APPLIED PHYSICS LETTERS VOLUME 85, NUMBER 20 15 NOVEMBER 2004

Wave fields measured inside a negative refractive index metamaterial

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(Received 2 June 2004; accepted 21 September 2004)

Measured spatial variations of field phase and amplitude versus position inside a wire/split ring resonator negative refractive index metamaterial are presented that show that the phase velocity inside a negative index material points toward the source and consequently that phase velocity and energy flow are reversed in such a medium. These measurements directly demonstrate the fundamental behavior of electromagnetic waves in a negative refractive index material. The same internal field measurements are also used to accurately and reliably measure the effective bulk properties of the negative index metamaterial. © *2004 American Institute of Physics.*
[DOI: 10.1063/1.1823595]

Negative refractive index behavior in properly designed metamaterials has been experimentally demonstrated through transmission maxima at theoretically expected frequencies[1] and through measurements of negative beam refraction at an interface of positive and negative index materials.[2–4] These measurements are essentially incontrovertable evidence that negative refractive index materials can be built.

These measurements have all been based on electromagnetic fields outside the negative index material (NIM), and consequently the fundamental behavior of the wave fields inside a NIM has only been inferred in experiments. A measurement that shows directlythat phase velocity and energy flow are in opposite directions is possible from internal NIM wave fields. Such a measurement would be experimental confirmation of the fundamental physics of wave fields in a NIM. But beyond providing insight and completing the physical picture of NIMs, internal field measurements enable the probing of important parameters not easily accessible through external field measurements, such as measuring the effective bulk properties with minimal ambiguity, and determining where in the metamaterial NIM the fields behave as though the medium is a continuous medium.

In this letter, we present measurements of wave fields inside a NIM metamaterial that show directly that the phase velocity inside the NIM points toward the source and is consequently antiparallel to the energy flow. The wave fields behave as though the metamaterial NIM is a continuous medium only one unit cell from the edge of the metamaterial. It is further shown that the internal fields can be matched to theoretical calculations to measure the effective bulk electromagnetic properties (specifically, permittivity and permeability) of the NIM in an overdetermined and thus robust and convincing way.

Our aim was to measure fields like those in Fig. 1, which shows the theoretical sinusoidal steady state electric field amplitude and phase spatial distribution in a three slab configuration (air/NIM/air) with a plane wave normally incident from air. The NIM considered here is slightly lossy and not perfectly matched to free space, resulting in the amplitude decay in the NIM and the partially standing wave in the first air section. If the NIM had relative permittivity and permeability $\epsilon_r = \mu_r = -1$, then the amplitude would be uniform with position and the phase would be linear in each medium. But even with loss and imperfect matching, the reversed phase velocity of the wave in the NIM is evident from the reversed phase gradient (since $v_p^{-1} = \omega^{-1} d\phi/dz$). This negative index behavior should be clear as long as the standing wave ratio in the NIM is not too high and thus the fields propagate in predominantly one direction.

Our experimental configuration was as follows. An HP8720C Network Analyzer is used to measure all of the *S* parameters and internal fields presented here. A 0.6 m straight section of aluminum WR340 rectangular waveguide (TE_{10} cutoff frequency is 1.7 GHz) was used to contain the NIM slab. Small access holes were drilled in the waveguide to provide access for a rigid coax probe inserted into the medium to measure electric field amplitude and phase. Rectangular flange adapters completed the input and output ports.

We designed and built a NIM slab to have a negative refractive index near 2.6 GHz, at the center of the single mode band of the waveguide. Figure 2 shows the relevant dimensions of the material inclusions and a photograph of the resulting wire–SRR–copper foil slab. Split ring resonators (SRRs)[5] were iteratively designed using Ansoft HFSS (with a target resonant frequency of 2.6 GHz) and were fabricated on low-loss Rogers TMM3 substrate. 30 AWG wire

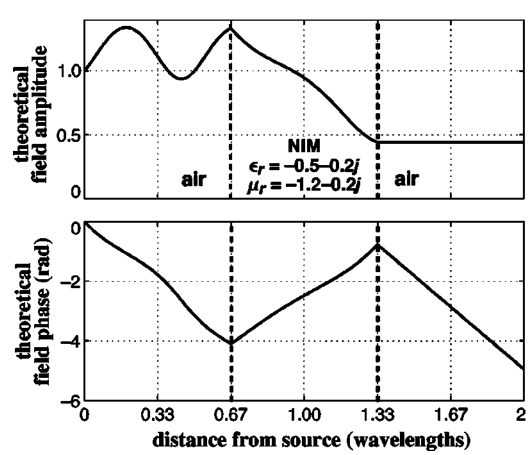

FIG. 1. Theoretical plots of the field phase and amplitude distributions in a representative normal incidence three slab (air/NIM/air) problem. The assumed relative NIM parameters are shown. The reversed phase gradient in the NIM is the targeted measurement and would confirm negative index behavior.

