Cloaking with optimized homogeneous anisotropic layers

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(Received 7 November 2008; published 5 February 2009)

We present a method to reduce the scattering from arbitrary objects by surrounding them with shells composed of several layers of homogeneous anisotropic materials. An optimization procedure is used to find the material parameters for each layer, the starting point of which is a discretized approximation of a coordinate transformation cloaking shell. We show that an optimized, three-layer shell can reduce the maximum scattering of an object by as much as 15 dB more than a 100-layer realization of a coordinate transformation cloaking shell. Moreover, using an optimization procedure can yield high-performance cloaking shell solutions that also meet external constraints, such as the maximum value of permittivity or permeability. This design approach can substantially simplify the fabrication of moderate-size cloaking shells.

DOI: 10.1103/PhysRevA.79.023806 PACS number(s): 42.25.Fx, 41.20.Jb, 42.25.Gy

Significant research has recently focused on developing new methods to minimize the interaction between given objects and electromagnetic waves. Pendry *et al.* [1] showed that carefully designed inhomogeneous and anisotropic shells can prevent electromagnetic radiation from penetrating inside them and, more importantly, cancel the scattering off these shells, making them and their interior effectively transparent to electromagnetic waves. What is appealing about this coordinate transformation approach is its generality: it can be used to conceal objects of any size and shape.

Even though numerical simulations [2] and further theoretical analysis [3] confirmed the efficacy of this method, experimental demonstrations have proven more challenging. One effort [4] involved a cylindrical shell surrounding a metal cylinder and demonstrated the basic physics of such structures, namely, that waves can be steered around the structure. However, in this work approximations were made [2] that significantly reduced the scatter reduction performance of the shell. Other work has derived different approximations to ideal cloaking shell parameters that also sacrifice performance for fabrication simplicity [5–7].

The difficulty of fabricating cloaking shells specified by coordinate transformation theory stems from the requirements on the material that composes it: the shell has to be anisotropic with its permittivity and permeability varying continuously with position over a broad range of values. Its physical implementation will always require some form of discretization of these continuous profiles. For example, Schurig *et al.* [4] used a ten-layer stepwise approximation of the ideal parameters in their experiment.

Beginning with the notion that anisotropy appears to be the most important ingredient of cloaking shells that are not electrically small, one might ask whether cloaking shells composed of anisotropic layers can be designed through another approach. Here we show that scatter reducing shells composed of a relatively small number of homogeneous layers can be designed through an optimization procedure that uses a coordinate transformation shell as the initial condition. The performance of fewer than five optimized layers can equal or even exceed the performance of a 100-layer discrete approximation of the smoothly homogeneous shell designed through coordinate transformations.

For simplicity, we will focus on the two-dimensional cylindrical shell for electromagnetics, but the analysis presented here can be applied for other geometries and wave types as well, such as acoustics [8]. Figure 1 shows a perfect electric conductor (PEC) cylindrical object of radius a surrounded by a shell of outer radius b. It has been shown [2] that one set of relative material parameters that completely cancel the scattering from the structure is (in cylindrical coordinates),

$$\epsilon_r(r) = \mu_r(r) = \frac{r-a}{r}, \quad \epsilon_\phi(r) = \mu_\phi(r) = \frac{r}{r-a},$$

$$\epsilon_z(r) = \mu_z(r) = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}, \tag{1}$$

where z is the invariant direction, and r and ϕ are the radial and azimuthal coordinates, respectively.

