Anisotropic acoustic metafluid for underwater operation

Bogdan-Ioan Popa,a) Wenqi Wang, Adam Konneker, and Steven A. Cummer
Department of Electrical and Computer Engineering, Duke University, Durham, North Carolina 27708, USA

Charles A. Rohde, Theodore P. Martin, and Gregory J. Orris
U.S. Naval Research Laboratory, Code 7160, Washington, DC 20375, USA

Matthew D. Guild
NRC Research Associateship Program, U.S. Naval Research Laboratory, Washington, DC 20375, USA

(Received 24 September 2015; revised 19 January 2016; accepted 15 February 2016; published online 30 June 2016)

The paper presents a method to design and characterize mechanically robust solid acoustic metamaterials suitable for operation in dense fluids such as water. These structures, also called metafluids, behave acoustically as inertial fluids characterized by anisotropic mass densities and isotropic bulk modulus. The method is illustrated through the design and experimental characterization of a metafluid consisting of perforated steel plates held together by rubber coated magnetic spacers. The spacers are very effective at reducing the effective shear modulus of the structure, and therefore effective at minimizing the ensuing coupling between the shear and pressure waves inside the solid effective medium. Inertial anisotropy together with fluid-like acoustic behavior are key properties that bring transformation acoustics in dense fluids closer to reality. © 2016 Acoustical Society of America.

I. INTRODUCTION

Transformation acoustics1–3 and metamaterials that behave acoustically as fluids provide significant control over the propagation of sound,4–29 and consequently have numerous applications in underwater acoustic systems and ultrasound medical imaging. Nevertheless, most experimental acoustic metamaterials reported to date have been designed to work in air. Reported experiments in water-based media, in which they could have a significant technological impact, are much rarer. The reason for this is the complex physics of inhomogeneous solid mixes in dense fluids that are typically negligible in gases. These complications raise a unique set of challenges that are yet to be addressed effectively.

One of the challenges is the adequate control of the metamaterial effective shear modulus. Shear and pressure waves couple strongly at the interface between inhomogeneous media having material parameters of similar order of magnitude,30 and the coupling tends to significantly reduce the degree of anisotropy.31 For instance, it has been shown that a structure having mass density tensor components differing by a factor of \( \approx 5 \) in the absence of shear is virtually isotropic as soon as shear is enabled.31 Since the success of transformation acoustics relies on obtaining highly anisotropic structures, it is important to avoid this effect. It was suggested that operating in a laminar layer of water avoids this issue altogether.14 But if we operate in unbound water one solution is to use pentamode metamaterials, which are artificial media that exhibit stiffness anisotropy.32–37 Such materials, however, tend to be delicate structures that rely on a network of thin filaments supporting much heavier inclusions. In the same spirit, the shear modes can be reduced if the solid materials are surrounded by layers of background fluid.31 Alternatively, one can use soft materials and avoid solids altogether.38 All these approaches are not mechanically robust which makes these structures challenging to work with in practice.

A second difficulty stems from the lack of standard procedures to characterize experimentally acoustic metamaterials in water. Air-based metamaterials are much more convenient from this perspective. From an application point of view, waveguide based measurements8,17 have been shown to be a reliable method to retrieve experimentally the effective material parameters of fabricated metamaterial samples. However, it is not trivial to adapt these methods to water-based designs. This is mainly because sound in water couples strongly with the waveguide walls, and this effect needs to be carefully taken into account.39 Free-space measurements have been used to characterize electromagnetic metamaterials and conceivably they could be adapted to acoustics. However, they typically require samples of many wavelengths in diameter to mitigate the diffraction from the sample edges,40 which make the samples prohibitively large.

Our goal is to address these two limitations. First, we show how robust structures consisting of arrays of perforated steel sheets connected by rubber padded spacers can be engineered to have anisotropic mass density and a sufficiently small shear modulus in a broadband of frequencies, and thus behave essentially as metafluids suitable for transformation acoustics. Second, we present and demonstrate an unbounded medium method to retrieve the effective material parameters of metamaterial samples as small as two wavelengths in diameter. The frequency band of interest covers the interval 10 to 20–kHz and is chosen based on the dimensions of the water tank used to make the measurements, namely, \( 6 \text{ m} \times 6 \text{ m} \times 4 \text{ m} \).

