Dynamics of causal beam refraction in negative refractive index materials

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A finite difference simulation of a causally excited electromagnetic Gaussian beam incident on an interface between free space and a physically realizable negative refractive index material shows that negative refraction of finite beams does occur. A discontinuity in the phase normal direction is established very quickly when the beam front reaches the interface of positive and negative index materials. Once the beam enters the negative index material, the causal wave-front propates with the group velocity consistent with theory. During the transient portion of the beam development, the beam undergoes small direction changes, which can be explained as a consequence of higher frequencies present during the beam turn-on. After this transient, the theoretically predicted sinusoidal steady state is reached. © *2003 American Institute of Physics.*
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Negative refraction is the transmission of waves across a material boundary in which the perpendicular (to the interface) component of the wave vector changes direction. Veselago first predicted negative refraction at a sharp interface between positive and negative refractive index half spaces,[1] along with a variety of unusual effects that would occur in a negative index material. Negative refraction, and the resulting wave focusing, are no longer of just academic interest because of the physical constructability of these materials[2] and the observation of negative refractive index effects in periodic structures like photonic band-gap materials.[3,4]

Valanju *et al.* recently argued, with qualitative and mathematical discussion, that negative refraction only occurs in the sinusoidal steady state of pure plane waves and that negative refraction of a causal, finite beam does not occur in the dispersive materials in which a negative refractive index can exist.[5] Smith *et al.*, by noting the distinction between the directions of group velocity and interference pattern velocity, resolved this discrepancy and showed analytically that finite beams can refract negatively by considering the sinusoidal steady state of two interfering beams.[6]

The interaction of a causal, finite beam with an interface of a positive index material (PIM) and a negative index material (NIM) is complex and it is not obvious how the negatively refracted fields dynamically evolve in the NIM. It is shown below how negative beam refraction occurs dynamically (as opposed to in the sinusoidal steady state) at a PIM/NIM interface using a full-wave, finite difference approximation to the Maxwell equations in a causal and realizable negative refractive index material. This simulation shows that causal and finite beams refract negatively at such an interface, and that the transmitted wave fields refract negatively within a few wave periods of the beam front arriving at the interface. The rapid establishment of the wave-vector discontinuity at the material interface also shows how the qualitative argument against negative refraction in Fig. 1(b) of Ref. 5 does not apply.

We consider two dimensional ($\partial/\partial y=0$) fields in a two interface material geometry. The material properties are only z dependent. Free space ($n=1$) is present in the ranges $z<0.163$ m and $z>0.326$ m. Between $z=0.163$ m and $z=0.326$ m is a slab of lossy material with electric and magnetic plasma responses that give the medium a permittivity and permeability of

$$\epsilon=\epsilon_0\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2-j\omega\nu_e}\right), \tag{1}$$

$$\mu=\mu_0\left(1-\frac{\omega_{\mathrm{pm}}^2}{\omega^2-j\omega\nu_m}\right), \tag{2}$$

where ω_{pe} and ω_{pm} are the electric and magnetic plasma frequencies, respectively, and ν_e and ν_m are the electric and magnetic loss factors (or collision frequencies). We choose $\omega_{pe}=\omega_{pm}=\omega_p$ and $\nu_e=\nu_m=\nu$, resulting in a frequency-dependent index of refraction

$$n(\omega)=1-\frac{\omega_p^2}{\omega^2-j\omega\nu}. \tag{3}$$

We choose $\omega_p=2\sqrt{2}\pi\times10^{10}$ s^{-1} and $\nu=10^8$ s^{-1}, resulting in a medium with a refractive index of $n=-1-0.0032j$ at the 10-GHz source frequency. The resulting material is thus a physically realizable and causal medium that exhibits a bandwidth of slightly lossy negative refraction.

At the $z=0$ plane, an electric current sheet $J_y(x,0)=f(t)\exp[-(x/x_0)^2]\sin(2\pi\times10^{10}t+kx)$ is imposed, where $f(t)$ is a function that turns on smoothly in about six wave periods and k is a constant (122.7 rad/m) that shifts the beam direction approximately 36° away from the z direction. The smooth turn on reduces broadband dynamics and facilitates the interpretation of the simulation. A 10-GHz source frequency is chosen to facilitate comparison with experiments with negative index (or left-handed) materials, which have thus far only been constructed at microwave frequencies. The presented results are applicable to any wavelength and frequency through linear time and distance scaling.

