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Preserving omnidirectionality in optimized asymmetric transformation optics designs

Bogdan-Ioan Popa and Steven A Cummer

Department of Electrical and Computer Engineering, Duke University, NC 27708, USA

E-mail: bogdan.popa@duke.edu

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Abstract

Optimization techniques are efficient methods to simplify the design of transformation optics devices. Asymmetric devices obtained through these methods typically lose omnidirectionality and perform well only when illuminated from a finite set of directions. We present here an optimization approach that results in simplified, omnidirectional designs of fairly large bandwidths. The method leverages a class of coordinate transformations that result in transformation media whose material parameters follow identical isocontours. We show that discretizing these media in a finite number of layers that follow the common isocontours and optimizing the material parameters inside each layer is an effective way to significantly simplify the design complexity while preserving most of the original performance including omnidirectionality.

Keywords: optimization, transformation optics, material parameter isocontour

(Some figures may appear in colour only in the online journal)

1. Introduction

Transformation optics [1, 2] and its acoustic analogue, transformation acoustics [3–5], comprise a set of tools that provide the ability to manipulate physical waves almost arbitrarily. The application that fueled the popularity of these techniques is the invisibility cloak, a device that cancels the scattering from the cloaked object [1–9]. At the same time, the invisibility cloak showcased the full range of challenges encountered while using transformation optics. Namely, the prescribed ideal media (transformation media) are inhomogeneous and anisotropic, having components of the material parameter tensors that can cover the full range of values from zero to infinity. In addition, the material parameter gradients can assume very large values. Although these ideal transformation media are too complex for the current technological limits, it became apparent that simplifications to the ideal materials lead to realizable designs that preserve the basic functionality while trading-off some of their performance [10–19].

To this end, it has been shown [20] that the performance penalty can be reduced significantly if optimization techniques are used to tune the material parameters obtained inside the transformation materials. More specifically, these optimization-based methods simplify the transformation medium complexity by approximating it as a small number of homogeneous regions (usually layers of various thickness) in which the material parameters are degrees of freedom in an optimization algorithm that seeks to minimize a certain design metric. In the case of cloaking this metric could be the scattering cross-section or forward-scattering.

Many optimization strategies have been developed since then that targeted various transformation optics systems and various levels of simplifications [21–27]. Since the complex transformation media is commonly implemented using metamaterials, the metamaterial structure itself became subject to optimization in an attempt to produce the simplest possible realizable structures [28–30].

Nevertheless, optimization-based simplifications to transformation optics designs have their own challenges. One of the biggest difficulty is that the design efficacy of asymmetric devices becomes dependent on direction, which is detrimental in many situations. Almost all optimization techniques reported so far involve devices having cylindrical or spherical symmetry and thus are immune to this issue [20–22, 24–27]. However, if the symmetry is broken because device geometry or material is allowed to be asymmetrical, the efficacy of the optimization reduces. More specifically, it