---

a)Electronic mail: bogdan.popa@duke.edu
Unless specified otherwise, all fluid material parameters are expressed throughout the paper relative to the parameters of water, namely, mass density $\rho_w = 1024\text{ kg/m}^3$ and bulk modulus $B_w = 2.3\text{ GPa}$.

II. EFFECTIVE SHEAR MODULUS REDUCTION

Metamaterials made of perforated solid sheets are a convenient way to generate a significant amount of anisotropy and have been used successfully in air.\textsuperscript{7,15,24,27,29} The degree of anisotropy increases monotonically with the decrease of the perforations' diameter, making them relatively straightforward to design.

Figure 1(a) shows a $6 \times 6 \times 6$ unit cell metamaterial sample made of 1.2 mm thick steel plates having perforations of 3.4 mm in diameter. Using the procedure in Ref. 8 the samples were designed to have a uniaxial mass density tensor, $\overline{\rho}$ with components relative to water density of $\rho_x = 1.7$ and $\rho_y = \rho_z = 1.1$. We chose these values because they provide a significant degree of anisotropy of 1.5, useful in many water-based transformation acoustic designs, and which can be measured reliably. The unit cell has a cubical shape, is 1 cm along the $x$, $y$, and $z$ directions, and contains one perforated steel sheet perpendicular to the $x$ direction. The unit cell is marked in the figure with dotted lines.

In the first phase of the design the plates are isolated mechanically from each other by layers of fluids to cancel the shear modulus and obtain a fluid-like metamaterial.\textsuperscript{31} The advantage of this method is threefold. First, the method makes no assumptions on the physics of the metamaterial under test. This allows us to focus on the main behavior of the metamaterial and neglect second order effects such as viscous and thermal absorption without loss of generality. If a loss mechanism was included in simulations, the loss would be reflected in non-zero imaginary parts of the effective material parameters as seen in the experimental results presented in Sec. IV. Second, the metamaterial sample is measured in an environment very similar to that of the intended application. Third, the multiple measurements needed to obtain all the material parameters provide enough redundancy to verify our main assumption that the shear modulus is negligible.

Two Comsol Multiphysics simulations were used to compute the metamaterial effective mass density and bulk modulus along the directions in which the mass density tensor components are different. In each simulation a plane wave is sent in the probed direction and is normally incident on a three unit cell thick metamaterial sample of infinite transverse extent. The reflection and transmission coefficients are inverted to obtain the mass density component along the direction of incidence together with the bulk modulus. Since the metamaterial considered has uniaxial $\overline{\rho}$ only, two such simulations are needed, each providing the bulk modulus independently. If any of the assumptions made in the model break down, namely, that the metamaterial has negligible shear modulus and the underlying assumption that the structure behaves as an effective homogeneous medium, the two retrieved moduli will diverge.

The dotted curves in Fig. 2 show the extracted effective material parameters for the two directions of propagation considered. The thick curves correspond to the probing wave normally incident on the plates, while the thin curves correspond to the direction of incidence parallel to the sheets. To facilitate comparison with the simulations of the realizable structure specified in the second phase of the design, these curves are replicated on all the panels shown in the figure.

The bulk moduli retrieved in both simulations assume approximately the same value within a 5% error margin. This confirms that the metamaterial behaves acoustically as a fluid. The difference is highest toward 20 kHz because at these frequencies the cell becomes slightly larger than 1/8 of a wavelength. As a result, we approach the gray region in which metamaterials behave somewhere in between homogeneous materials and sonic crystals. The same gray region has been observed for air-based acoustic metamaterials.\textsuperscript{5} At the same time, the simulations confirm that the dynamic mass density tensor is anisotropic with an anisotropy factor defined as $\rho_x/\rho_y = 1.51$.

