Frequency tunable electromagnetic metamaterial using ferroelectric loaded split rings

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Measurements of a frequency tunable magnetic metamaterial using metallic split rings loaded with barium strontium titanate thin film capacitors are presented. The resonant frequency of this medium is voltage tunable across a 140 MHz band centered at 1.75 GHz. *S*-parameter measurements in a microstrip waveguide reveal that the effective relative permeability of the slab has a roughly Lorentzian shape that reaches minimum values between −2 and −3 for biases from 0 to 5 V. The permeability of the slab can tune between positive and negative values, making it useful in applications requiring a state switchable magnetic permeability. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2898575]

Since Veselago's theoretical study of negative index media[1] and the study of Pendry *et al.* of wire structures[2] and split ring resonators (SRRs),[3] artificial electromagnetic materials have been developed that exhibit unique characteristics. The majority of previous research in metamaterials has examined passive linear devices with fixed parameters. Control of the effective electromagnetic parameters of a metamaterial is possible through externally tunable components. Studies have examined the ability to control the response of individual particles using tunable devices such as varactor diodes,[4–6] photoconductive semiconductors,[7] and barium strontium titanate (BST) thin films.[8] Zhao *et al.* reported a multielement tunable magnetic medium using SRRs immersed in liquid crystals, which requires that the SRRs be enclosed in leak-proof packaging.[9] In the experiment, a small (2%) tunable range was achieved. He *et al.* reported a tunable negative index medium using copper wires and ferrite sheets.[10] The negative permeability behavior is completely dependent on the location and bandwidth of the ferrimagnetic resonance, which restricts the negative index band. In addition, a coil or permanent magnetic is needed to supply the magnetic field bias for tuning. A medium of BST-loaded SRRs is an alternative and convenient approach since it encapsulates all of the tunability within the SRR circuit, making large-scale printed circuit board fabrication of such structures simple and practical. Such a tunable medium could find use in many applications since the ability of a material to switch between reflective, transparent, and absorptive states is useful in microwave devices and components such as rf power splitters/combiners, filters, tunable phase shifters, and electronic beam-steering antennas.

We demonstrate experimentally a multielement, frequency tunable magnetic metamaterial composed of SRRs loaded with voltage tunable BST thin film capacitors. BST (BaSr TiO$_3$) is a perovskite ferroelectric crystalline material that exhibits a tunable electric permittivity when biased with a dc electric field. The resonant frequency of each ring is

tuned by applying a bias voltage across each film. The tight element-to-element tolerance of BST thin films makes it an attractive material for use in a tunable metamaterial. Several unit cells were created using 8 μm^2 BST thin films (50/50 BST: stoichiometry Ba$_{0.5}$Sr$_{0.5}$TiO$_3$), part of a capacitor die provided by the Gennum Corporation. The thin films were deposited at a thickness of 1100 Å on platinum electrodes, subsequently interconnected with aluminum and gold on a 1 mm thick alumina substrate. A nine cell slab composed of these BST-loaded SRRs was constructed with dimensions shown in the center photograph of Fig. 1. Small metal pins were used to form a dc bias bus that provided equal bias to each unit cell in addition to giving the slab mechanical support. Gold wire bonds (roughly 1 μm in diameter) were used to connect the thin film pads to the copper rings, and two surface mount 127 kΩ resistors were used to isolate the dc bus from the SRR. The fabricated slab was resonant with center frequency f_0=1.75 GHz and could tune over roughly a 140 MHz band from 1.67 to 1.81 GHz. Since the BST thin film requires a dc bias voltage to tune its capacitance, another capacitor in series with the BST thin film must be used to prevent shorting the bias voltage through the ring. The left photograph of Fig. 1 shows the placement of this isolation gap capacitance C in series with the BST capacitor C_{BST}. The value of C will affect the zero-bias resonant frequency as well as the tunable range of the SRR particle. We define the tunable range as