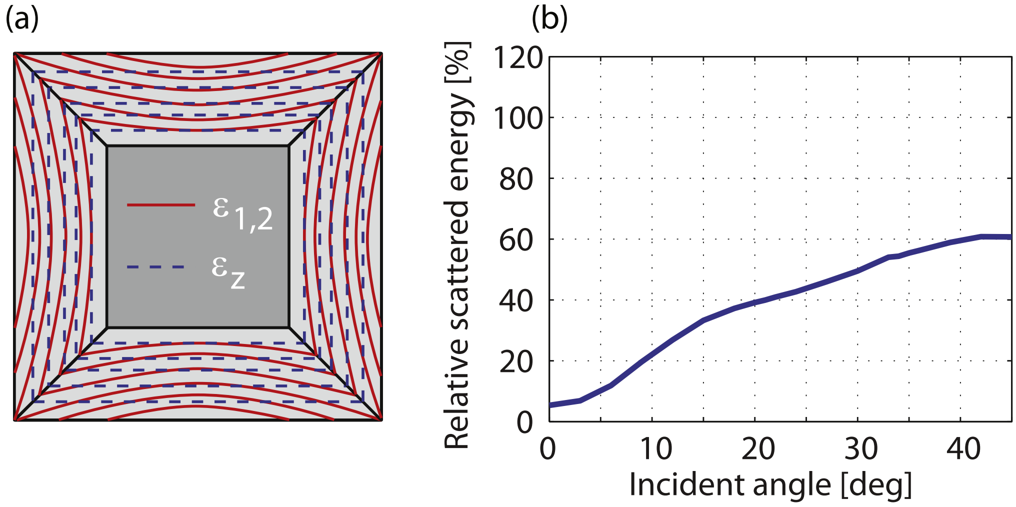

Figure 1. Performance of a transformation optics device designed through standard optimization methods. (a) The isocontours of the in-plane (solid) and out-of-plane (dashed) components of the material parameter tensors; (b) relative scattered power versus angle of incidence. The 100% line corresponds to energy scattered by the bare object.

has been shown that full invisibility cloaks lose effectiveness for most directions of incidence but a very small discrete set of directions [22, 28, 30].

We present a design strategy that addresses this limitation. The method leverages the power of optimizations to simplify the design structure while maintaining most of the original performance including the design omnidirectionality. To illustrate this technique we consider the representative case of the invisibility cloak.

Throughout the paper we consider designs in which the relative permittivity, $\overline{\overline{\epsilon}}_r$, and permeability, $\overline{\overline{\mu}}_r$, tensors are equal, as prescribed by transformation optics. As such, in the following we will consider only the values of the permittivity, with the implied assumption that $\overline{\overline{\epsilon}}_r = \overline{\overline{\mu}}_r$. We consider the case of a two-dimensional cloak in which ϵ_1 and ϵ_2 are the in-plane permittivity components along the principal axes and ϵ_z is the out-of-plane component.

2. Optimized transformation optics and material parameter isocontours

In this section we show that, if we want to apply optimization techniques to simplify transformation optics designs, then all material parameters inside the transformation medium must have identical contours of constant value (isocontours) to preserve design performance regardless of angle of incidence.

We start by considering the representative case of an asymmetric cloak designed with a typical coordinate transformation [31], and thus call this device the ‘standard cloak’. Figure 1(a) shows a reflective square object (dark gray) surrounded by a square cloaking shell (light gray region). The object is asymmetric with respect to arbitrary rotations around the geometrical center of the square, which makes this geometry very challenging from a cloaking point of view because an impinging wave sees different features of the object depending on the angle of incidence. For instance, a wave incident on one of the object’s corners will have very little backscattering towards the source. On the other hand, edge incidence will result in significant backscattering. In addition, the sharp object corners enhance the scattering signature of the object making it more difficult to cloak when employing various simplifying approximations to the ideal parameters prescribed by transformation optics.

Ideal cloaks designed using transformation optics produce devices whose performance is independent from the actual object to be cloaked. However, any approximation to the ideal design will generate devices for which the incident wave leaks into the cloaked region and interacts with the object inside. As a result, the boundary of the cloaked region needs to be taken into account in the optimization [32]. To prevent wave leaking into the cloaking region, in our designs we assume that the inner boundary of the cloaking shell is made of perfect electric conductor.

Simplifications of cloaking media through optimizations start with discretizing the geometry in a small number of homogeneous regions, usually concentrical layers, in which the ideal material parameters have relatively little variation or, at least, similar gradients. However, this step fails in the general case of asymmetric geometries because various material parameters can have very dissimilar gradients. This is the case of the square cloak. Figure 1(a) shows several curves along which the in-plane, ϵ_1 and ϵ_2, and out-of-plane, ϵ_z, components of the permittivity tensor are constant according to a simple transformation optics design that first introduced the square cloak [31]. We notice that $\epsilon_{1,2}$ and ϵ_z have isocontours of different shapes. Approximating the ideal cloaking material as a collection of homogeneous layers will cut through these isocontours regardless of the layer geometry. As a result the gradients of the material parameters inside each layer will, in general, be different from what transformation optics prescribes. The implications of that can be seen by analyzing the wave equation for inhomogeneous media,

