Subwavelength diffractive acoustics and wavefront manipulation with a reflective acoustic metasurface

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(Received 31 August 2016; accepted 31 October 2016; published online 18 November 2016)

Acoustic metasurfaces provide useful wavefront shaping capabilities, such as beam steering, acoustic focusing, and asymmetric transmission, in a compact structure. Most acoustic metasurfaces described in the literature are transmissive devices and focus their performance on steering sound beam of the fundamental diffractive order. In addition, the range of incident angles studied is usually below the critical incidence predicted by generalized Snell’s law of reflection. In this work, we comprehensively analyze the wave interaction with a generic periodic phase-modulating structure in order to predict the behavior of all diffractive orders, especially for cases beyond critical incidence. Under the guidance of the presented analysis, a broadband reflective metasurface is designed based on an expanded library of labyrinthine acoustic metamaterials. Various local and nonlocal wavefront shaping properties are experimentally demonstrated, and enhanced absorption of higher order diffractive waves is experimentally shown for the first time. The proposed methodology provides an accurate approach for predicting practical diffracted wave behaviors and opens a new perspective for the study of acoustic periodic structures. The designed metasurface extends the functionalities of acoustic metasurfaces and paves the way for the design of thin planar reflective structures for broadband acoustic wave manipulation and extraordinary absorption. Published by AIP Publishing.

http://dx.doi.org/10.1063/1.4967738

INTRODUCTION

Recent years have witnessed a great expansion of acoustic metamaterial functionality, among which acoustic metasurfaces have attracted significant attention owing to their compact planar subwavelength structure and unprecedented wave modulation capabilities. Various applications favor acoustic metasurfaces for their design flexibilities and exceptional wavefront shaping capabilities, including acoustic imaging, acoustic sensing, and noise control.

The governing mechanism behind most grating metasurfaces is diffraction as described by the generalized Snell’s law (GSL), which enables novel wavefront manipulation techniques by introducing an abrupt phase variation along a surface. By engineering the phase gradient and the angle of incidence, the exiting wavefronts can be tailored. For some incident angles, a critical condition can occur, in which no diffractive order reflects and the incident wave would be converted to surface wave along the metasurface. Although most functionalities realized by proposed acoustic metasurfaces are based on such dispersive wavefront modulation and multiple diffraction orders are permitted by metasurfaces with effective grating structures, research has been primarily focused on the zero order beam, and studies of wave phenomena at large angles beyond critical have not been established. A comprehensive analysis of subwavelength diffraction by a reflective metasurface and a complete prediction of diffraction patterns regarding multiple orders allowed by a periodic phase-modulating structure, especially beyond the critical incidence, have not been reported so far. A number of transmissive metasurfaces have been demonstrated, however, explorations on reflective metasurfaces are quite limited. Extraordinary reflection, acoustic focusing, and surface wave conversion have been investigated as counterparts to transmissive cases, but a few experimental investigations have been made.

With these challenges in mind, in this work we derive theoretically the modes reflected by an arbitrary subwavelength surface with periodically varying phase distribution, accounting for higher order diffracted beams. Then, we demonstrate both analytically and experimentally a subwavelength reflective acoustic metasurface composed of labyrinthine acoustic metamaterials, whose diffraction pattern is successfully predicted by the proposed theory at all angles of incidence. The designed metasurface exhibits various wavefront shaping properties, including anomalous reflection, negative reflection, and surface wave coupling, while preserving a bandwidth of approximately 10% of the central frequency. Notably, extraordinary absorption is observed in experiments for higher order diffracted waves owing to their retention in the lossy medium and is confirmed by transient simulations. The presented analysis fills in the gap between the GSL and practical observed diffraction patterns. The inclusion of multi-order diffractions in the metasurface regime enlightens the research from new perspectives, especially surface-wave- and enhanced-absorption related applications, and the proposed metasurface may lead to development of novel devices such as broadband acoustic absorbers, acoustic holograms, and acoustic bandpass filters.
THEORETICAL ANALYSIS