However, the structure giving these material parameters is ideal in that the steel plates are isolated mechanically from each other through layers of water. Although this is a very effective method to cancel the effective shear modulus, the resulting metamaterial is not mechanically stable and would collapse under its own weight. Supporting materials need to be added in order to make it more robust. Figure 1(b) shows the same metamaterial with solid spacers in place represented by the highlighted cylinders. While they stabilize the structure, these favor the propagation of various types of mechanical waves throughout the metamaterial.

Next, we investigate the effect of spacer density on the extracted effective material parameters. Figure 2(a) shows the retrieved mass density and bulk modulus for the two directions of incidence and three spacer densities. For high

![Image](Image 57x127 to 170x259)

FIG. 1. (Color online) Schematics of the acoustic metamaterial samples. (a) The ideal fluid-like structure made of perforated steel plates suspended in water. The dotted lines show the boundary of one metamaterial unit cell. (b) The same structure as in (a) in which the plates are connected through solid spacers. Assuming periodic replication of this metamaterial in all three directions, the figure shows a spacer density of 6 unit cells/spacer.
Spacer densities (4 cells/spacer) the bulk moduli diverge in virtually the entire band of interest, which indicates that the metamaterial fluid model is not applicable. As the spacer density decreases, the metamaterial starts to behave acoustically as a fluid in certain bands. For example, at a density of 6 cells/spacer, the case illustrated in Fig. 1(b), the metamaterial is fluid-like in the band 11–14 kHz and at 8 cells/spacer the band increases to 10–15 kHz.

However, the usable bandwidth increases slowly with spacer density and for small spacer densities the structure may lose robustness. For instance, we can imagine a situation in which we need metamaterials characterized by bulk moduli or mass densities smaller than that of water. In this case the steel plates are replaced by other, softer materials such as various metal or ceramic foams. Low spacer densities mean that there is an increasing danger of plate bending due to the lower stiffness. A new strategy needs to be developed that allows higher spacer densities.

Rubber has been proposed as a central material in the design of water-based transformation acoustics devices due to its high Poisson’s ratio, \( \nu \). However, rubbers having \( \nu \) close enough to 0.5 to effectively cut the shear modes in the metamaterial structure are typically soft in nature and limit the robustness of the overall metamaterial if used in bulk, and therefore limit the applicability of this approach.

Instead, the strategy employed here revolves around coating the spacers with thin rubber pads. The pads are not only suitable to reduce the metamaterial shear modulus, but also have a small footprint that does not affect the mechanical robustness of the design. Moreover, the spacers and pads are small enough that the coated spacers do not support resonant modes in the band of interest by themselves. Figure 2(b) shows the extracted material parameters of the spacer supported metamaterial having the spacers padded with 1 mm thick rubber characterized by a Poisson’s ratio of 0.49, density of 1700 kg/m\(^3\), and bulk modulus of 25 MPa.

Even at high spacer densities the metamaterial functions as a fluid in a large bandwidth as indicated by the matching extracted bulk moduli in these bands. For example, at 4 cells/spacer the metafluid bands are 10–14 and 16–18 kHz, while at 6 cells/spacer the metamaterial works well in almost the entire band of interest between 10 and 18 kHz. In addition, all material parameters agree very well with the target parameters represented by the dotted lines, which confirms the efficiency of rubber to attenuate shear waves. The sharp variations in the extracted material parameters visible at certain frequencies in all the plots correspond to various resonant modes in the plates. The rubber removes some of these but not all of them. However, they are very narrowband and in practice we expect these modes to be greatly attenuated due to viscous and thermal losses inherent in real materials and which were not taken into account in the simulations.

### III. Measurements of the Effective Material Parameters

An important step in the design process is the ability to measure experimentally the effective material parameters of metamaterial samples. Waveguide based measurements proved to be a very effective method to retrieve the effective
A numerical retrieval for an isotropic fluid characterized by mass density and bulk modulus of air-based acoustic metamaterials. However, adapting these to a water background is not trivial because the waveguide material cannot be approximated as infinitely rigid, and its elastic properties will strongly influence the metamaterial response.