$\nabla \times (\overline{\overline{\mu}}^{-1}\nabla \times \mathbf{E}) = \omega^2\overline{\overline{\epsilon}}\mathbf{E}$, written in index notation as

$$\tilde{\epsilon}_{ijk}\,\tilde{\epsilon}_{lmn}\left(\nu_{kl,j}E_{n,m} + \nu_{kl}E_{n,mj}\right) = \omega^2\epsilon_{ij}E_j, \qquad (1)$$

where $\tilde{\epsilon}_{ijk}$ is the Levi-Civita symbol, ν_{ij} are the components of the inverse permeability tensor ($\nu_{ij}\mu_{jk} = \delta_{ik}$ with δ_{ik} being the Kronecker delta), and $\cdot_{,j}$ represent derivative with respect to the coordinate whose index is j. If ξ_{ij} are the components of the inverse permittivity tensor ($\xi_{ij}\epsilon_{jk} = \delta_{ik}$) the above equation can be further written:

$$\tilde{\epsilon}_{ijk}\,\tilde{\epsilon}_{lmn}\xi_{pi}\left(\nu_{kl,j}E_{n,m} + \nu_{kl}E_{n,mj}\right) = \omega^2E_p. \qquad (2)$$

This equation is valid in any coordinate system. We express it in the principal axes characterized by coordinates $(1, 2, z)$ in which the material tensors are diagonal:

$$\sum_{\bar{k}}\tilde{\epsilon}_{\bar{p}j\bar{k}}\,\tilde{\epsilon}_{\bar{k}mn}\xi_{\bar{p}}\left(\nu_{\bar{k},j}E_{n,m} + \nu_{\bar{k}}E_{n,mj}\right) = \omega^2E_{\bar{p}}, \qquad (3)$$

where the indexes assume values from the set $\{1, 2, z\}$ and the bar over an index means that the index does not follow the normal summation rules used in index notation. For these indexes the summation is explicitly specified. Since the material parameter tensors are diagonal, it follows that $\xi_{\bar{p}} = \epsilon_{\bar{p}}^{-1}$ and $\nu_{\bar{k}} = \mu_{\bar{k}}^{-1}$ in the above equation. Consequently, the wave equation becomes

$$\sum_{\bar{k}}\tilde{\epsilon}_{\bar{p}j\bar{k}}\,\tilde{\epsilon}_{\bar{k}mn}\frac{1}{\epsilon_{\bar{p}}}\left(-\frac{\mu_{\bar{k},j}}{\mu_{\bar{k}}^2}E_{n,m} + \frac{1}{\mu_{\bar{k}}}E_{n,mj}\right) = \omega^2E_{\bar{p}}, \qquad (4)$$

in which we recognize the components of the gradient $\nabla \ln \mu_{\bar{k}}$:

$$\sum_{\bar{k}}\tilde{\epsilon}_{\bar{p}j\bar{k}}\,\tilde{\epsilon}_{\bar{k}mn}(\epsilon_{\bar{p}}\mu_{\bar{k}})^{-1}\left[-\left(\nabla \ln \mu_{\bar{k}}\right)_j E_{n,m} + E_{n,mj}\right] = \omega^2E_{\bar{p}}. \qquad (5)$$

Transformation optics gives the material parameters that satisfy the wave equation for any spatial distribution of the electromagnetic field. When we apply optimization techniques to simplify the transformation media, we simplify the geometry of the design (i.e. use a finite number of homogeneous layers) and perturb the material parameters inside these layers to account for the geometry change. Does this approach work in the general case? To answer this question we note that the wave equation can be written as a weighted sum of the field dependent terms

$$F_{jkmn} = -\left(\nabla \ln \mu_k\right)_j E_{n,m} + E_{n,mj}, \qquad (6)$$

where the index $\bar{k}$ in equation (5) was replaced in the above expression by k because dropping the upper bar does not interfere with the summation rules of index notation.