We consider the TE (transverse electric) polarization for which only ϵ_z, μ_r, and μ_ϕ are relevant. We assume that the cloak can be assembled from M concentric layers of homo-

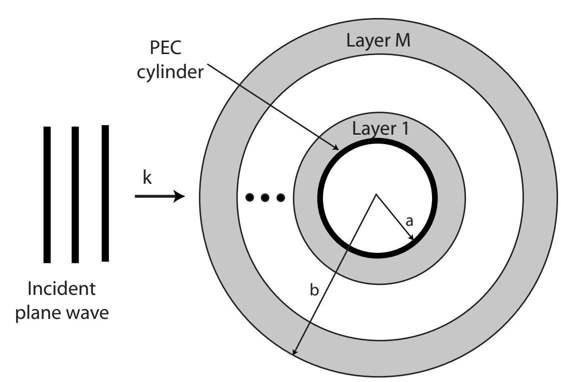

FIG. 1. Cylindrical perfect electric conductor surrounded by a multilayer shell and illuminated with a plane wave. The input and output radii of the shell are a and b, respectively.

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TABLE I. Optimized material parameters for the three-layer cloak.

	Initial guess			Optimized parameters		
Layer	ϵ_z	μ_r	μ_ϕ	ϵ_z	μ_r	μ_ϕ
1	1.60	0.021	47.15	3.32	0.006	47.21
2	4.61	0.061	16.38	6.06	0.039	16.88
3	7.40	0.098	10.23	7.99	0.10	10.63

geneous materials as shown in Fig. 1. Each layer is characterized by permittivity and permeability tensors that are constant with position inside the layer. Our goal is to find a set of parameters that minimizes the scattering off the cloak. One choice is to use the stepwise approximation of Eqs. (1) as in Ref. [4]. Using this approach and TE polarization, as was pointed out before [9,10], the boundary conditions at the inner interface of the cloak with the PEC object induce significant scattering. We therefore expect that other choices of material parameters may improve cloaking performance.

The $\exp(+j\omega t)$ time convention is assumed throughout. Consider a plane wave having the electric field $E_{\rm inc}=\hat{z}\exp(-jk_0 r\cos\phi)$ incident on the object and shell depicted in Fig. 1. The symmetry of the problem allows us to compute the fields inside and outside our structure analytically, by employing the procedure outlined in [11].

Thus, the incident plane wave can be expanded into a sum of Bessel functions of the first kind as

$$E_{\rm inc} = J_0(k_0 r) + 2\sum_{n=1}^{\infty} j^n J_n(k_0 r)\cos(n\phi) \tag{2}$$

while the scattered field in the $r>b$ region can be written in terms of Hankel functions of the second kind as

$$E_{\rm sc} = \sum_{n=0}^{\infty} A_n H_n^{(2)}(k_0 r)\cos(n\phi). \tag{3}$$

The fields in layer m inside the shell are given by

$$E_m = \sum_{n=0}^{\infty} [B_{mn} J_\nu(k_m r) + C_{mn} Y_\nu(k_m r)]\cos(n\phi), \tag{4}$$

where $m=\overline{1,M}$, Y_ν is the Bessel function of the second kind, $k_m=\omega\sqrt{\epsilon_{z,m}\mu_{\phi,m}}$, and $\nu=n\sqrt{\mu_{\phi,m}/\mu_{r,m}}$. The coefficients A_n, B_{mn}, and C_{mn} can be found by imposing the continuity of tangential E and H fields across the boundaries of each layer.

Once we know the fields inside and outside our structure, we can compute the figure of merit used throughout this paper: the radar cross section per unit length also known as scattering width (SW), which is defined as $\sigma(\phi)=2\pi R|E_{\rm sc}(\phi,R)/E_{\rm inc}|^2$, where R is the distance from the object where the far-field scattered field $E_{\rm sc}$ is evaluated. Since in the far-field region $E_{\rm sc}$ is inversely proportional to R, σ is independent of R as long as R is big enough.

The question is whether we can derive a set of material parameters for the layered shell that would reduce the scattering more than if we simply discretize the profiles given by Eqs. (1). The answer is affirmative and we outline the procedure for the general case of an M-layer shell, after which we apply the method to a specific example.