In order to understand diffraction patterns permitted by a phase-varying metasurface, we consider a model based on

cells possessing effective medium properties, as depicted in Fig. 1. The model consists of several periods of acoustic metamaterials with rigid boundaries on all sides except the top ones. This structure has two periodicities: one is associated with the width of each cell, with period \( d \), and the other reflects the repetition of the group of cells with array periodicity \( \Gamma \). The refractive index of each unit cell has an increment of \( \Delta n \) with regard to the refractive index of its previous cell, i.e., \( n_i = n_{i-1} + \Delta n \), which can be further simplified to \( n_i = n_0 + i\Delta n \).

The assumed uniform plane wave incident field is

\[
u_{inc}^d(x,y) = e^{-jk_0x \sin \theta_i} e^{jk_0y \cos \theta_i}.
\] (1)

The superscript \( d \) indicates that the incident wave propagates in air, and \( k_0 \) denotes the wave vector of the incident field in air. The field transmitted into the metasurface is given by

\[
u_{inc}^m(x,y) = e^{-jk_0x \sin \theta_i} e^{jk_0n_0y \cos \phi}.
\] (2)

The superscript \( m \) indicates the wave now propagates in the metasurface. Since unit cell dimensions are small compared to the wavelength of interest, we can assume that \( \cos \phi = 1 \). Then, the internal metasurface fields can be simplified to

\[
u_{inc}^m(x,y) = e^{-jk_0x \sin \theta_i} e^{jk_0n_0y}.
\] (3)

Assume that the air-metasurface interface is at \( y = 0 \). Adapting Floquet’s Theorem, the scattered field on the air-metasurface interface can be written as

\[
u_{scat}^d(x,y) = \sum_{n=-\infty}^{\infty} p_n e^{-jk_0x \sin \theta_i} e^{jk_0n_0y} e^{-j\theta_i n_0 \frac{2\pi}{\Gamma}}, \quad n \in \mathbb{Z}.
\] (4)

The superscript \( d \) denotes the width of each cell which is the cell-scale periodicity considered here, \( n \) denotes the diffraction orders permitted by the structure with regard to the periodicity provided by the arrangement of cells, \( p_n \) is the amplitude of the \( n \)th order diffracted waves, and \( k_{\text{scat}} \) is the wave vector of those diffracted waves in the \( y \) direction. The transverse wave vectors of the diffracted waves are given by

\[
k_{\text{scat}} = k_0 \sin \theta_i + \frac{k_0 \Delta n \cdot 2D}{d} - \frac{2\pi n}{d}, \quad n \in \mathbb{Z}.
\] (5)

The phase shift achieved by each cell is \( \Delta \phi^\text{phase} = \frac{k_0 \cdot 2D \cdot \Delta n}{d} \), so \( \Delta \psi = \Delta \phi^\text{phase} = \frac{k_0 \cdot 2D \cdot \Delta n}{d} \), and thus Eq. (5) can be rewritten as

\[
k_{\text{scat}} = k_0 \sin \theta_i + \frac{\Delta \psi}{d} - \frac{2\pi n}{d}, \quad n \in \mathbb{Z}.
\] (6)

Using the dispersion relation for air, the normal component of the wave vectors is given by

\[
k_{\text{scat}} = \left\{ \begin{array}{ll}
\sqrt{k_0^2 - k_{\text{scat},y}^2}, & k_0 \geq k_{\text{scat}}

i \sqrt{k_{\text{scat},y}^2 - k_0^2}, & k_0 < k_{\text{scat}}.
\end{array} \right.
\] (7)

Then, if we further take the periodicity of the metasurface array into consideration, apply Floquet’s Theorem again, the acoustic field scattered at the air-metasurface interface is given by

\[
u_{\text{scat}}^\Gamma(x,y) = \sum_{m=-\infty}^{\infty} p_m e^{-jk_0x \sin \theta_i} e^{j(k_{\text{scat},y} \frac{m\Gamma}{\Delta n} y)}, \quad n \in \mathbb{Z}.
\] (8)

