomena, including a superfocusing effect. Veselago showed theoretically that a planar slab with index of $n=-1$ could focus the rays from a nearby electromagnetic source to an image on the opposite side of the slab.[1] In a more recent analysis,[2] it was shown that in addition to the far-field components associated with a source being brought to a focus by the slab, the nonpropagating near-field components could also be recovered in the image. It was, therefore, proposed that the image created by a planar slab could, in principle, contain *all* of the information associated with the source object, thereby achieving resolution well beyond that of the diffraction limit. For this reason, the slab was described as a *perfect lens*. We maintain this description here, referring specifically to a planar slab of continuous material with $\mu=-1$ and $\varepsilon=-1$ (no losses) as a perfect lens.

The resolution enhancement associated with the perfect lens was a surprising result, stimulated by the experimental demonstration[3] of a *left-handed* material at microwave frequencies, for which $\varepsilon<0$ and $\mu<0$. However, far from being a continuous material, the measured sample was comprised of two interlaced periodic arrays of copper elements, one array being composed of split ring resonators,[4] and the other, wires.[5]

The values of the electromagnetic parameters and the spatial periodicity render the experimental sample distinct from the idealized perfect lens. The question then arises as to whether or not focusing beyond the diffraction limit can actually be observed using any practically obtained or fabricated material, or even can be simulated using standard numerical methods (e.g., finite difference or finite element) which inevitably approximate the ideal situation. We explore the inherent limitations associated with realizable materials, and the expected impact on the focusing properties of a slab.

The planar geometry we consider here allows a straightforward analysis to be implemented, as was used in Ref. 2. The fields from an arbitrary electromagnetic source are expanded in a Fourier series over homogeneous and inhomogeneous plane waves. The influence of the slab on each plane wave component can easily be determined by a standard transfer matrix technique. Restricting the field variation to one transverse direction, with the electric field having *S* polarization, we find the field expansion

$$\mathbf{E}(x,z,t)=\sum_{k_x} E(k_x)\exp(ik_z z+ik_x x-i\omega t)\hat{y}, \tag{1}$$

where k_z and k_x are the components of the wave vector normal to and parallel to the slab, respectively. Outside the slab, the wave equation leads to the usual dispersion relation relating the frequency ω and the components of the wave vector, or $k_z=\sqrt{\omega^2/c^2-k_x^2}$. Those modes for which $|k_x|<\omega/c$ are propagating, while those for which $|k_x|>\omega/c$ decay evanescently along the propagation direction (z). As was shown in Ref. 2, *it is the latter inhomogeneous modes that are responsible for image resolution beyond the diffraction limit*. The ability to recover these components in an image is what distinguishes a negative index structure from all other positive index materials.

For each plane wave component, we can determine a transfer function, defined as the ratio of the field at the image plane to that at the object plane. For an *S*-polarized plane wave incident on a slab of thickness d and arbitrary values of ε and μ, the transfer function has the form

$$\tau_S=e^{ik_z d}\left\{\begin{array}{c} e^{q_z d}\left[\frac{1}{2}+\frac{1}{4}\left(\frac{\mu k_z}{q_z}+\frac{q_z}{\mu k_z}\right)\right] \\ +e^{-q_z d}\left[\frac{1}{2}-\frac{1}{4}\left(\frac{\mu k_z}{q_z}+\frac{q_z}{\mu k_z}\right)\right]\end{array}\right\}^{-1}, \tag{2}$$

where we have now defined $q_z=\sqrt{k_x^2-\varepsilon\mu\omega^2/c^2}$, and redefined $k_z=\sqrt{k_x^2-\omega^2/c^2}$. The expression for *P*-polarized waves is similar to Eq. (2), with the explicitly appearing μ replaced by ε. We apply all of our arguments to the *S*-polarization terms in this letter, as the results for *P* polarization follow trivially.