An alternative method is to employ the same reflection/transmission measurement method in free space. However, the large wavelengths in a water-based medium mean that the metamaterial samples need to be quite large to avoid diffraction induced artifacts. Here we show that unbounded medium reflection/transmission measurements coupled with a field averaging technique allows us to accurately measure the material parameters of small samples several wavelengths in diameter.

Figure 3(a) shows a thin sample of ideal fluid having dimensions $30 \times 30 \times 2 \text{ cm}$, isotropic mass density $\rho$, and bulk modulus $B$, submerged in water. The sample is ensonified by a point source located $2 \text{ m}$ in front of the sample. The sound produced by the source at the sample location can be well approximated by a plane wave normally incident on the sample. The waves reflected by and transmitted through the sample are measured on the highlighted planes situated $2 \text{ cm}$ in front of and, respectively, behind the sample.

Figure 3(a) shows the simulated reflected and transmitted amplitude patterns at $12 \text{ kHz}$ obtained for a fluid characterized by $\rho = 1.5$ and $B = 1.1$. As expected, the diffraction from the edges of the sample generates a non-uniform field pattern on the measurement planes, and makes the retrieval of material parameters non-trivial. The naive approach of probing the transmitted and reflected fields at one point in front and behind the sample as is typically done in a waveguide would fail here because the measurement is very sensitive to the measurement positions relative to the sample, sample dimensions, and frequency. However, since diffraction redirects and spatially redistributes the acoustic energy, we expect the energy density passing through the measurement planes, averaged over an area having the same dimensions as the sample, to be significantly less influenced by these factors.

We test this idea in a simulation in which we set the fluid sample material parameters to known values and use the reflection/transmission measurement method to recover and compare to the actual values. In the following we are using the refractive index, $n = \sqrt{\rho/B}$ and relative impedance $z = \sqrt{\rho B}$ as alternative material parameters.

Figure 3(b) shows the retrieved material parameters of a transparent sample ($z = 1$) whose refractive index was varied between 1 and 2.5 in steps of 0.5, and Fig. 3(c) shows the same plots for $n = 2$ and a relative impedance varied from 1 to 2.5 in steps of 0.5. In all these simulations the retrieved parameters matched very well within the actual parameters which confirms the effectiveness of the method. The retrieved parameters $n$ and $z$ tend to be slightly smaller than the real values by a margin of less than 5% due to a small fraction of the incident energy being redirected away from the averaging planes. One notable difference was the case corresponding to $n = z = 1$. In this case the impedance shows a dip around $18.5 \text{ kHz}$. As we will see in Sec. IV this is a numerical artifact caused by the quarter of wavelength thickness of the material under test.

IV. EXPERIMENTAL DEMONSTRATION OF WATER-BASED METAFLUID

The next step is to employ the theoretical apparatus described above and design, implement, and characterize a water-based anisotropic metafluid suitable for transformation acoustics in a water environment. We chose $\rho_x = 1.5$, $\rho_y = 1.1$, and $B = 1.1$ due to the design’s easiness of fabrication while maintaining a significant amount of anisotropy. As before, the metamaterial unit cell is a cube having $1 \text{ cm}$ edges, and the perforation area is $23\%$ of the steel sheet area inside one cell. Since we need to measure two different components of the mass density tensor, we need two metamaterial samples that represent thin slices through a bulk metamaterial along the directions of different density components. Figure 4(a) shows photographs of the two samples. The one on the left (sample 1) is used to measure $\rho_x$ and $B$, and the sample in the middle of the figure (sample

![FIG. 3. (Color online) Diffraction from small metamaterial samples. (a) Simulation setup showing the sample, source, and measurement planes, together with the amplitude of the field simulated on the transmission and reflection sides of the sample at $12 \text{ kHz}$. (b) The effective refractive index $n$ (top) and impedance $z$ (bottom) numerically retrieved for an isotropic fluid characterized by $z = 1$ and the listed values of $n$. (c) The effective refractive index $n$ (top) and impedance $z$ (bottom) numerically retrieved for an isotropic fluid characterized by $n = 2$ and the listed values of $z$.](image-url)
procedure described in Fig. 3(a), and is identical to that which inherits the low shear modulus and robustness of rubber. In key places, we obtained a metamaterial that avoids this tradeoff. Rubber is a natural material therefore the more fragile the metamaterial becomes. In our design we avoid this tradeoff. Rubber is a natural material that has a very low shear modulus, yet it is robust. By gluing the rubber in key places we obtained a metamaterial that inherits the low shear modulus and robustness of rubber.