We use the standard second-order leapfrog method[7] to simulate the electromagnetic fields in this domain. The electric and magnetic plasma responses are easily included through additional equations for electric and magnetic current using the differencing scheme of Ref. 8. This method is

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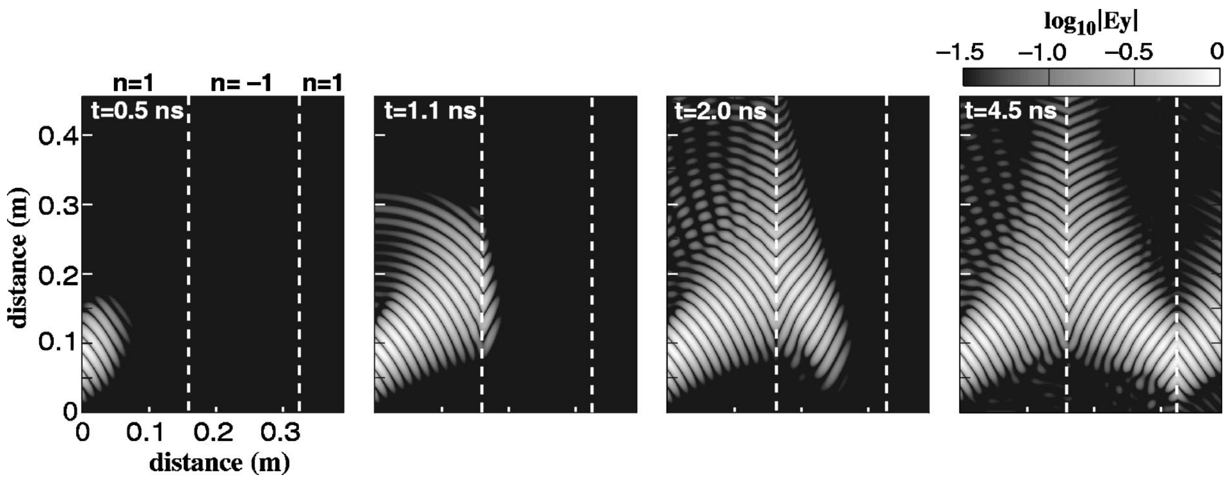

FIG. 1. Images of the instantaneous $|E_y|$ during the development of the Gaussian beam. The solid lines show the material boundaries. The beam refracts negatively very quickly at the two PIM/NIM interfaces and clearly shows that finite, causal beams are refracted negatively in realizable NIM materials.

nearly identical to that used in Ref. 9. The simulation uses a time step of $\Delta t = 1.5$ ps and a spatial step $\Delta x = 0.652$ mm, and a primary computational domain 1000 by 650 cells. The perfectly matched layer absorbing boundary condition[10] is used to simulate free-space open boundaries at $z = -0.0326$ m and $z = 0.391$ m. Because of the difficulty in designing stable absorbing boundary conditions in the two-plasma-response (TPR) material, we use periodic boundary conditions on the constant x edges of the computational domain. This makes the effective source a spatially periodic series of gaussian beams. Simulations with a variety of periodicities show that, for the presented simulation, the individual beams are narrow enough that they do not interact significantly. Thus the presented results are essentially those for a single gaussian beam.

The source is turned on at $t = 0$ and the simulation run for 3000 time steps (45 wave periods) to reach a sinusoidal steady state. Figure 1 shows the spatial distribution of the amplitude of the instantaneous $E_y(t)$ at four different times during the beam development. Note that the variations are the 10-GHz sinusoidal oscillations, not spatial variations from the interference of two different frequencies. It is clear that the beam refracts negatively in the NIM almost instantaneously after arriving at the first PIM/NIM interface. The beam front propagates through the NIM and arrives at the second NIM/PIM interface, where it again refracts negatively. Although it is not evident in these field images, the wave phase runs backwards in the NIM, as expected, such that the phase fronts in the beam propagate back towards the interface. The beam front moves in the opposite direction away from the interface, however, implying oppositely directed phase and energy velocities, as predicted theoretically.[1]

There is essentially no reflection at the interface due to the matching of the incident and refracted angles and wave impedances that occurs at an $n = 1/n = -1$ interface. Lens-like focusing is also evident in the simulated fields, as the beam diverges in free space and converges in the $n = -1$ material. This focusing property of an $n = -1$ slab due to the negating of the free space propagation phase was noted by Veselago[1] and is distinct from the theoretical perfect focusing first demonstrated by Pendry.[11]