We can control the values of these terms to some degree using the parameter μ_k. However, the gradient direction $\nabla \ln \mu_k$ is set by the layer geometry and is not controlled by the optimization process. Not being able to match the gradient directions prescribed by transformation optics when the material isocontours are dissimilar means that we are not be able to shape F_{jkmn} for every field distribution E_n according to what transformation optics requires, which is bound to decrease performance.

This can be seen when we increase the number of homogeneous layers to improve performance. In the limit of infinite number of layers we do not recover the ideal transformation optics medium if the material parameter tensor components have different isocontours.

3. Optimized standard cloak

We quantify next this decrease in performance for the optimized standard square cloak. We employ a typical optimization technique in which we choose to approximate the cloak as four homogeneous layers of same thickness, as shown in figures 2(a) and (b). The material parameters in each layer are varied iteratively using the sequential quadratic programming (SQP) method [33] to minimize the relative scattered energy. This optimization method was chosen because it is widely used in similar types of problems and is already implemented in many commercial software packages. The optimization was performed for an electromagnetic wave incident on the object edge and coming from a point source situated seven wavelengths away from the object (see figure 2(a)). We call this angle of incidence 0°.

The electromagnetic field propagating through the cloak is obtained in numerical simulations performed using Comsol Multiphysics, a full-wave finite element method (FEM) solver of Maxwell's equations, and the optimization algorithm was implemented in Matlab. The mesh employed by the FEM solver was created so that there are at least ten mesh elements per wavelength and at least three elements in each cloak layer. The optimization was constrained to provide solutions for which the relative permittivity and permeability have maximum values of 20. Consequently, the maximum mesh element size inside the cloaking region was set to account for these maximum values. The difference between the fields obtained using this mesh and a mesh twice finer is less than 4% which is sufficient for our purpose.

The scattered field is computed by subtracting the free-space field, E_0, simulated in the absence of the object from the field, E, obtained in the presence of the object. The relative scattered energy is computed as $\Psi = \left(\int |E - E_0|^2 \mathrm{d}\phi\right)/\left(\int |E_0|^2 \mathrm{d}\phi\right)$, where the integrals are taken along a circle of radius 9 wavelengths and centered in the middle of the object. The optimization algorithm minimizes Ψ, which essentially minimizes the total scattered energy propagating in the far-field.

The optimization algorithm results in a set of material parameters that provides a very good cloaking performance as illustrated in figure 2(a). The wave passes around the cloaked object with minimal scattering. However, the performance reduces significantly if the angle of incidence changes. Figure 2(b) shows the same cloak when the incident wave impinges at an angle of 33° with respect to the horizontal. In this case, the scattering increases dramatically and there is a significant shadow forming behind the object, as well as a

