# Conformal array design with transformation electromagnetics

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We apply the theory of transformation electromagnetics to source arrays and show that a complex conformal antenna array can be made to behave like a geometrically different array when surrounded with a properly designed transformation electromagnetics medium. Numerical simulations are presented to show how a nonuniform circular array can be made to radiate and receive as a uniformly spaced linear array. In this way, transformation electromagnetics provides a method by which all of the advantages of simple arrays in array processing, such as beamforming, can be retained in an array whose elements are constrained to a complex geometry. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3158614]

Pendry *et al.*[1] and Leonhardt[2] first described the concept of transformation optics in which coordinate transformations of electromagnetic fields can be physically realized with a complex medium whose parameters are defined explicitly by the transformation. That concept has been extended to show how electromagnetic sources, namely currents and charges, can be manipulated using the same coordinate transformation approach. Luo *et al.*[3] conceptually demonstrated how a coordinate transformation medium could be used to make one current distribution radiate like an entirely different one. Conformal antennas are one potential application of such an approach, in which currents may be constrained to a given surface but one wishes to have these currents radiate as if they were in a different location or had a different shape.[3] Kundtz *et al.*[4] implemented numerical simulations that confirmed that complex current distributions can be made to radiate like simple ones when surrounded by a properly designed transformation optics medium.

Here, we demonstrate how the source transformation approach can be applied to manipulate arrays of sources in a way that is both practical and straightforward to test experimentally. Specifically, we show through numerical simulations how a conformal source array can be surrounded by a transformation optics medium so that it behaves as if it were a linear array of uniformly spaced elements. If Ref. 3 focused on how individual sources change shape as a result of the transformation, we focus here on the array behavior of such design. This includes phasing the conformal elements as if they were a uniform linear array. This has consequences for array processing techniques, such as adaptive beamforming,[5] in which computations are fast and efficient for a uniform linear array but significantly more complicated for more complex array geometries. Thus, with this approach, array elements can be placed in constrained locations, such as along the outside of a closed volume, but the array behaves in every sense and can be processed as if it were a simple uniform linear array.

We illustrate our method through the following example set for simplicity in a two dimensional space. Consider the phased array represented by the black dots in Fig. 1 and positioned in free space. We call it the "reference array," and it is made of $N=10$ equally spaced line sources oriented in the $z$ direction and spanning a distance $d=2\lambda$, where $\lambda$ is the wavelength at which the phased array is designed to operate. For these dimensions, the coordinates of the line sources are given by $x_n=0$ and $y_n=-d/2+d(n-1)/(N-1)$ where $n=1\ldots N$ is the source index. A constant phase shift of 40° between adjacent sources is applied to form a beam at 30° with respect to the direction perpendicular on the array. This implies that the current density through the $n$th element is given by $\mathbf{J}_n=\hat{\mathbf{z}}\exp\{j[-\pi+2\pi(n-1)/(N-1)]\}\delta(x)\delta(y-y_n)$. Figure 2(a) shows the fields produced by this reference array in a numerical simulation performed with COMSOL MULTIPHYSICS and confirms the desired behavior.

Now, suppose we want to design a different phased array that behaves identically to the reference array but whose elements are positioned along the surface of a cylinder of radius $R_1$, which represents the exterior of an object on which we wish to place the array. We further assume that whatever material shell is needed around the elements is at least $d/2$ in radius. In this way, the overall antenna defined by the elements placed at $r=R_1$ and the material shell extending from

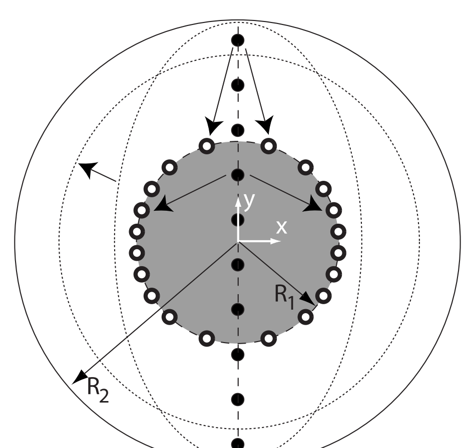


FIG. 1. The original phased array (black dots) designed to form a beam at 30° in free space is modified to perform identically in the presence of the shaded cylinder. The modified design (black and white dots) is obtained using the coordinate transformation approach. Our transformation maps ellipses into circles: the dashed line maps into the dashed circle of radius $R_1$, the dotted ellipse maps into the dotted circle, and the solid circle of radius $R_2$ maps into itself.

