Design and demonstration of broadband thin planar diffractive acoustic lenses

Wenqi Wang, Yangbo Xie, Adam Konneker, Bogdan-Ioan Popa, and Steven A. Cummer

Department of Electrical and Computer Engineering, Duke University, Durham, North Carolina 27708, USA

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We present here two diffractive acoustic lenses with subwavelength thickness, planar profile, and broad operation bandwidth. Tapered labyrinthine unit cells with their inherently broadband effective material properties are exploited in our design. Both the measured and the simulated results are showcased to demonstrate the lensing effect over more than 40% of the central frequency. The focusing of a propagating Gaussian modulated sinusoidal pulse is also demonstrated. This work paves the way for designing diffractive acoustic lenses and more generalized phase engineering diffractive elements with labyrinthine acoustic metamaterials.

Acoustic lenses manipulate wave propagation and control energy transfer through modulating the phase and amplitude of the incident wave. They are widely applied in acoustic imaging, therapeutic ultrasound, and sonar systems. Traditional acoustic lenses are often constructed with homogeneous materials chosen from a limited material library and face challenges in increasing throughput and reducing device footprints. During the past few years, the emergence of acoustic metamaterials, together with the related design tools, such as transformation acoustics and computational optimization algorithms, provide alternative solutions to acoustic lenses designer.

Diffractive lenses operate by generating interference and diffraction patterns through phase engineering. Their mechanism is different from gradient-index (GRIN) lenses, which rely on the gradually varying refractive index to accumulate the phase delay needed. Diffractive lenses thus are usually advantageous with their thinness, high transmission, and larger degree of design freedom. However, achieving a broad range of phase modulation while retaining high transmission puts demanding constraints on the wave modulating elements. Another challenge lies in the operating bandwidths as the properties of diffractive elements are usually sensitive to frequency, which can limit the lensing effect to a narrow frequency range.

With both challenges in mind, we chose tapered labyrinthine acoustic metamaterials for designing broadband diffractive lenses. The space-coiling geometry of this type of unit cells renders them inherently broadband for lenses. The effective refractive index approximates the ratio between the inner meandering path and the unit cell diameter and thus the phase change in the unit cell is proportional to the operating frequency, which will result in a broadband diffractive device. Here, we adopt a frequency normalized phase modulation $\Delta \phi(x) / \omega$ in the design so that the broadband performance of both the unit cells and the lenses can be quantified with this parameter. Figure 1(a) demonstrates the performance of a typical tapered labyrinthine unit cell. Its frequency normalized phase modulation varies with only a small slope over a frequency span of 1400 Hz around the central frequency of 3000 Hz and the amplitude of its transmission coefficient $|S_{21}|$ remains a relatively constant value around 70%.

Two lenses are demonstrated here. The first lens can transform cylindrical radiation to a plane wave, or vice versa (referred to later as point-to-plane lens, whose ray trajectory is shown in Figure 1(b)). The second lens can transform the radiation from a point source to an image spot (referred to later as point-to-point lens, whose ray trajectory is also shown in Figure 1(b)). Both lenses have the same thickness, which ranges approximately from 0.5 to 0.75 wavelengths across the frequency range of interest (2400 Hz–3600 Hz). Simulated and measured results are compared, demonstrating that both...
lenses proposed exhibit excellent lensing effect as desired, high energy throughput, and bandwidth of more than 10% of the central frequencies.

Both lenses presented here are composed of a series of carefully designed and optimized labyrinthine unit cells. Labyrinthine acoustic metamaterials\textsuperscript{17–20} acquire their properties mostly from their inner meandering geometries, thus avoiding the drawbacks of narrow bandwidth and high absorption which are typical of most of the locally resonant metamaterials (such as those based on Helmholtz resonators or elastic membranes). In the lens design process, analytical estimates are formulated to predict the phase modulation needed for each type of the lens. Then finite element analysis (FEA) is used for designing the unit cells suited for the lenses and simulating the lensing performance. Experimental prototypes were fabricated using fused filament fabrication (FFF) 3D printing and the field mapping measurements are performed in our lab-made two-dimensional parallel plate acoustic scanning measurement system.\textsuperscript{16,21}