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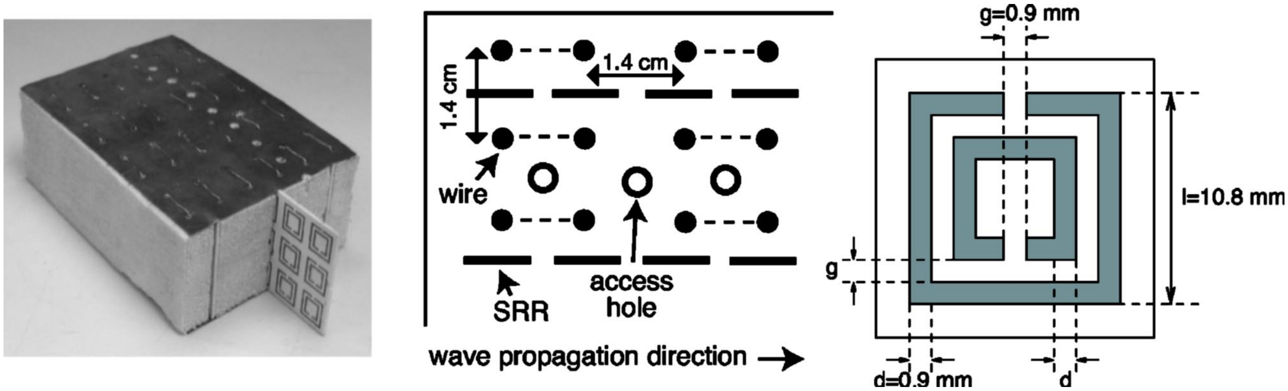

FIG. 2. Left: a photograph of the NIM slab used in these experiments. Middle: a plan view of the NIM slab. Right: dimensions of an individual SRR in the slab.

was looped through a styrofoam core (for structural stability) bounded by copper sheets to produce a thin wire medium with a unit cell dimension of 1.4 cm and therefore with an effective plasma frequency of 3.94 GHz.[6] The SRRs were then slid into grooves cut in the foam core. The resulting structure fits snugly in the waveguide, and the copper foil ensures that each wire makes reliable electrical contact with the waveguide walls. This slab was inserted in the center of the waveguide for measurement.

The field structure inside the wire–SRR metamaterial is complicated due to quasistatic fields associated with the electrically small inclusions. To minimize their effect on the measurements, the access holes are offset from the center of the waveguide to place the measurement points as far from the SRRs and wires as possible. Also, the holes are spaced one unit cell length of the metamaterial so that the measured fields are equivalent at each point.

For ease of fabrication, our NIM slab only contains SRRs that respond to the transverse magnetic field in the TE_{10} waveguide mode, and the material is thus anisotropic. The wires and wave electric field point in the $\hat{x}$ direction, and the SRRs are in the x–z plane to respond to the y-directed transverse magnetic field. If we model this as a homogeneous material with relative permittivity ϵ_x and relative permeabilities μ_z in the propagation direction and μ_y in the transverse direction (these are the only fields present in the TE_{10} mode), then, assuming $\exp(jwt)$ time dependence and a spatial dependence of $\exp(-jk_z z)\exp(-jk_y y)$, the resulting wave dispersion relation is

$$\frac{k_z^2}{\mu_y} + \frac{k_y^2}{\mu_z} = \epsilon_x \frac{\omega^2}{c^2} \quad (1)$$

and thus

$$k_z^2 = \epsilon_x \mu_y \frac{\omega^2}{c^2} - \frac{\mu_y}{\mu_z} k_y^2. \quad (2)$$