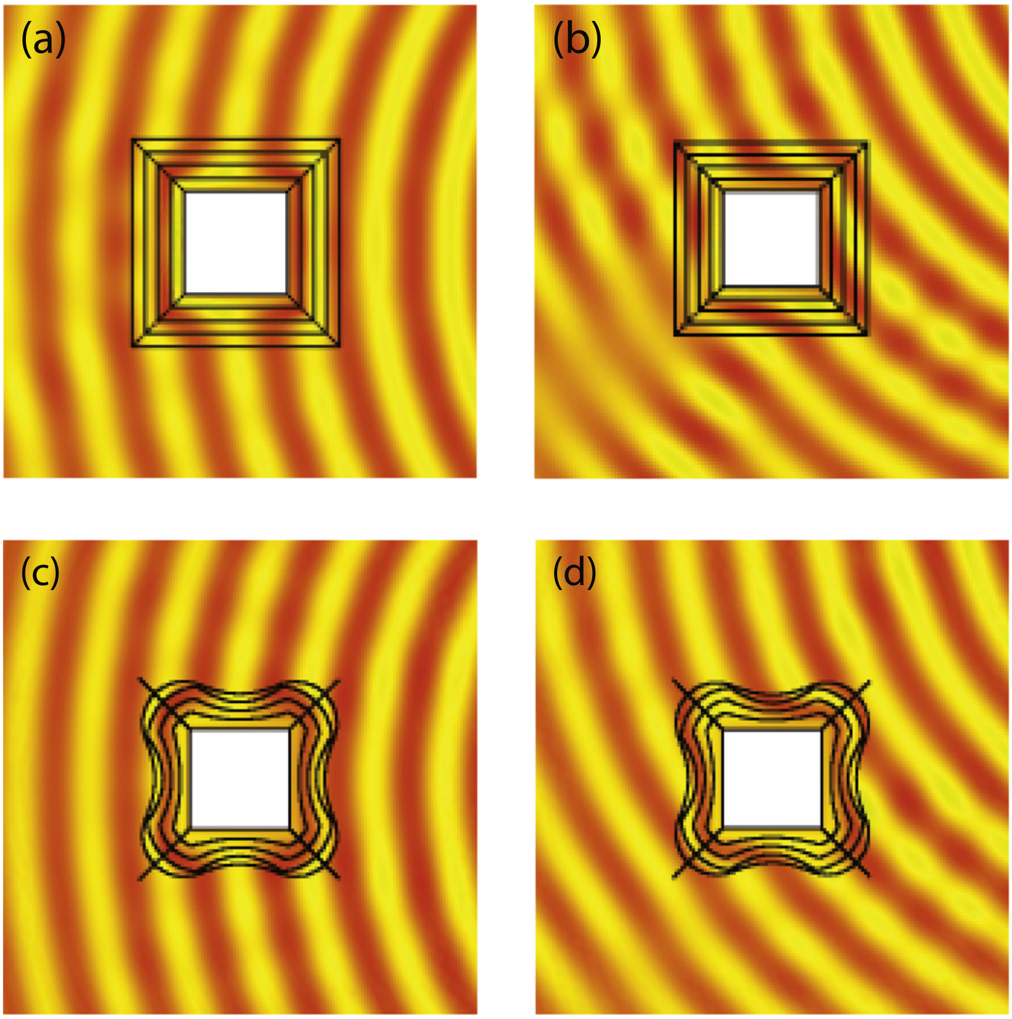

Figure 2. Fields simulated in the vicinity of the standard and isocontour cloaks for two angles of incidence. (a) Standard cloak, incident wave is perpendicular on the cloak edge; (b) standard cloak, incident wave is incident on the tip of the cloak; (c) isocontour cloak, incident wave is perpendicular on the cloak edge; (d) isocontour cloak, incident wave is incident on the tip of the cloak.

distinct interference pattern generated by the waves reflected from the sides of the cloaked object.

To quantify the scattering on angle dependence, we plot in figure 1(b) the energy scattered by the cloaked object relative to the energy scattered by the bare object versus angle of incidence, $E_s(\theta)$. The energy scattered by the uncloaked object is represented by the 100% line. The geometrical symmetry of the problem reduces the representative range of θ to the $0°$–$45°$ range. As expected, at the optimization angle (0°) the cloak performs very well ($E_s \approx 7\%$); however, E_s increases monotonically to 60% as the angle of incidence increases.

This is the direction dependence issue affecting most optimization-based asymmetric designs. A possible way to reduce this performance variability is to consider multiple directions of incidence in the optimization algorithm. However, it has been shown that this approach levels the cloak scattering cross-section at the expense of a significant performance reduction because, as the number of constraints in the optimization increases, the number of degrees of freedom stays the same [22].

4. Optimized isocontour cloak

As mentioned earlier, the performance variation with direction of incidence is in large part caused by not matching the gradients $\nabla\epsilon_1$, $\nabla\epsilon_2$, and $\nabla\epsilon_z$ with the quantities given by transformation optics. However, transformation optics gives the freedom of choosing the underlying transformation that generates the material parameters. It has been proven that for two-dimensional devices we can always find a transformation that results in designs in which all material parameter tensor components assume identical isocontours [34]. This translates to $0°$ angles between each pair of material parameter gradients. The key benefit is that, unlike the standard cloak analyzed in the previous section, these gradients can be exactly matched in a layered simplification of the ideal design provided the layers follow the isocontours. To illustrate this idea we optimize the cloak having material parameters of similar isocontours, which we call in the following the 'isocontour cloak'.