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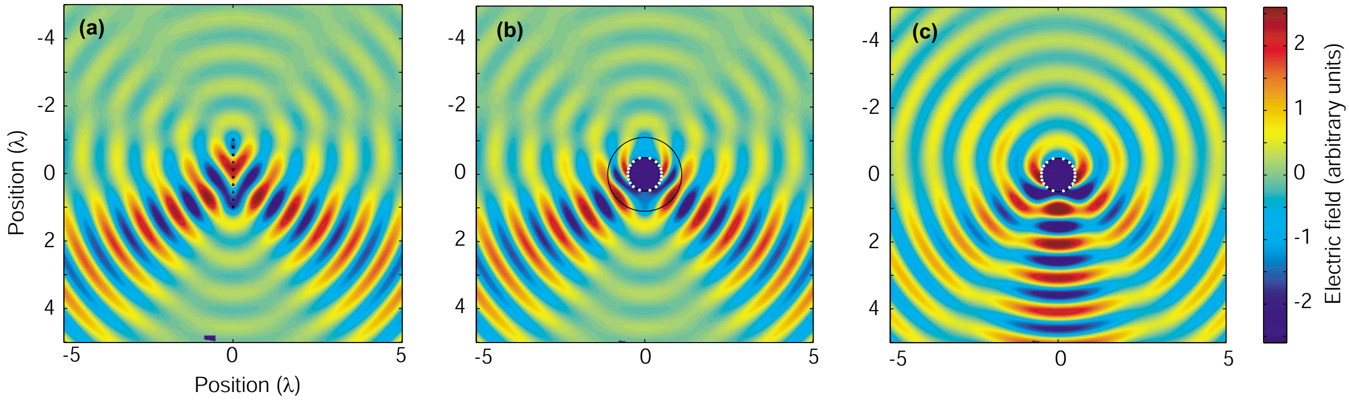


FIG. 2. (Color online) (a) Electric field produced by a phased array made of ten line sources spanning a length of 2λ, designed to radiate in free space at an angle of 30°; (b) Transformed phased array modified under geometrical constraints. The shell between the two cylinders is made of an inhomogeneous material designed using coordinate transformations; (c) Electric field in the absence of the shell.

$R_1 < r < R_2$, as illustrated in Fig. 1, is not smaller than the reference array whose behavior we are trying to duplicate.

This problem can prove difficult to solve using conventional methods for complicated geometries but fits naturally in the context of the coordinate transformation theory. This method requires us to choose a coordinate transformation that maps the space of the reference phased array (we call it the original or virtual space) to the space of the modified array built around the cylinder of radius $R_1$ (we call it the transformed or real space). By convention, quantities expressed in the transformed space will be followed by a prime sign. A transformation that leads to a design that satisfies the constraints listed above is given by

$$x = a(r')\cos(\phi'); \quad y = b(r')\sin(\phi'); \quad z = z', \tag{1}$$

where $r' = \sqrt{x'^2 + y'^2}, \cos(\phi') = x'/r'$, $\sin(\phi') = y'/r'$, and

$$a(r') = \frac{R_2}{R_2 - R_1}(r' - R_1),$$

$$b(r') = \frac{R2 - d/2 - \delta d}{R_2 - R_1}(r' - R_1) + d/2 + \delta d. \tag{2}$$

For $r' > R_2$, the original and transformed spaces are identical. The transformation maps ellipses in the original space to circles in the transformed space (see Fig. 1). The small $\delta d$ quantity assures that no sources are located on the two ends of the segment where the shell material parameters will prove singular. As a result, each source in the original space corresponds to two different sources in the real space. The current density vector of each mapped source, $J'$, is given in terms of the current density of the original source by $J' = AJ|J|/|AJ|$, where $A$ is the Jacobian matrix[3,6]

$$A = \frac{\partial(x',y',z')}{\partial(x,y,z)} = \left[\frac{\partial(x,y,z)}{\partial(x',y',z')}\right]^{-1}. \tag{3}$$

In order to shield the region $r' < R_1$, which is undefined in our coordinate transformation, from the phased array radiation we set its boundary to be a perfect magnetic conductor, and consequently taking into account the image theorem, we need to halve $J'$. In our case, the sources are oriented in the $z$ direction therefore the expressions for current densities simplify to $J'_{\pm n} = J_n/2$ for every $n$ between 1 and $N$. The $\pm n$ index symbolizes that each source in the original space is mapped to two sources in the transformed space. The above equation tells us that we can change the angle of the beam radiated by the modified phased array simply by changing the phase difference between adjacent line sources in the same way we would do for the uniform and linear reference array.

According to the coordinate transformation theory, the deformation of the original space determines a change of the relative permittivity and permeability tensors in the transformed one inside the shell delimited by the cylinders of radii $R_1$ and $R_2$.[7] We note that, similarly, coatings based on transformation optics techniques have been used to improve antenna designs in varied ways.[8,9] We have

$$\bar{\bar{\epsilon}}' = \bar{\bar{\mu}}' = (\det A)^{-1} A A^T, \tag{4}$$

where $A^T$ is the transpose of the Jacobian matrix. Figure 3 shows the relevant required profiles inside the shell. We assumed $R_1 = 0.5\lambda$, $R_2 = 1.1\lambda$, and $\delta d = 0.1\lambda$.