Therefore if $\mu_z = 1$ and $\epsilon_x, \mu_y < 0$, k_z is real and thus the phase and energy propagate in the directions they would in an isotropic NIM in the absence of a waveguide ($k_y = 0$). We conclude that the wave fields in our anisotropic material in a waveguide should exhibit negative refractive index behavior. Such a material is actually an indefinite "never-cutoff" medium.[7]

Figure 3 shows the measured frequency-dependent S parameters for this three slab configuration. Little wave energy enters the slab (i.e., S_{11} is near unity magnitude) except in a narrow frequency band centered at 2.63 GHz, precisely where the negative index band is expected. This transmission band suggests (but does not prove) negative index behavior between 2.60 and 2.65 GHz.[1] The series of transmission peaks between 2.30 and 2.45 GHz come from the well-understood transmission maxima when the material slab is an integer number of half-wavelengths thick. This was confirmed by observing the number of spatial field minima inside the slab, and is expected since the slab permeability increases sharply below the resonant frequency (and thus wavelength decreases sharply with frequency).

We then measured the electric field in each access hole of the waveguide (some in the NIM slab and some in the air) at the apparent negative index frequencies with the hope of seeing field amplitude and phase spatial distributions similar to the ideal in Fig. 1. Figure 4 shows the measured field phase and amplitude spatial distributions at 2.6175 GHz, the frequency with the best impedance match and thus most linear field phase. The field amplitude shows a small standing wave ratio (SWR) in both air regions ($\leqslant 2.2$), indicating that reflections from the terminating adapter and from the NIM slab are small and therefore each of the three regions contains one dominant wave direction.

The wave phase gradient in each slab (compare to Fig. 1) shows clearly that the air regions contain wave fields with a phase velocity pointed away from the source (i.e., the phase is progressively delayed away from the source), while the NIM slab contains wave fields with a phase velocity that points back toward the source (i.e., the phase is progressively advanced away from the source). The energy in all regions must flow away from the source; consequently this measurement shows that phase velocity and energy flow are antiparallel inside a NIM. By probing the internal physics of NIMs

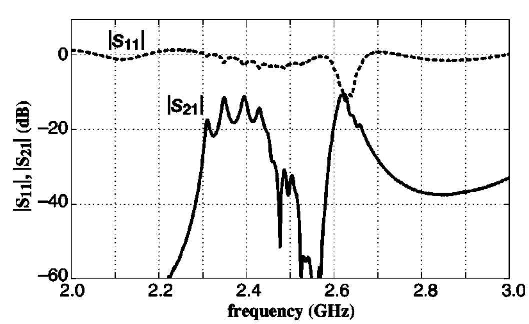

FIG. 3. Measured S parameters for the three slab waveguide configuration.

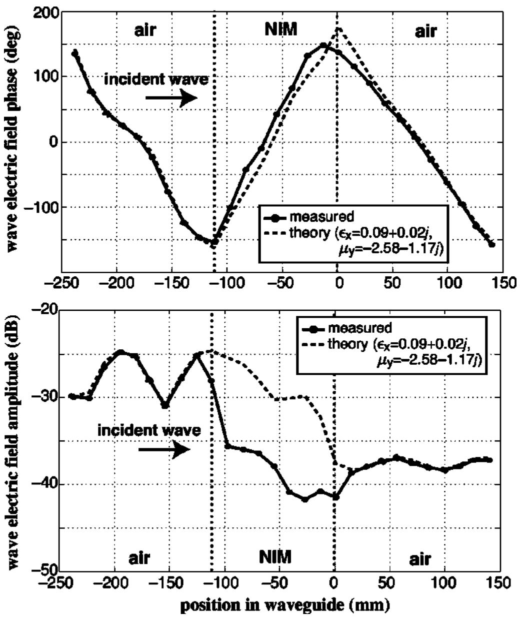

FIG. 4. Measured electric field phase (top) and amplitude (bottom) spatial distributions. The reversed phase gradient in the NIM confirms oppositely directed phase velocity and energy flow. Also shown are theoretical fits to the data used to measure the effective bulk permittivity and permeability of the NIM.

directly, this measurement shows that, inside the NIM, the wave fields behave as expected both theoretically and from previous NIM transmission experiments. These measurements also show that wave fields in the discrete metamaterial behave as if the medium were continuous even only one unit cell from the interface.