The measurement setup follows closely the simulation procedure described in Fig. 3(a), and is identical to that employed in Ref. 10. Namely, the metamaterial under test is submerged in water and ensonified by an underwater circular piston source (NRL-USRD F33 produced in-house by the US Navy) placed at 2.25 m in front of the metamaterial on the axis perpendicular on the metamaterial center. The input signal driving the source is a pulse containing ten periods of a sinusoidal signal of frequency $f$. The left-hand-side of Fig. 4(b) shows the signal when $f = 17$ kHz. The signal has a relatively broadband of at least 10% of the center frequency (middle panel), which means that 11 sets of measurements using input signals centered on frequencies between 10 and 20 kHz and spaced 1 kHz apart will cover the entire bandwidth of interest and provide sufficient redundancy to verify the measurement validity.

The Gaussian transverse profile of the generated pressure wave was measured in absence of the sample, at the sample position. A Gaussian fit of the measured beam profile was performed to find the center of the incident sound beam. The center of the samples was placed to coincide with the measured acoustic beam axis. The samples were mounted in such a way that they were fixed in place (i.e., not moved) for all transmission and reflection measurements.

A Bruel and Kjær 8103 cylindrical hydrophone with a 50 mm length and 9.8 mm diameter scans 2 planes parallel to the metamaterial and situated 4 cm in front and behind the metamaterial. This hydrophone was placed at the end of a water filled PVC boom, orientated with the long axis of the hydrophone in the vertical (z axis) direction. The collected hydrophone signal was amplified with a Stanford Research Systems SIM910 amplifier of gain 100, and the resulting signal was filtered with a Frequency Devices, Inc. bandpass filter having 500 Hz–50 kHz bandpass. The signal was then sampled at 1 MHz and digitized with a National Instruments data acquisition system. The scanning operation is performed with a fully automatic Velmex BiSlide 3D scanning system whose precision (repeatability) is 5 μm. The system has an absolute accuracy of 38 μm over its 1.2 m travel range. Each scan plane was collected using a global coordinate system which maintained this precision and accuracy.

Figure 5 shows the reflected and transmitted wave amplitudes relative to the amplitude of the incident wave measured with the incident pulse centered on 17 kHz. These field amplitudes match very well simulations of an ideal fluid characterized by $\rho = 1.5$ and $B = 1.1$ which confirm that most of the field variation is caused by diffraction from the metamaterial edges and to a lesser degree by flexural waves in the steel sheets. A notable difference is the higher amplitude measured in the center of the scanned area than what was obtained in the ideal fluid simulation. This may be caused by the measurement of the transmission-side plane being slightly shifted in the propagation direction, and not exactly 4 cm behind the sample as considered in the

![Figure 4](image-url)

**FIG. 4.** (Color online) Photograph of fabricated samples and measurement procedure. (a) The two metamaterial samples dubbed sample 1 (left) and sample 2 (middle) and used to measure $\rho_1$ and $B$, and, respectively, $\rho_2$ and $B$. The right-hand side shows a photo of one of the rubber coated neodymium magnet spacers. (b) Schematic of the metamaterial submerged in water and ensonified by an underwater speaker. The incident pulse (left) has a broadband covering more than 10% of the pulse center frequency (middle).
simulation. In addition, the flexural modes supported on the steel plates may generate a higher contrast between the regions of lower and higher amplitude for the measurements compared to the simulations. We recall that the latter assumed that the samples under test are ideal fluids.