The superscript \( \Gamma \) denotes the periodicity of the metasurface array considered here, \( m \) denotes the diffraction orders generated regarding to the periodicity of the cell array, \( p_m \) represents the amplitudes of each order of waves scattered from the structure, and \( k_{\text{scat}}^\Gamma \) is the wave vector in the \( y \) direction of each order. The wave vectors of the diffracted waves from the whole structure can then be written as

\[
k_{\text{scat}}^\Gamma = k_{\text{scat}} + \frac{2\pi m}{\Gamma} = k_0 \sin \theta_i + \frac{\Delta \psi}{d} - \frac{2\pi m}{\Gamma}.
\] (9)

The angle of reflection thus satisfies

\[
\theta_r = \sin^{-1} \left( \frac{k_{\text{scat}}^\Gamma}{k_0} \right)

= \sin^{-1} \left( \sin \theta_i + \frac{\Delta \psi}{k_0 d} - n \frac{\lambda}{d} - m \frac{\lambda}{\Gamma} \right), \quad m, n \in \mathbb{Z}.
\] (11)

Let \( \xi \) be the phase gradient term \( \frac{\Delta \phi}{\Gamma} \), and let \( G_d = \frac{2\pi}{\Gamma} \), \( G_{\Gamma} = \frac{2\pi}{\Gamma} \), rearrange Eq. (11), we have

\[
\theta_r = \sin^{-1} \left( \frac{k_{\text{scat}}^\Gamma}{k_0} \right)

= \sin^{-1} \left( \sin \theta_i + \frac{\Delta \psi}{k_0 d} - n \frac{\lambda}{d} - m \frac{\lambda}{\Gamma} \right), \quad m, n \in \mathbb{Z}.
\] (11)
The above formula gives full predictions of the diffracted beams permitted by an arbitrary acoustic metasurface with a fixed phase gradient. If \( m \) and \( n \) are both set to zero, i.e., only 0th order is examined, the derived formula can be simplified to the generalized Snell’s law, \( k_0(\sin \theta_r - \sin \theta_i) = \xi - nG_d - mG_T, \quad m, n \in \mathbb{Z}. \) (12)

The space coiling geometry of such cells renders them inherently broadband for phase manipulating applications. The designed set of labyrinthine cells provides phase delays at intervals of \( 1/6\pi \), covering the full \( 2\pi \) range with one single layer at 3000 Hz. The cells were designed based on the finite element analysis (FEA), and experimental prototypes were fabricated using fused filament fabrication (FFF) 3D printing with acrylonitrile butadiene styrene (ABS) plastics. 24

A reflective metasurface composed of above described unit cells approximates a phase gradient of \( \Delta \phi = 3.2 (2\pi \text{rad m}^{-1}) \), where \( \Gamma = 12d \). According to Eq. (9), \( |k_{yn}| > |k_0| \) holds for any non-zero values of \( n \) owing to its subwavelength geometry. However, various radiating diffraction orders coexist based on the choice of \( m \), as shown in Fig. 3(a). The reflected angle of the scattered modes can be reduced to

\[
\theta_r = \sin^{-1}\left(\sin \theta_i + \frac{\xi}{k_0} - mG_T\right), \quad m \in \mathbb{Z}. \quad (13)
\]

Different diffraction patterns can be obtained by varying the angle of incidence. Fig. 3(a) shows the analytical relationship between incident angles and reflected angles for various orders. The diffraction patterns can be affected by two factors: one is the angle of incoming sound wave with regard to 0th order critical incidence, and the other is the significance of impact imposed by higher order waves on the fundamental one. A full description of all propagating diffraction orders at a specific incident angle can be obtained numerically: first extract the near field distribution at the air-metasurface interface from simulations of the reflection field and then take a spatial Fourier transform of the exported data to obtain far field amplitudes and phases. Fig. 3(b) shows the result acquired using such method at 50° incidence, at which the fundamental order is evanescent, and thus more clearly illustrates the properties of the higher order diffraction. By comparing it with Fig. 3(a) (the brown line indicates propagating modes at incidence of 50°), it is clear that the diffracted directions of each mode are in good agreement with those computed from analytical derivations. Thus, we are confident in saying that the presented analysis is validated with simulations.