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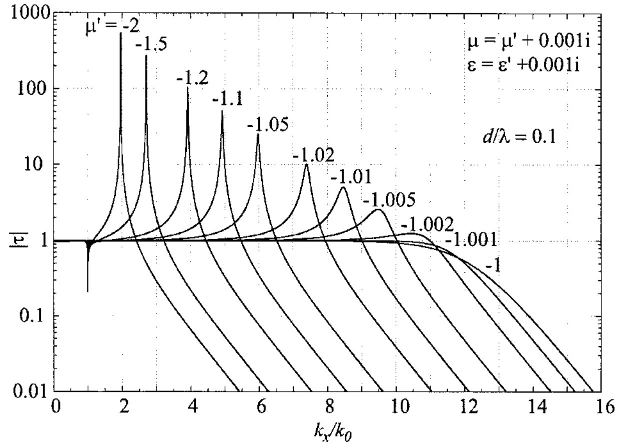

FIG. 1. Transfer function for a left-handed slab. For the perfect lens, the transfer function would be unity for all k_x. However, the deviation of either the real or imaginary part of μ limits the range of k_x, so that the slab acts as a low-pass filter. The losses, inherent to left-handed materials, also remove the singularities that appear in the transfer function.

When μ is positive, the first term in brackets will dominate the behavior of the transmitted wave for sufficiently large d and $|k_x|>\omega/c$, with the evanescent components decaying exponentially through the slab. When both ε and μ are equal to -1, however, the normally dominant solution vanishes, and $\tau_s=1$ for every component from the source field—homogeneous or inhomogeneous, thus, exactly reproducing the source field in the image plane. But the balance is delicate; any deviation from the perfect lens condition, however small, will result in an imperfect image that degrades exponentially with slab thickness d, until the usual diffraction limit is reached. This sensitivity has been noted by other researchers.[6,7]

The effect on the image of deviations from the perfect lens condition for a given slab thickness can be estimated by determining those values of parameters which cause the two terms in Eq. (2) in brackets to be roughly equivalent. For small deviations from the perfect lens condition, we can find an approximate expression for the resolution of the slab. For $\varepsilon=-1$ and $\mu=-1+\delta\mu$, the two terms in the denominator of Eq. (2) are of approximately the same magnitude when $2k_zd=-\ln|(\delta\mu/2)^2|$. Assuming that this limit occurs when the value of k_x is large, we have $|q_z|\sim|k_z|\sim|k_x|$, and we can replace q_z with k_x. At the maximum k_x, the minimum resolvable feature will be on the scale of $\lambda_{\min}=2\pi/k_x$. We, thus, find the resolution enhancement, $R=k_x/k_0$, of the lens as a function of small deviations in the permeability is

$$R\equiv\frac{\lambda}{\lambda_{\min}}=-\frac{1}{2\pi}\ln\left|\frac{\delta\mu}{2}\right|\frac{\lambda}{d}. \tag{3}$$

The validity of this approximate expression can be seen by comparison with Fig. 1. For example, for a deviation of $\delta\mu=0.005$ and $\lambda/d=10$, the numerically computed transfer function in Fig. 1 shows that the range of k_x values near unity is approximately up to $k_x\sim10k_0$; Eq. (3) predicts $R\sim9.5$.

The dependence of the resolution enhancement R on the deviation from the perfect lens condition is critical. The ratio λ/d (wavelength to slab length) dominates the resolution, the logarithm term being a relatively weakly varying function. For example, if $\lambda/d=1.5$, we find that to achieve an R of 10, $\delta\mu$ must be no greater than $\sim6\times10^{-19}$! However, for $\lambda/d=10$, $\delta\mu$ can vary by as much as ~0.002 to achieve the same resolution enhancement. Deviations in either the real or the imaginary part of the permeability will result in the same resolution. The effect of varying ϵ (for S polarization) is much smaller than that associated with μ, and we do not discuss the effects of this variation further.

The transfer function has poles that occur when:

$$\tanh\left(\frac{q_zd}{2}\right)=-\frac{\mu k_z}{q_z}\quad\text{or}\quad\coth\left(\frac{q_zd}{2}\right)=-\frac{\mu k_z}{q_z}. \tag{4}$$

Equation (4) corresponds to the dispersion relations for slab plasmon polaritons.[8,9] For certain values of k_x, these dispersion relations are satisfied, and resonant surface modes can exist on the slab. The direct excitation of these surface modes for imaging applications is undesirable, as the corresponding k_x components will be disproportionately represented in the image. Yet, the existence of these resonances is essential, as the recovery of the evanescent modes can be seen as the result of driving the surface plasmon far off resonance. The appearance of resonances can be seen in Fig. (1), which shows the transfer function as a function of k_x.