The effective electromagnetic properties (i.e., permittivity and permeability) are measureable from these fields. Beam refraction experiments can measure the real part of n with precision, but loss is harder to quantify. Controlled S parameter measurements can measure both real and imaginary parts of ϵ and μ, but they are exactly determined by the measurements and therefore it is hard to identify experimental errors or an incorrect assumed material model. In contrast, permittivity and permeability are overdetermined by the internal fields at many different positions and therefore at least some of these ambiguities can be resolved.

We estimate ϵ_x and μ_y in Eq. (1) (recall that $\mu_z=1$ in our material) through the following procedure. We first assume that the edges of the effective NIM slab are half of a unit cell length away from the wires at the edges of the slab. The SWR and the positions of field minima and maxima in the first free space section uniquely determine the complex wave impedance seen by the wave at the first air/NIM interface. Because the NIM is somewhat lossy (approximately 10 dB of one-way attenuation), multiple reflections in the NIM can be neglected and this impedance is the complex wave impedance of the NIM itself. The phase gradient and attenuation in the NIM determine the real and imaginary parts, respectively, of k_z. It is then straightforward to solve for the two unknown complex parameters (ϵ_x and μ_y) from the two measured complex parameters k_z and wave impedance. Using this as a starting point, we then iteratively adjusted ϵ_x and μ_y to give excellent agreement with the measured fields when all the complexities (including the reflection from the transmission-side adapter) are included. The initial parameters were modified less than 10% during the iteration. The resulting parameters are $\epsilon_x=0.09+0.02j$ and $\mu_y=-2.58-1.17j$, and Fig. 4 shows the good agreement between the measured fields and those that are theoretically expected from these parameters. The measured amplitude inside the NIM exhibits the correct spatial variation but is roughly 5 dB smaller than expected for a bulk medium. We attribute this to the quasistatic fields from the discrete inclusions; simulations indicate that the field amplitude can be enhanced or suppressed depending on exactly where in the unit cell the fields are sampled.

To roughly quantify the uncertainty in the measured parameters, we find changes of ±10% in μ_y are still essentially consistent with the measured fields. The permittivity ϵ_x is much less well-determined because if $|\epsilon_x|<1$, the $(\mu_y/\mu_z)k_y^2$ term dominates in the dispersion relation (2); consequently the influence of ϵ_x on the measured wave propagation parameters is weak. We attribute the lack of a strongly negative permittivity to the well-documented difficulties of controlling the permittivity in a wire–SRR medium. Houck *et al.* experimentally observed a wire–SRR medium response significantly different from that expected,[4] and Li *et al.* found that, empirically, two wires per SRR can be required to generate a significant negative permittivity.[8]

Note that even though our estimated ϵ_x has a weakly positive real part, the wave still propagates in a "backwards" sense and supports our measurements of the basic physics of wave propagation in a NIM. This is easily seen from the transverse wave impedance $E_x/H_y=\mu_y\mu_0\omega/k_z$ in this anisotropic medium. If μ_y and μ_z have opposite signs, then Eq. (2) shows that even for small positive ϵ_x, the wave number k_z is predominantly real. In this case, the sign of μ_y dictates the phase velocity and energy flow directions; for example, if k_z is positive and μ_y is negative, then the phase velocity is in the $+z$ direction, but the impedance is negative and thus the energy flow is in the $-z$ direction.

In conclusion, measured wave fields inside a negative index metamaterial explicitly demonstrate that phase velocity and energy flow are oppositely directed. Internal field measurements also show that wave fields in metamaterial composed of discrete inclusions behave like those in a bulk, continuous medium even a short distance (one unit cell length) from the edges of the metamaterial. These effective bulk properties (permittivity and permeability) of the medium can be extracted from the internal field measurements with reasonable precision and minimal ambiguity.

[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[2] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[3] C. G. Parazzoli, R. B. Greegor, K. Li, B. E. Koltenbah, and M. Tanielian, Phys. Rev. Lett. **90**, 107401 (2003).
[4] A. A. Houck, J. B. Brock, and I. L. Chuang, Phys. Rev. Lett. **90**, 137401 (2003).
[5] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[6] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, J. Phys.: Condens. Matter **10**, 4785 (1998).
[7] D. R. Smith and D. Schurig, Phys. Rev. Lett. **90**, 077405 (2003).
[8] K. Li, S. J. McLean, R. B. Greegor, C. G. Parazzoli, and M. H. Tanielian, Appl. Phys. Lett. **82**, 2535 (2003).