**FIG. 2.** Schematic and structure of an acoustic reflective metasurface. (a) One period of the acoustic labyrinthine cells designed that provides required phase delays and constitutes the acoustic metasurface. (b) Concept schematic of the wave modulation capability of a planar acoustic metasurface: the blue region represents arbitrary sound hard boundary and the green region represents the metasurface region and is composed of unit cells as depicted in (a). (c) A demonstration of several functionalities that can be achieved with such acoustic metasurface reflectors by varying the angle of incidence, including extraordinary reflection, negative reflection, and surface mode conversion.
EXPERIMENTS

We now explore the behavior of labyrinthine reflective metasurfaces experimentally with support from numerical simulations. The measurements were done in a custom two-dimensional acoustic scanner. A Gaussian modulated plane wave was generated from a transducer array, and a microphone was swept over the two dimensional measuring area in a point-by-point fashion to record the transmitted wavefronts. Two separate measurements were done in order to get the scattered field distribution at one specific incidence: one with metasurface samples inserted into the scanner, which collects data of the whole field with both incident and scattered waves; and the other without samples, which measures the incident field. The two measurements were conducted under identical measurement conditions. The scattered field was calculated by subtracting the measured incident field from the total field. We consider several fundamentally different regimes, beginning with the “simple” case of an incident angle below the critical angle and propagating higher order beams exhibiting very low amplitudes.

In this scenario, we only need to consider \( m = 0 \), and the derived formula Eq. (13) gives identical predictions of the diffraction patterns as predicted by the GSL. When a plane wave is emitted from the transducer array and impinges on the metasurface region at a tailored incident angle \( \theta_i \), the reflected wave will exit at an angle \( \theta_r = \sin^{-1}(\frac{C_1}{C_0} \sin \theta_i) \).

This case is examined for a range of frequencies in Fig. 4.
Fig. 4(a) shows a numerical simulation of reflection from the full labyrinthine structure, with an angle of reflection unequal to the angle of incidence as expected from the generalized Snell’s law. The angle of incidence is 30°, and Fig. 4(b) shows the measured incident field. Measurements of the reflected fields conducted within 2800 Hz and 3200 Hz are shown in Figs. 4(c)–4(e), demonstrating the expected anomalous reflection over a bandwidth of more than 10% of the central frequency.

Another phenomenon that can be achieved in this regime is negative reflection. As can be seen in Fig. 3(a), when the angle of incidence is negative, or \( \sin^{-1}\left(\frac{C}{C_0}\right) < \theta_i < 0 \), the main reflected beam can be at a positive angle and thus on the opposite side of the wave normal. Figure 5 shows the measured incident and scattered fields for a \(-10^\circ\) angle of incidence. As expected, the zero order reflected beam is retro-reflected with high efficiency.

We enter another regime by further increasing \( \theta_i \) until it approaches the critical angle of the fundamental order. In this case, the reflected wavefront for this order will be steered towards the metasurface. At critical incidence, \( k_{in} = k_0 \), the angle of reflection is \( \theta_i = 90^\circ \). Further increasing \( \theta_i \) will generate the surface wave whose surface wavenumber \( k_{sw} > k_0 \) and perpendicular wavenumber \( k_{in}^r = \frac{k_{in}^0}{\sqrt{(k_{in}^0)^2 - k_0^2}} \), which is purely imaginary, according to Eq. (10). From Eq. (13), the critical angle can be calculated as \( \theta_c = \sin^{-1}(1 - \frac{1}{\sqrt{2}}) = 40^\circ \) based on the proposed design. Beyond the critical angle \( \theta_c \), the surface waves generated become evanescent owing to the rapid decaying of the component in the perpendicular direction. It should be noted that, under this condition, both experimental measurements and simulations that embed in-lab environmental loss yield a relatively clean demonstration of the converted surface modes, with no noticeable high-intensity excess fields, as in Figs. 6(b) and 6(c). However, if lossless environment is assumed in simulation, a combined phenomenon of negative reflection and surface wave conversion is observed, as in Fig. 6(a). The backward reflected waves may correspond to the high order modes generated from our designed structure; nevertheless, its presence in lossless simulations but not in experiments intrigued us to dig deeper into the unknown properties of those high orders. This seeming “inconsistency” between simulation and measurements will be explored in the subsequent part of the paper.