$$\delta = \frac{1}{2\pi\sqrt{LC}} \left| \frac{1}{\sqrt{\dfrac{C_{BST}}{C_{BST} + C}}} - \frac{1}{\sqrt{\dfrac{C_{BST}\Delta}{C + C_{BST}\Delta}}} \right|, \qquad (1)$$

where L is the self inductance of the SRR trace, C is the capacitance of the gap, and C_{BST} is the zero-bias capacitance of the BST thin film. The dimensionless parameter Δ takes into account the decrease in capacitance with increasing bias, where typically $\Delta \approx 0.5$ for Ba$_{0.5}$Sr$_{0.5}$TiO$_3$ and $\Delta \approx 0.25$ for Ba$_{0.7}$Sr$_{0.3}$TiO$_3$.[11] Given that we are designing a ring that must be electrically small ($\sim\lambda_0/10$), L is constrained and the resonant frequency is dominated by the series combination

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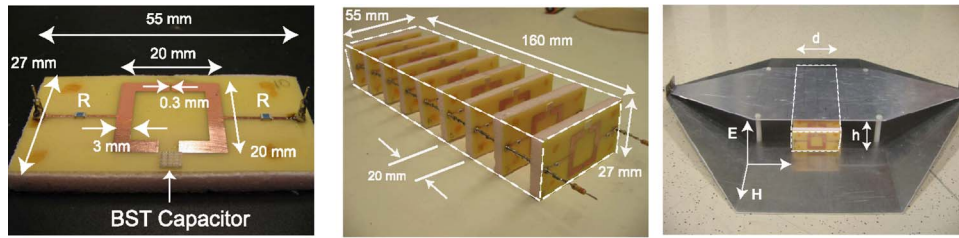a)Electronic mail: cummer@ee.duke.edu.

FIG. 1. (Color online) Left: View of an individual unit cell of the tunable slab. The resistor $R(127 \text{ k}\Omega)$ is used to isolate the split ring from the bias lines. Center: Photograph showing the fabricated tunable magnetic metamaterial slab used in experiment. Right: Photograph of the tunable slab inserted in the microstrip waveguide for effective permeability retrieval, where $d=55$ mm and $h=30$ mm.

of C and C_{BST}. Since we seek a design that responds strongest to an incident magnetic field, we want to maximize the area enclosed by the ring while minimizing its self inductance.[12] The full wave electromagnetic field solver Ansoft HFSS was used to iteratively find the dimensions of such a ring that would be resonant between 1 and 2 GHz, and the final design is shown in Fig. 1. The copper ring is 20 μm thick and sits on a 250 μm thick FR4 substrate. Ansoft Q3D EXTRACTOR was used to estimate the circuit parameters of this ring design: $L \approx 45$ nH, $C \approx 0.24$ pF, and $R \approx 0.25 \ \Omega$. The $Ba_{0.5}Sr_{0.5}TiO_3$ voltage-dependent capacitance was used in the simulation, where measurements of a single BST thin film capacitor was made by the Gennum corporation using an HP4284A LCR meter at 1 MHz. High frequency measurements of the same devices using rf probes show that, for a given bias voltage, the capacitance is uniform across our frequency range. It was found that the capacitance tunes from 1.84 pF (0 V) to 0.91 pF (5 V), and this was confirmed experimentally since the observed resonant frequencies of the SRRs matched closely with the simulation. The capacitance tunes down with increasing bias since the permittivity varies with the applied electric field E as $\epsilon_r \sim E^{-2/3}$ for bulk BST.[13,14] A parametric sweep in HFSS was performed using measured values of the single BST sample, and the resulting transmission curves (S_{21}) for various BST biases are shown in the left panel of Fig. 2. Using $\Delta=0.5$, $L=45$ nH, $C=0.24$ pF, and $C_{BST}=1.84$ pF yields $\delta \approx 142$ MHz. These simulation results support the roughly 8% tunable bandwidth obtained in the experiment. Magnitude and phase measurements of the transmission and reflection coefficients were used to extract the effective electrical constitutive parameters of the medium.[15] A microstrip waveguide (shown in the right photograph of Fig. 1) of height $h=30$ mm (Transverse Electromagnetic Mode (TEM) for $f<5$ GHz) was used to produce a TEM wave normally inci-