The details of designing isocontour cloaks in general, and a square cloak in particular, are presented in [34] and are briefly summarized here. Assuming that (x, y, z) are the coordinates describing the physical space of the actual transformation optics device and (u, v, w) are the coordinates of the virtual space of ideal electromagnetic field behavior (empty space in the case of cloaking), an isocontour cloak is obtained using the coordinate transformations

$$u = xf(x, y),\ v = yf(x, y),\ w = z. \tag{7}$$

The function $f(x, y)$ satisfies the eikonal equation

$$|\bar{r}||\nabla f| = k, \tag{8}$$

where $\bar{\mathbf{r}}$ is the position vector in the physical space and k is a constant chosen so that the contour $f(x, y) = 1$ is completely contained in the cloaking shell volume.

Once we have the isocontour square cloak that conforms to the initial volume of the standard cloak shown in figure 1(a), we reduce its complexity using the same approach as employed for the standard cloak. More specifically, we replace the cloaking shell media by four layers of homogeneous material parameters. The layers follow the isocontours of the material parameters and are illustrated in figure 3(a) using different colors. For the purpose of this demonstration the layers thicknesses were chosen such that they assume the same value at the middle of each object edge.

We find ϵ_1, ϵ_2, and ϵ_z inside each layers using the same SQP optimization algorithm employed for the standard optimized cloak. The iterative process starts with the initial permittivity and permeability inside each layer chosen to be the transformation optics ideal values in the middle of the layer. At the end of the iteration SQP finds the material parameters that minimize the scattered energy as an electromagnetic wave is incident at an angle of 0° with respect to the horizontal. We obtain good cloaking performance not only for waves incident at the optimization angle (figure 2(c)) but other angles of incidence as well (figure 2(d)). A comparison between figures 2(b) and (d) reveals that the isocontour cloak behaves significantly better than the optimized standard cloak.

5. Results and discussion

A comparison between the standard and isocontour optimized cloaks (see figure 3(b)) shows that the energy in the scattered field decreases by ≈25% for the latter when the angle of incidence is 0°. However, the variation of the scattered energy with angle of incidence remains significant. This happens because for the particular angle of incidence chosen in the optimization, $\theta_{\mathtt{opt}} = 0°$, (called 'optimization angle') the probing wave does not necessarily couple to all the modes of the structure, and consequently the minimization algorithm overlooks some of these modes. This issue is solved by either including $\theta_{\mathtt{opt}}$ in the optimization process itself or using multiple directions of incidence in the optimization procedure.

We illustrate the first method here in a brute force manner. To this end, we varied the optimization angle in steps of 11°, and for each angle we reapplied the optimization procedure. Figures 3(b)–(f) show the relative scattered energy obtained for the optimized standard and isocontour cloaks.

The standard cloak shows the same features regardless of optimization angle. Namely, the performance is good at the optimization angle but becomes significantly worse as we move away from it. The extreme case is obtained for $\theta_{\mathtt{opt}} = 45°$. When the incident wave impinges on the cloaked object under $\theta_{\mathtt{opt}}$, the cloak behaves very well and reduces the scattered energy to 5% of the bare object. However, at 12° incidence the device scatters even more than the uncloaked object.

The isocontour optimized cloaked does not suffer from this issue. The cloak reduces the scattered energy by 80% for all $\alpha_{\mathtt{opt}} > 10°$. In particular, $\alpha_{\mathtt{opt}} = 22°$ reduces the scattered energy to below 10% of that of the bare object as illustrated in figure 3(d). The material parameters inside the four layers are given in table 1. Even though they are highly anisotropic as usually encountered in transformation media, their values are within the bounds of what can be obtained with current technologies.