To maximally reduce object visibility, one should minimize $\max_\phi \sigma(\phi)$. However, since the forward scattering of an object that is not electrically small is usually largest [12], we solve the simpler problem of optimizing for the permittivity and permeability tensors that minimize the forward scattering (i.e., the shadow). As we will see next, this heuristic gives a strongly reduced SW in all directions. Mathematically, we want to find the ϵ and μ values that minimize the function $\sigma(\phi=0,\epsilon_z^{(1\cdots M)},\mu_r^{(1\cdots M)},\mu_\phi^{(1\cdots M)})$, where the superscripts signify that we have one set of material parameters for each one of the M layers. This is a classical optimization problem that can be solved with a variety of algorithms. In this paper, we use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, which is already implemented in software tools such as MATHEMATICA and MATLAB. The BFGS method provides a local minimum for f around a specified starting point, $X_0=(\epsilon_{z,0}^{(1\cdots M)},\mu_{r,0}^{(1\cdots M)},\mu_{\phi,0}^{(1\cdots M)})$, whose choice is very important for the success of this algorithm. Since it already gives good results, we choose X_0 to be the discretized version of Eqs. (1), namely, $\epsilon_{z,0}^{(i)}=\epsilon_z(r=[R_i+R_{i+1}]/2)$ (similar expressions for $\mu_{r,0}^{(i)}$ and $\mu_{\phi,0}^{(i)}$), where R_i and R_{i+1} are the inner and, respectively, outer boundaries of layer i, $R_1=a$, and $R_{M+1}=b$. The local minimum in $\sigma(\phi=0)$ gives the material parameters for each of the M layers.

We illustrate the procedure for an object that has the dimensions specified in [2], namely, $a=10$ cm, which makes it 1.33 wavelengths at the working frequency of 2 GHz. We choose to design a thin shell of 1.3 cm, i.e., $b=11.3$ cm. For this size, three layers provide enough degrees of freedom (nine in total, i.e., ϵ_z, μ_r, and μ_ϕ for each layer) to significantly reduce the scattering width of our object. Table I shows the starting point and material parameters found by the optimization algorithm, while Fig. 2 shows the scattering width normalized to the object diameter versus angle for our design (curve d) and, for comparison, for three other shells whose material parameter profiles are the three-, nine-, and 100-level staircase approximation of Eqs. (1).

This example demonstrates that the optimization method presented here not only enables the design of shells composed of a reduced number of layers, but also can significantly improve their performance. In this case, the maximum scattered field is approximately 15 dB smaller for the optimized three-layer cloak than for the 100-layer discretization of the coordinate transformation cloak. Figure 3 illustrates the contribution of these scattered fields to the total fields:

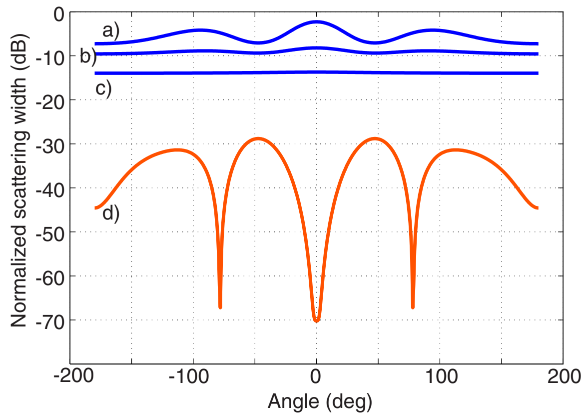

FIG. 2. (Color online) The scattering width normalized to the object diameter for the three-layer (a), nine-layer (b), and 100-layer (c) approximations of the analytical cloak compared to the optimized cloak (d).

the optimized version perturbs the incident field considerably less than the staircase approximation to the analytical cloak. We also notice that, even though a μ_ϕ of 47 in the innermost layer of the shell is difficult to achieve with current materials, it is still significantly lower than what would be required for the nine-layer (μ_ϕ=139) or 100-layer (μ_ϕ=1539) approximations of the analytical cloak.