These reflection and transmission measurements are inverted using the averaging method described above to obtain the samples’ effective material parameters. The resulting material parameters are presented in Fig. 6. Except for the resonant looking feature around 18 kHz, that are discussed below, the effective bulk moduli retrieved from the measurements of both samples match very well in the entire band of interest. This confirms that the metamaterial shear modulus has been virtually canceled by the rubber padded spacers. Consequently, the metamaterial behaves acoustically as a fluid, so it can properly be called a metafluid. As expected, the measured mass density tensor components are different based on the direction of propagation, which proves that the metafluid has the prescribed mass anisotropy. In addition, the effective material parameters are approximately constant in the entire band of frequencies, which shows excellent bandwidth performance. At the same time, the samples have almost no loss as represented by the small imaginary values of the material parameters.

There are, however, some discrepancies between the design parameters and measurements. First, the experimentally retrieved material parameters are, at some frequencies, 15% of the target values. The reason for this inaccuracy lies partly with the field averaging method which, as we showed earlier, results in consistently lower retrieved material parameters due to some energy in the transmitted and reflected fields being diffracted away from the measured areas. In addition, we notice from the amplitude distribution shown in Fig. 5 that metafluid samples were not perfectly centered on the middle of the scanned areas and at the same time they are slightly rotated by a couple of degrees clock-wise. This reduces the averaged field values because of the areas of small fields around the sample edges that would not be averaged if the sample was centered.

The second discrepancy is the resonant looking shape obtained for sample 2 material parameters around 18 kHz, and, to a lesser degree, obtained for sample 1 parameters at 16.5 kHz. To understand the cause of this discrepancy we have to look deeper at the inversion equations that convert reflection (R) and transmission (T) coefficients into the relative impedance (z), namely, \( z = [(1 + R)^2 - T^2]/(1 - R)^2 - T^2 \). There are situations in which both the denominator and numerator approach zero at the same time. As a result the retrieved impedance becomes very sensitive to measurement errors, and the result is the resonant looking behavior seen in Figs. 3 (when \( z = 1 \) and \( n = 1.5 \)) and 6.

One such situation occurs when both \( R \) and \( T \) are real and the measured material is transparent. In this case \( R \approx 0 \) and \( |T| \approx 1 \). Sample 2 meets these requirements at 18 kHz. It is almost transparent having \( B \approx \rho_1 \approx 1 \), and, at this frequency, the wavelength inside the metafluid is twice the sample thickness, in which case \( T \approx -1 \). To further test this theory we assumed that the metafluid thickness was 3.6 cm instead of the actual 4 cm in the inversion formulas. This 20% reduction in thickness results in a 20% increase of the half-wavelength frequency, and, as expected, shifted the resonant behavior to \( \approx 20 \text{kHz} \) for sample 2 and \( \approx 18 \text{kHz} \) for sample 1.

V. CONCLUSIONS

In conclusion, we presented theoretically and demonstrated experimentally a method to design anisotropic metafluids that are suitable for transformation acoustics in a water environment, which at the same time have a robust mechanical structure made of solid materials. The method relies on first designing a theoretical metafluid in which its solid components are isolated through layers of the background fluid. A second design pass adds supporting spacers coated with rubber that significantly reduce the effective
shear modulus of the metamaterial. The fabricated material does not cancel the shear of the steel plates themselves. Simulations show that this shear component is responsible for flexural waves in the plates that result in narrowband resonant modes. However, these resonances are sparsely distributed along the frequency spectrum and cause no issues in the presented design.

If needed, the effect of the steel plates’ shear modes can be mitigated using the same method employed to reduce the metamaterial shear. Namely, each steel plate can be divided into smaller patches of various sizes, for instance having areas of $12 \times 12$ unit cells. Thin rubber gaskets glued in between these patches connect the patches together in composite plates that do not support significant vibration modes along the surface of the plates.

Finally, we demonstrated an unbounded medium experimental retrieval procedure for the metafluid material parameters that involves small metafluid samples. The method involves measuring the reflected and transmitted fields on planes situated in front and behind the samples, and calculating the sample reflection and transmission coefficients based on the fields averaged along these planes. The coefficients are then inverted using a standard method proven to be very effective in acoustics.

The design procedure and experimental characterization method presented here open the road for transformation acoustic designs in water-based media.

ACKNOWLEDGMENTS

This work was supported by Grant No. N00014-12-1-0460 from the Office of Naval Research.