The omnidirectional performance is not obtained by chance, but it is an inherent feature of the design. Suppose there is an object for which its optimized isocontour cloaking shell is unidirectional. By increasing the number of layers in the cloak we increase the number of degrees of freedom that allow us to control an increasing number of eigen-modes of the structure. This leads to a smaller scattereing cross-section while preserving the original omnidirectionality because in the limit in which the number of layers approaches infinity we obtain the ideal transformation optics design. This contrasts with the standard optimized cloak for which increasing the number of layers does not generally result in the ideal transformation optics design and consequently there is no guarantee that omnidirectionality can even be achieved.

The optimization method proposed here achieves a significant reduction of the cloaking media complexity at the expense of bandwidth reduction. Figure 4 shows the efficiency of the isocontour cloak at frequencies situated ±33% away from the optimization frequency. Below the design frequency the cloak retains its performance in part because the electromagnetic size of the cloak reduces with frequency. On the other hand, above the design frequency the efficiency decreases but the scattered energy stays approximately 50% below that of the bare object for all angles of incidence. The main reason for this is that at this size, four cloaking layers do not provide enough degrees of freedom to cancel all the significant eigenmodes of the object. Nevertheless, the omnidirectionality is preserved even at a frequency 33% away from the design frequency.

6. Conclusions

We have described an optimization method that significantly reduces the complexity of transformation optics designs of arbitrary shape while maintaining an excellent performance. Unlike other similar optimization strategies reported so far, our method does not compromise on omnidirectionality.