From a sensitivity point of view, random perturbations of up to 1% applied simultaneously to all the optimized ϵ and μ parameters given in Table I change the scattering width by approximately ±5 dB for most angles. This makes the optimized shell fairly sensitive to changes in the desired parameters, but even with these perturbations the performance of the optimized shell is significantly better than that of the discretized analytical shell. We also emphasize the importance of choosing a good starting point in the BFGS optimization algorithm. If, for instance, we choose all the initial permittivity and permeability components to be unity, the final values given by the iterative algorithm result in a shell that, even though it reduces the overall scattering compared to the bare cylinder, has a maximum scattering cross section that is 30 dB bigger than the optimized cloak specified in Table I.

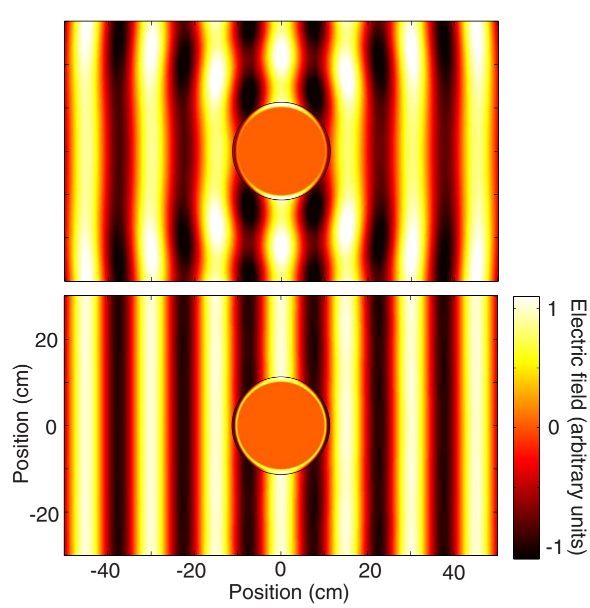

FIG. 3. (Color online) Total electric field for the nine-layer approximation of the analytical cloak (top) and the optimized three-layer cloak (bottom).

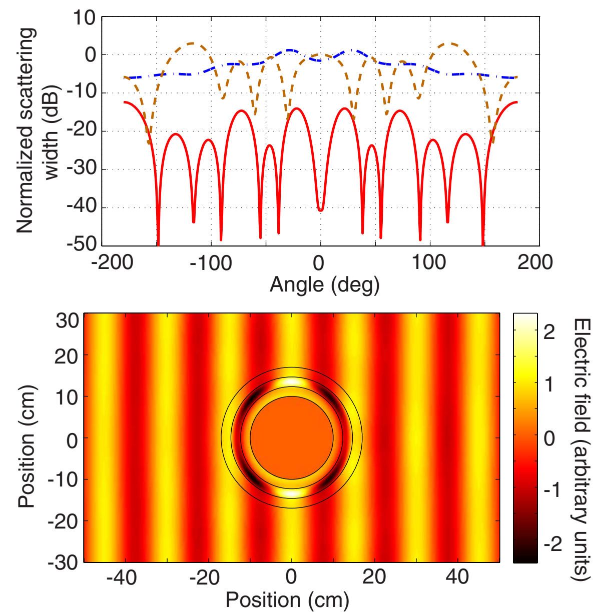

FIG. 4. (Color online) Top: The scattering width vs angle for the three-layer approximation of the analytical cloak (dash-dotted), nonmagnetic cloak (dashed), and three-layer optimized nonmagnetic cloak (solid). Bottom: The total fields inside and around the nonmagnetic optimized cloak.

The optimization procedure presented above shows that for an object of 1.33 wavelengths in diameter, a three-layer optimized cloak is more than 100 times better than a three-layer approximation of the analytical cloak. If we double the size of the object (i.e., 2.66 wavelengths in diameter), the improvement provided by optimization of three layers is only a factor of 7. With another factor of 2 (i.e., 5.32 wavelengths) optimization yields a factor of 2 improvement. This size dependence of the optimization improvement is expected; as the object becomes larger, we need to minimize increasingly more coefficients in the Hankel function expansion of the scattered field [see Eq. (3)]. Three layers do not provide enough degrees of freedom to dramatically improve the cloaking of a five-wavelength-wide object. However, by increasing the number of layers, we expect to obtain significant improvements through optimization for larger objects.