A point-to-plane lens capable of converting a cylindrical radiation to a plane wave is required to possess a frequency normalized phase modulation profile (the phase change between the exiting and the entrance interfaces of the lens) along the interface

\[
\Delta \phi(x) \omega_0 \omega = \sqrt{\left(\frac{f}{2}\right)^2 + f^2} - \sqrt{x^2 + f^2} \frac{c}{\omega_0},
\]

where \(f\) is the focal length, \(L\) is the width of the lens, \(x\) is the distance from the center to a point along the interface, \(c\) is the speed of sound in air. A lens composed of 48 labyrinthine unit cells approximating the phase modulation profile given by Eq. (1) was designed and fabricated. Figure 2(a) shows the simulation results at 3000 Hz, where the conversion from cylindrical radiation to a plane wave is clearly seen. Figures 2(b)–2(d) shows the measurement results at 2800 Hz, 3000 Hz, and 3200 Hz, all of which agree well with the simulation. The field patterns at these three different frequencies indicate that the lens can operate over a bandwidth of more than 400 Hz, which is over 10% of the 3000 Hz central frequency. Such operational bandwidth is broader than most of the resonant metamaterial-based modulation devices.

A point-to-point lens projecting cylindrical radiation to a focal spot. (a) Simulated pressure field pattern (real value) at 3000 Hz. (b) Simulated pressure amplitude pattern at 3000 Hz. (c) Measured field patterns at 2800 Hz, 3000 Hz, and 3200 Hz (left: amplitude distribution, right: real value of the pressure field). (d) Measured amplitude profiles along \(y = 0.6\) m at 2800 Hz, 3000 Hz, and 3200 Hz, compared with that in the free space.
A point-to-point lens capable of transforming a cylindrical radiation to a focal spot must have the following phase modulation profile:

$$\Delta \phi(x) = \frac{\omega_0}{\omega} \left( \sqrt{\left(\frac{L}{\lambda}\right)^2 + f_1^2} + \sqrt{\left(\frac{L}{\lambda}\right)^2 + f_2^2} - \sqrt{\frac{x^2}{c^2} + f_1^2} - \sqrt{\frac{x^2}{c^2} + f_2^2} \right),$$

where $$f_1$$ is the length from the point source to the lens, $$f_2$$ is the length from the lens to the focal spot, and other variables have the same meaning as defined above. The lens is composed of 48 labyrinthine unit cells with phase modulation approximating the lens design Eq. (2).

The simulated field patterns are plotted in Figures 3(a) and 3(b), where a cigar-shaped focal spot can be clearly identified. The measured field patterns are plotted in Figure 3(c), which match the simulated results well. Comparing the field patterns at 2800 Hz, 3000 Hz, and 3200 Hz, we can conclude that the operating bandwidth of the point-to-point lens is more than 400 Hz (note that we expect 1400 Hz bandwidth from the cell design). Figure 3(d) plots the measured amplitude along $$y = 0.6 \text{m}$$ for each of these three frequencies, whose energy density at the focal point is magnified by a factor of approximately 15 compared to that of the free space propagation.

The time evolution of a broadband pulse after being modulated by the point-to-point lens is also studied here. A Gaussian modulated sinusoidal pulse with a central frequency of 3000 Hz and a full-width-at-half-maximum (FWHM) bandwidth of 1365 Hz was chosen as the input. The field pattern as a function of time was measured and three representative frames are shown in Figure 4. A frame at the initial stage of the formation of the focused pulse after the modulation of the lens is first plotted in Figure 4(a), and its time is defined as a reference point. Two other frames at $$t_0 + 566.9 \mu s$$ and $$t_0 + 1133.8 \mu s$$ are shown, respectively, in Figures 4(b) and 4(c). The pulse remained compact along its propagation along the approximately 40 cm distance in free space. The generation of such high power density acoustic pulse, similar to the recently proposed “sound bullet”, might be useful for various non-destructive therapeutic treatments.

The two lenses demonstrated here serve as examples of possible wavefront shaping devices that can be designed with the phase modulating labyrinthine metamaterials. More generalized holographic diffractive elements adopt similar phase engineering and can be designed in similar approaches. To further improve the performance of the lenses presented here, optimization algorithms such as genetic algorithm can be utilized for the purpose of reducing undesired scattering, increasing throughput, etc.

In conclusion, we present in this paper the design and tests of two broadband subwavelength-thin planar diffractive acoustic lenses, both of which are constructed with tapered labyrinthine unit cells. The measured lensing effects are in excellent agreement with the simulations, and broadband measurements show an operating bandwidth of more than 40% of the central frequency. Our work provides a labyrinthine metamaterial based design approach for low profile acoustic lenses and might be useful for applications such as therapeutic ultrasound, acoustic imaging, and sonar systems.

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