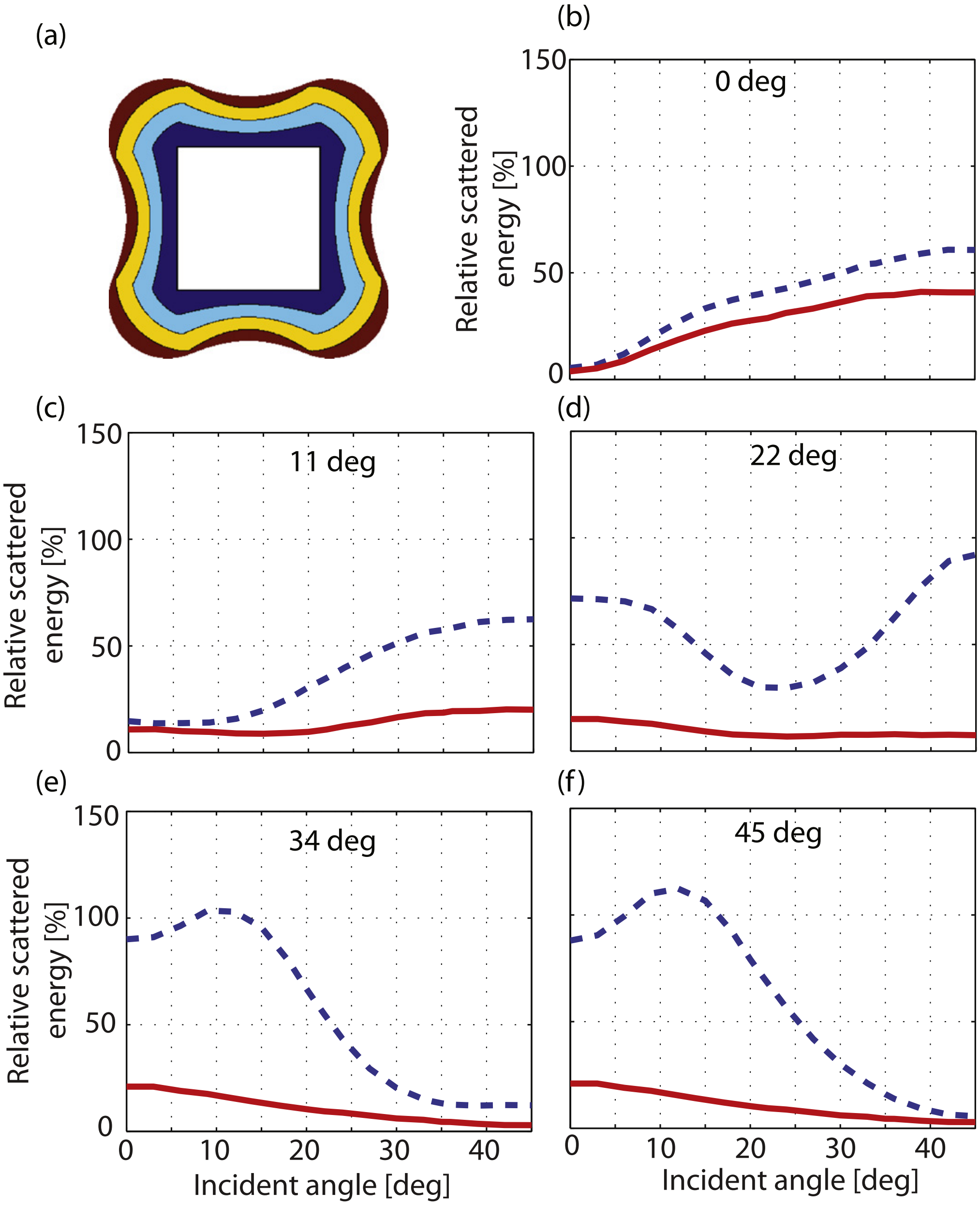

Figure 3. Comparison between optimized standard cloak and isocontour cloak. (a) The geometry of the isocontour cloak composed of four homogeneous layers; (b)–(f) relative scattered energy computed for the standard optimized cloak (dashed) and optimized isocontour cloak (solid) for five angle of incidence: 0°, 11°, 22°, 34°, and 45°, respectively. The 100% line corresponds to energy scattered by the bare object.

Table 1. Optimized material parameters inside the optimized isocontour cloak. Layer 1 is the inner most layer.

Layer	ϵ_1	ϵ_2	ϵ_z
1	11.83	0.029	0.31
2	4.73	0.102	1.32
3	3.68	0.198	2.15
4	3.96	0.306	2.55

Regardless of geometrical constraints and asymmetries, the final design performs well while illuminated from any direction. We showed that the design should necessarily be based on a class of coordinate transformations that result in material parameters of same isocontours. The ideal transformation optics material is discretized in a finite number of homogeneous layers, and the parameters of each layer are optimized in order to maximize a specified performance metric.

Asymmetric geometries typically have asymmetric eigenmodes. The optimization needs to take into account all these modes to preserve omnidirectionality. Consequently, it is important that the angle of incidence used during the optimization process be part of the optimization procedure itself as an extra degree of freedom. This can be done in a number of ways. For the purpose of this paper and to better illustrate the issue, we employed a brute force approach in which the optimization was repeated for various angles of incidence and

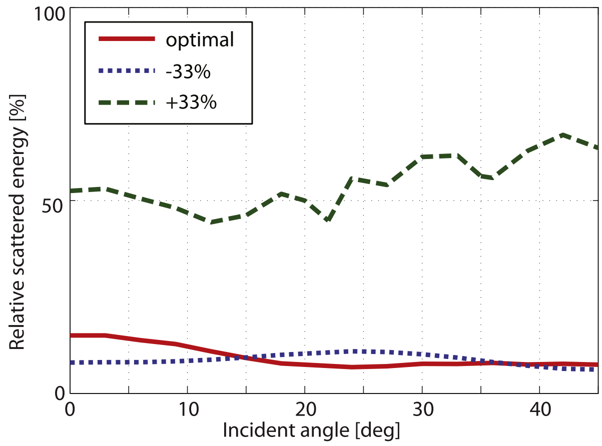

Figure 4. Relative scattered power at the optimal frequency (solid), 33% lower (dotted), and 33% higher (dashed) than the optimal frequency.

showed that the incident angle used in the optimization played an important role in the final performance of the cloak.

The price paid by the reduction in design complexity is a decrease in bandwidth. However, for most designs this decrease is outweighed by the narrow band of the materials implementing the design. The method was illustrated in the design of a cloaking shell covering a square object and its performance was contrasted with other optimization methods.

The approach described here opens the door for optimization based designs applied to devices of arbitrary shape.

Acknowledgments

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