While losses severely limit the obtainable resolution of the focus for a negative index slab, periodicity, as exists for example in structured *metamaterials*[3] or in the sampling associated with finite-difference simulations, also imposes a significant resolution limitation. The effect of periodicity is minimal for propagating plane waves whose wavelength is much larger than the repeated unit-cell size; however, periodicity has a significant effect on the recovery of the nonpropagating components having large transverse wave number.

We can introduce periodicity into this analysis in an approximate manner. The wave equation in a medium, assuming that **E** is polarized in the y direction, is

$$\frac{\partial^2E_y}{\partial x^2}+\frac{\omega^2}{c^2}n^2(x)E_y=q_z^2E_y. \tag{5}$$

We have assumed that the periodic variation in the index is only in the transverse (x) direction. Under these assumptions, $n^2(x)=n^2(x+a)$, where a is the repeat distance.

Due to the periodicity in $n^2(x)$, Eq. (5) in Fourier space will involve sums over reciprocal lattice vectors $g_n=2n\pi/a$. However, to obtain a rough estimate of the limitation that periodicity imposes on the resolution enhancement, we solve Eq. (5) for an index having the form

$$n^2(x)=1+2\Delta\sin\frac{2\pi x}{a}. \tag{6}$$

Using this form in Eq. (5) and assuming the periodic modulation is sufficiently weak that only two bands need be considered, we find the modified dispersion relation by evaluating the determinant

$$\begin{vmatrix} k_0^2-k_x^2-q_z^2 & k_0^2\Delta^2 \\ k_0^2\Delta^2 & k_0^2-(k_x-g)^2-q_z^2 \end{vmatrix}=0, \tag{7}$$

where $g=2\pi/a$ and $k_0=\omega/c$. When $\Delta\ll1$, the bands only weakly couple, and we ignore the effects of higher mode

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excitation to find an expression for q_z. When the two terms in the denominator of Eq. (2) are roughly equal, we then have the condition

$$q_z d = -\ln\left|\frac{1}{2}\frac{k_0^2\Delta^2}{k_z^2}\right| + \sinh^{-1}\left|\frac{g^2-k_x^2}{2k_0^2\Delta^2}\right|. \tag{8}$$

In the limit $k_0\Delta/g \ll 1$, the resolution enhancement is approximately

$$R \equiv \frac{\lambda}{\lambda_{\min}} = \frac{1}{2\pi}\ln\left(\frac{\lambda^2}{a^2\Delta^4}\right)\frac{\lambda}{d}. \tag{9}$$

A similar resolution enhancement limit to that obtained by a variation in material parameter is thus imposed by the introduction of periodic modulation. Similar to Eq. (6), the effects of periodicity enter logarithmically, and again lead to a critical dependence. As an example, an R of 10 for a slab with $\lambda/d=10$ and $\Delta\sim 1$ requires a periodicity of $\sim\lambda/20$. While admittedly of limited quantitative use, Eq. (9) shows that the inherent periodicity in metamaterials will impose a limitation on the resolution of the lens. This same limitation will result also from numerical methods that model a continuous material by evaluating the fields at a finite number of sampling points periodically spaced. This limitation has undoubtedly complicated numerical attempts to observe the superfocusing effect.[10] A more recent numerical study has concluded that, within the parameter range determined from Eq. (6) and with a fine enough discretization grid, enhanced resolution can indeed be observed in finite-difference calculations.[11]

It should be noted that we have applied a definition for resolution in this work that may not be appropriate for all applications. For instance, calculations (not shown) suggest that the excitation of surface slab plasmons leads to a broad background in the image plane, with the subwavelength features superposed. However, for certain applications, the subwavelength information obtained from such an image may be of value.

Our conclusion from this analysis indicates that the perfect lens effect exists for a fairly restricted region of parameter space. Yet, demanding as these specifications are, achieving subwavelength resolution is possible with current technologies. Negative refractive indices have been demonstrated in structured metamaterials, and such materials can be engineered to have tunable material parameters so as to achieve the optimal conditions. Losses can be minimized in structures utilizing superconducting or active elements.

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