The distinct results obtained in simulations and experiments under critical incidence conditions imply that loss is an inevitable factor to be considered in both cases. In simulations, the loss factor can be embedded in the imaginary part of wave vectors and varied accordingly. In experiments, however, loss is determined by the surrounding environment, temperature, humidity, dust, etc. Thus, if a wave is very sensitive to loss, then chances are its amplitude attenuates so drastically in the lossy medium that it will not be observed in experiments. One option for visualizing and analyzing the characteristics of a loss-sensitive wave is through lossless simulations, as the one that was done for Fig. 6(a). To study the sensitivity of the higher order modes to loss, a lossless transient simulation was conducted with the proposed metasurface, the results presented in Fig. 7. It is shown that the formation time of the high order waves is approximately 7 times longer than the main order before exiting the structure, leading us to conclude that those orders get multiple reflected between the two boundaries of the metasurface. An
observed that at an incidence angle of 50°, the 5th order diffracted wave exhibits non-trivial amplitudes. A calculation of the incident angle for the 5th order mode to start propagating is \( \theta_c = 30° \), which is smaller than the critical incident angle \( \theta_c \). When the incident angle falls in the range \( \theta_i < \theta_c \), two propagating beams will exit the metasurface, as in Fig. 7. Beyond critical incidence, the fundamental order becomes evanescent, while the 5th order wave continues propagating, as shown in Fig. 6. The angle of reflection for the 5th order beam can be calculated as \( \theta_r = \sin^{-1}(\sin \theta_i + \frac{\theta_i}{2} - \frac{\theta_i}{T}) \), which is negative for our design parameters. Thus, an additional pattern of the backward reflection is observed for \( \theta_i > \theta_c \).

For demonstration purpose, we consider the 5th diffraction order as a representation among all higher orders owing to its dominant amplitude. The overall beam steering properties of the designed metasurface are generalized in Fig. 8(a), with the relationship between incident and reflected angles demonstrated for both the fundamental order and the 5th order diffracted waves. Because of the loss-sensitive nature of higher order waves and the non-radiating nature of the surface mode, a feature of high absorbance is observed in practice beyond critical incidence for the metasurface. The normalized total reflected power \( P_r^n \) from the designed metasurface over a defined boundary is calculated as

\[
P_r^n = \frac{P_r^n}{P_i^n}
\]

FIG. 7. Lossless transient simulation demonstrating exit delay of high order beams and coexistence of forward and backward propagation. (a) Scattered field when the incident wavefront first hits the metasurface (\( \theta_i = 40° \)). (b) After 0.7 ms, the main order starts to propagate outside to free space. (c) 4.7 ms later, the 5th order exits the metasurface. (d) Stable condition when both propagating orders coexist. The white arrows indicate the propagation direction of the fundamental order wave, and the yellow arrow indicates the propagation direction of the 5th order wave.

FIG. 8. Relationship between incidence and reflection. (a) Relationship between incidence angle \( \theta_i \) and reflected angle \( \theta_r \). The theoretical curve is calculated based on the proposed diffraction formula. (b) Relationship between normalized total reflected power \( P_r^n \) and incident angle. Reflection coefficient is calculated as the ratio between incident and reflected power flow. The blue region denotes the range of negative reflection, the green region denotes the range of anomalous reflection, the yellow regions represent the range of both anomalous and negative reflection, but only anomalous reflection can be observed in experiments; and the brown region represents the range of both surface wave conversion and negative reflection, and only non-radiating surface wave can be practically observed.

unexpected phenomenon arose from this finding: enhanced absorption.