The shell is made of anisotropic and inhomogeneous materials but the values of the $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ tensor components are well within the values that have already been achieved at radio frequencies using metamaterials. Very localized singularities in $\mu_{xx}$ are present at the two points where $r = 0.5$ and $x = 0$. As shown below, these are not essential to the operation of the array and probably do not have to be included in any

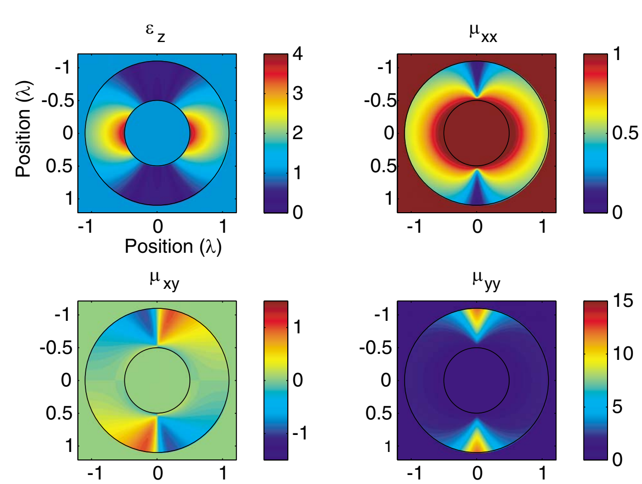


FIG. 3. (Color online) Material parameters inside the shell. For the transverse electric wave radiated by the phased array only the $\epsilon_{zz}$, $\mu_{xx}$, $\mu_{xy} = \mu_{yx}$, and $\mu_{yy}$ are relevant. Very localized singularities in $\mu_{xx}$ at $r = 0.5$ and $x = 0$ not shown on the plot.

physical realization of this shell. As the size of the reference array is reduced, the required parameter values become more extreme. In past related work,[3,4] the size of the reference antenna has always been less than $R_1$, and it is worth emphasizing that more easily realizable shell parameters arise when the reference antenna has a size between $R_1$ and $R_2$.

Notice that the $\mu_{xy}$ component being nonzero is a consequence of our choice of cartesian coordinate system $x'y'z'$. The permeability tensor can be locally diagonalized (i.e., off-diagonal terms set to zero) by rotating the coordinate system around the $z'$ axis. The practical implication of this observation is that the permeability profiles can be implemented using structures similar to the split ring resonator (SRR).[10] The orientation of the SRRs at each point inside the shell is dictated by the orientation of the $x'y'z'$ coordinate system that diagonalizes the $\bar{\bar{\mu}}$ tensor, as used experimentally in Ref. 11.

Figure 2(b) shows the simulated fields radiated by the resulting cylindrical array surrounded by a transformation medium shell and phased with 40° difference between adjacent elements, exactly as for the reference array. The mesh used in the simulation has over 70 000 elements in order to approximate well the continuous profiles given by Eq. (4). A comparison between these fields and those radiated by the reference array shows them to be virtually identical, which confirms the validity of our design.

The inhomogeneous shell around the shaded cylinder in Fig. 1 is essential for the design in order to cancel the interaction between the phased array and the cylindrical region and also to account for the curved shape of the array. Without the shell, the radiation pattern changes significantly, as can be seen in Fig. 2(c). Far-field simulations presented in Fig. 4 show the main lobe of the radiated power in the latter case at a different angle (90°) than the desired 30°. Figure 4 also includes the far-field pattern when the singularities in the shell parameters are removed and only the values shown in Fig. 3 are used in the simulation. The deviation from the ideal pattern is very small, indicating that these singularities are not critical to the operation of the transformed array.

To conclude, we have shown how conformal antenna arrays can be designed using transformation electromagnetics theory to yield an array that behaves identically to a uniform linear array, with all the benefits for array processing that come with that but is actually composed of array elements placed nonuniformly over a complex, nonlinear geometry. We illustrate this method with detailed numerical simulations for the particular case of a circular array designed to behave as a linear array. In this case, the resulting parameters of the needed transformation electromagnetics material shell are complex but within the range that can be fabricated using existing rf metamaterials approaches. Although a simple case was treated here, the approach can, in principle, be applied to arbitrarily complex conformal arrays.

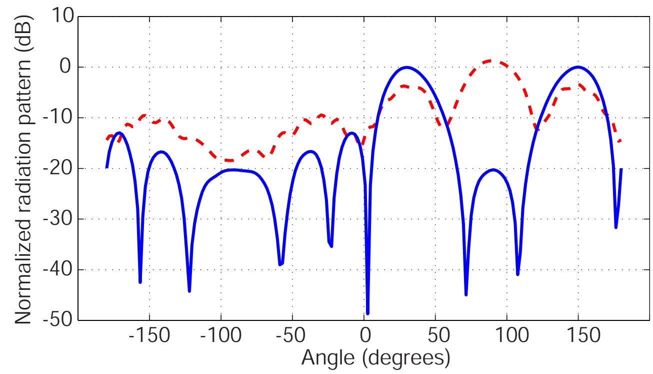


FIG. 4. (Color online) Far-field radiation patterns of the original phased array and array modified to satisfy geometrical constraints are identical (solid line). Without the anisotropic shell the radiation pattern changes significantly (dotted line). The main lobe is at 90° instead of the desired 30° (0° corresponds to the direction perpendicular to the reference array).

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