dent on the tunable SRR slab. The microstrip waveguide tapers toward the ports to provide a 50 Ω impedance everywhere. Since almost all of the field energy is confined in between the two plates of the waveguide in a linearly polarized uniform TEM wave and our slab fills the entire cross section of the microstrip, our system is equivalent to having a uniform plane wave normally incident on the slab in free space and, thus, the standard free-space extraction procedure[15] is valid for our system. To further confirm the validity of the retrieval procedure for our microstrip system, we performed an extraction on a rexolite slab (with known dielectric constant of 2.53 and low loss within our frequency range) that completely filled the transverse section of the waveguide, and the retrieved permittivity matched very well with the tabulated value of 2.53. The BST-loaded SRR slab was biased from 0 to 5 V at 1 V steps, and the resulting experimental S_{21} shown in the left panel of Fig. 2 is in close agreement with the simulation results. The experimental insertion loss is lower than in the simulation, but this raises no concerns since the ideal case of the simulation assumes an array of identical resonators. In reality, slight variations in the loss and resonant frequency of each BST-loaded SRR yield a transmission magnitude slightly different from the simulated transmission. The slab tuned approximately over a 140 MHz band, in excellent agreement with $\delta=142$ MHz predicted using Eq. (1). The effective magnetic permeability μ_r follows the Lorentzian form given by

$$\mu = \mu_0 \left(1 + \frac{F\omega^2}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}} \right), \tag{2}$$

where $\mu_r = \mu/\mu_0 = \mu_r' - j\mu_r''$, F is the oscillator strength of the medium, and Q is the quality factor of the medium, approximately equal to the quality factor of an individual unit cell.[16]
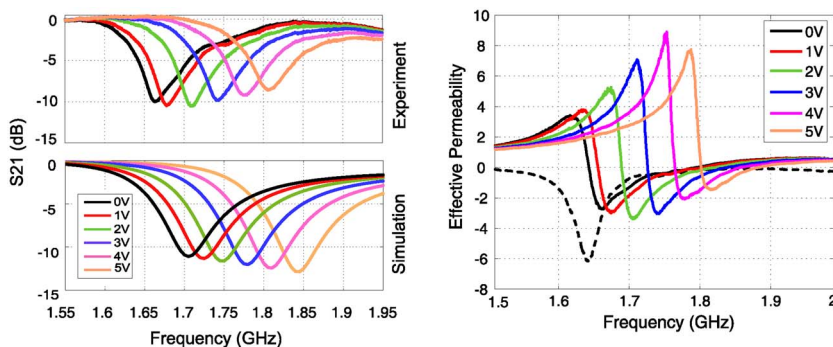

FIG. 2. (Color online) Left: S_{21} vs bias voltage for both simulation and experiment for biases from 0 to 5 V. Right: Extracted permeability for the nine cell slab used in the experiment. The imaginary part μ_r'' is shown for a 0 V bias, where the magnetic loss tangent $\tan \delta_m \approx 0.6$ at $\mu_r' = -1$.
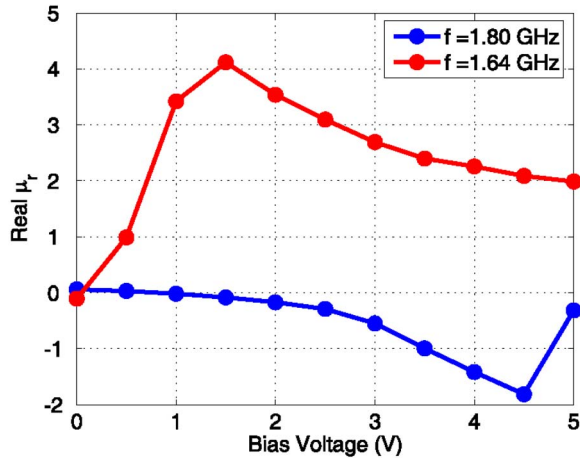
FIG. 3. (Color online) Experimental results showing the values μ_r' can attain at two frequencies for biases between 0 and 5 V. This plot shows that the medium has the capability to switch between positive and negative permeability states.