Extraordinary absorption is a long desired feature for acoustic devices, and it can be combined with the surface wave conversion in our metasurface design, yielding an extremely local field distribution for a range of incident angles. In a real experimental setup, even if the intrinsic absorption rate in the structure is minimal, because of the notably long exit time delay of the higher orders compared to the main one, its power can be significantly attenuated. As a result, in measurements, the surface wave is more noticeable and the higher order is hard to be seen, as in Figs. 6(b) and 6(c).

Now that the difference between results of lossless simulation and actual measurements has been accounted for, next to be explained is the existence of negative reflection in Figs. 6(a) and 7. From Fig. 3(b), it can be observed that at an incidence angle of 50°, the 5th diffraction order exhibits a significantly higher amplitude than any other propagating orders. The same calculations were performed on other angles of incidence, and the results indicate that the 5th order diffracted wave is dominant in amplitude, so long as it is non-evanescent. Thus, when the incident angle is larger than \( \theta_c \), the 5th order diffracted wave continues propagating, as shown in Fig. 6. The angle of reflection for the 5th order beam can be calculated as \( \theta_r = \sin^{-1}(\sin \theta_i + \frac{\theta_i}{2} - \frac{\theta_i}{T}) \), which is negative for our design parameters. Thus, an additional pattern of the backward reflection is observed for \( \theta_i > \theta_c \).

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\[
P_r^n = \frac{P_r^n}{P_i^n}
\]
\[
P_r' = \left[ \frac{1}{2} \mathcal{R}(p_i) \right] \mathbf{v}_r' \cdot \mathbf{n} \, dL,
\]

where \( p_i \) and \( p_r \) are the reflected and incident acoustic pressures, respectively, \( \mathbf{v}_r \) and \( \mathbf{v}_i \) are the acoustic particle velocity of the reflected and incident field, respectively, and \( \mathbf{n} \) is the unit normal vector to boundary \( L \). An asterisk (*) denotes the conjugate of a complex quantity. Fig. 8(b) demonstrates the calculated normalized total reflected power of the proposed metasurface with regard to a wide range of incident angles. It can be seen that around critical incidence, a very small perturbation of the input signal will lead to a significant drop in the ratio between incident and reflected power flows in the output signal. The reflection coefficient drops dramatically around the critical angle and eventually stabilizes around 0.1 for larger incident angles. Based on these facts, it is then intuitive to imagine that if a reflective metasurface could be constructed with normal incidence to surface wave conversion, it would make a compact high performance omnidirectional acoustic absorber.

**CONCLUSIONS**

To conclude, we have proposed a detailed theoretical analysis on multi-order diffractions from periodic phase-modulating metasurfaces. From the analysis, diffraacted angles can be calculated for all propagating modes at arbitrary angles of incidence. Then, a broadband planar acoustic reflective metasurface based on the labyrinthine acoustic metamaterials was designed, fabricated, and tested. The metasurface possesses various wavefront-shaping capabilities, including negative reflection at negative incidence, anomalous reflection below critical incidence, and surface wave conversion beyond critical incidence. All such phenomena are successfully predicted from the aforementioned theory and verified through simulations and measurements. Moreover, enhanced absorption for higher order diffracted beams is observed in experiments for the first time. Transient simulations relate this peculiar effect to the reduction of higher order modes in the lossy medium. The good agreement between theory, simulation, and experimental measurements further confirms the validity of the analysis. The derived formulas can serve as a guide for reference of future acoustic metasurface design. The reflective metasurface concept suggests new approaches for acoustic wave manipulation that could find use in devices such as acoustic lenses and acoustic holograms.

**ACKNOWLEDGMENTS**

This work was supported by the Multidisciplinary University Research Initiative grant from the Office of Naval Research (N00014-13-1-0631).