Optimization adds a great deal of flexibility to the constraints that can be imposed on cloaking shells. Equation (1) shows that for thin shells, the required values of μ_ϕ inside the innermost layers have to be large (in the previous example, it is μ_ϕ=47 inside the innermost layer), which can be an obstacle to fabrication. These extreme values can be reduced through optimization and still yield significant improvements over the discretized analytical shell. We demonstrate this in the following example by designing a nonmagnetic cloak that can potentially be implemented at optical frequencies.

TABLE II. Optimized material parameters for the three-layer nonmagnetic cloak.

	Initial guess			Optimized parameters		
Layer	μ_z	ϵ_r	ϵ_ϕ	μ_z	ϵ_r	ϵ_ϕ
1	1	0.064	5.90	1	0.10	4.49
2	1	0.40	5.90	1	1.38	9.99
3	1	0.80	5.90	1	0.70	0.89

From a practical perspective, even though it requires only three layers, the optimized shell described above is hard to fabricate: one needs to control both ϵ and μ; in addition, these parameters need to be quite big (for example, μ_ϕ needs to be 47 in the innermost layer), which is hard to achieve at high frequencies. For this reason, there is significant interest in designing cloaking shells that have certain properties that make them easier to fabricate. Some approximations to ideal parameters have been devised [2,5] to facilitate physical realizability, but most of these sacrifice performance to a high degree.

One such set of parameters that avoids using the harder to fabricate magnetic materials and can be used to create shells able to conceal objects detected using TM (transverse magnetic) polarized waves (only μ_z, ϵ_r, and ϵ_ϕ are relevant for this polarization) is [5]

$$\epsilon_r = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right)^2, \quad \epsilon_\phi = \left(\frac{b}{b-a}\right)^2, \quad \mu_z = 1. \quad (5)$$

Considering the same object as before (i.e., $a=10$ cm), Fig. 4 (top) shows the scattering width versus angle, $\sigma(\phi)$, for a relatively thick cloaking shell of outer radius $b=17$ cm made of a medium whose parameters are given by the above equations (dashed curve) compared to that of the three-layer approximation of the analytical cloak (dash-dotted curve). We chose a slightly thicker shell than before in order to obtain easier to achieve material parameters inside it. Neither of these approximations achieves especially good performance.

Optimization can dramatically improve the performance of a three-layer nonmagnetic cloak. The initial guess was given by Eqs. (5), and we added the additional constraint to simplify fabrication that no component of permeability or permittivity could be larger than 10. The resulting optimized cloak material parameters are given in Table II and the computed scattering width is shown in Fig. 4 (top). This optimized three-layer cloak gives an approximately 15 dB reduction in the scattering width over the cloak described by Eqs. (5), and Fig. 4 (bottom) illustrates the total electric field distribution around the shell and shows its good performance. We note that, as expected, stricter requirements on ϵ increase σ and consequently decrease to some extent the performance of the shell. For example, if we require $\epsilon \leqslant 5$, then $\max_\phi \sigma$ reaches 0 dB, and for $\epsilon \leqslant 8$, $\max_\phi \sigma \approx -5$ dB.

In conclusion, we present a practical optimization approach used to design layered shells of anisotropic materials that significantly reduce the scattering width of an object. This approach yields better performance and can also provide simpler material parameters compared to cloaking shells designed with coordinate transformation theory. As an example, we find that an optimized three-layer shell can perform much better than a 100-layer approximation to the analytical cloak. We also used this approach to design a three-layer nonmagnetic cloak made of relatively low permittivity anisotropic materials realizable even at optical frequencies that behaves significantly better than comparable designs.

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