From the right plot of Fig. 2, it is apparent that the retrieved permeability curves can be closely characterized by the Lorentzian form of Eq. (2) throughout the entire tuning band. Notice how the individual BST-loaded ring resonances tune uniformly with applied bias, which is a function of the minimal variability between individual BST capacitors. This tight control allows the entire medium to tune as if all the resonators were identical, which allows for a meaningful interpretation of effective losses and tunability.

Figure 3 shows the range of μ_r' of the tunable slab for two frequencies. This figure shows the ability of the metamaterial slab to tune across a range of positive (2–4) and negative (−2–0) effective permeabilities (μ_r'). This property of the tunable metamaterial medium renders it useful in applications requiring a material capable of tuning continuously between magnetic permeability states. As can be inferred from Fig. 3, waves inside the medium are either propagating ($\mu_r' > 0$) or evanescent ($\mu_r' < 0$) depending on the frequency and applied bias of the medium.

The performance of this tunable metamaterial is limited primarily by the intrinsic properties of the BST capacitors. The figure of merit for losses in our medium is the magnetic loss tangent magnitude, $|\tan \delta_m| = |\mu_r''/\mu_r'|$. From the properties of a single unit cell, we can estimate the minimum achievable loss tangent (occuring at $\mu_r' = -1$) using[16]

$$\min|\tan \delta_m| \approx 4/FQ, \qquad (3)$$

where F and Q are the resonator oscillator strength and quality factor, respectively. For the nine cell medium, $F = 0.1$ (determined analytically from the cell geometry) and $Q = 60$ (the measured average of all nine cells) yields $\min|\tan \delta_m| \approx 0.66$, in close agreement with the measured value of 0.6 (shown in the right panel of Fig. 2). The measured Q of an individual cell with the same trace, capacitor, and wirebond but without the BST capacitor was approximately 150, indicating that the internal BST losses are the dominant source of loss in this cell. We also observe an increase in effective loss with increasing bias voltage, which is due to the increase in effective series resistance of the BST thin film from the well

documented Q-rollover phenomenon that results in an increase in dielectric conductivity (loss) with applied electric field.[17]

The tunable range of these particles is also limited by the BST capacitors. To achieve a resonant frequency about 1.75 GHz from a particle about ten times smaller than a wavelength, a total capacitance of about 0.25 pF is needed. BST has a large dielectric constant and even physically small capacitors such as what was used here have capacitances (~ 1.8 pF at 0 V bias for our case) significantly larger than this. According to Eq. (1), it is difficult to achieve a large δ if $C \ll C_{BST}$. More tunability could be realized at frequencies lower than 1.75 GHz using these BST capacitors.

Experimental results demonstrate that a frequency tunable magnetic metamaterial using BST-loaded SRRs can be realized. A tunable slab with center frequency $f_0 = 1.75$ GHz tuned over 140 MHz ($\sim 8\%$ bandwidth), and the effective magnetic permeability μ_r' of the medium reached minimum values of −2 to −3 with a negative band spanning nearly 240 MHz (13.7% bandwidth) across the range of biases. Losses in the magnetic metamaterial were reasonable, where the loss tangent magnitude at $\mu_r' = -1$ was about 0.6. We identified the BST thin film capacitors as the primary source of loss in the medium, and this suggests that improvements in BST thin film fabrication to yield lower dielectric losses will allow for tunable magnetic metamaterials with smaller loss tangents. This demonstration offers great promise to future applications employing tunable metamaterials since we have shown that proper unit cell design can yield a medium with rings that can tune uniformly with applied bias.

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