Valanju *et al.*[5] argued qualitatively that negative beam refraction could not occur because beam phase normal would have to bend in a nonphysical way in order for the beam to smoothly change propagation direction across the PIM/NIM interface. The second panel in Fig. 1 shows that even in a dynamic situation, the fields arrange themselves so that the phase normal direction is discontinuous at the PIM/NIM interface, rather than smoothly varying. The non-steady-state negative beam refraction dynamics are thus very similar to those observable in ordinary (positive) refraction.

An interesting effect observable in this simulation is that the beam direction appears to change with time. In the third panel of Fig. 1, the beam angle angle of refraction appears larger than the beam angle of incidence, but after steady state is reached in the fourth panel, these two angles are equal, as expected. One can argue qualitatively that the higher temporal frequencies (those between 10 and 14.2 GHz for the simulation parameters) contained in the turn-on portion of the beam experience a refractive index between -1 and 0, and thus refract to a larger angle relative to the interface normal. Additionally, some propagating transverse spatial wave numbers contained in the turn-on portion of the beam will become evanescent because of the same smaller index magnitude, and thus the beam direction close to the interface can be different than the beam direction elsewhere before the sinusoidal steady state is reached. Once the steady state is reached, then all wave numbers experience the same index ($n = -1$) and do not change from propagating to evanescent across the interface.

A close look at individual phase fronts shows that their velocity is the group velocity predicted from the material dispersion relation. Figure 2 shows a closer view of the electric field spatial distribution at two times during the beam development in the NIM. Marked on each image is the position of a single wave peak at each of the two times. In the 1.05 ns between the images, this peak moves from (0.187,0.098) to (0.282,0.048). This propagation corresponds to 1.02×10^8 m/s, which is almost exactly the analytical group velocity of $c/3$ in a TPR medium at the $n = -1$ frequency {this is easily derived from Eq. (3) and the expression for group velocity $v_g = c/[d(n\omega)/d\omega]$}. The fields during the beam turn-on dynamics also show that the beam direction

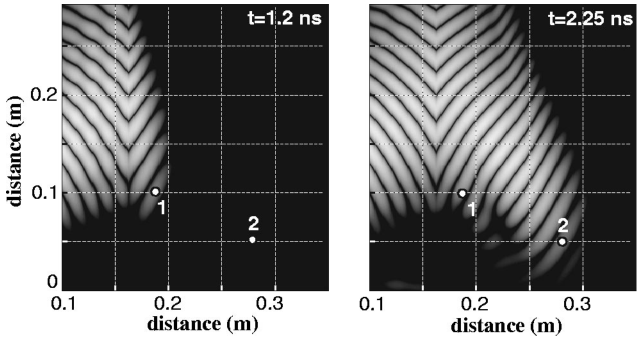

FIG. 2. Close view of $|E_y|$ at two points in time. A single wave peak travels from point 1 to point 2 during this period with a velocity almost exactly the theoretically expected group velocity of $c/3$.

and the phase normal direction do not appear to be parallel or antiparallel. This is especially clear in the third panel, in which the line through the center of the beam is not perpendicular to the individual phase fronts. The medium is isotropic and this is thus not a steady-state effect; rather, it is part of the causal and finite beam dynamics, through the reasons mentioned earlier, that disappears once sinusoidal steady state is achieved.

Simulations with varying degrees of loss (ν) show that the results are essentially identical for $\nu < 10^8$ s^{-1}. Higher loss introduces some beam attenuation through the TPR material, but does not fundamentally alter the observed negative refraction.

There are multiple causal material models that can produce a frequency band of negative refractive index,[12] and the TPR material simulated in this work is one of them. Some differences in the beam dynamics and group velocity would be present in negative index materials with different dispersion properties, but negative beam refraction occurs in all of them.

We conclude, on the basis of the presented electromagnetic simulations, that negative refraction does occur for causal finite beams. For a beam that turns on relatively slowly and thus does not have an especially broad bandwidth, the required phase-normal discontinuity at the PIM/NIM interface is established very quickly, and the beam front propagates through the NIM at the expected group velocity. Interesting small variations in the beam direction are observed during the transient portion of the field development, but the expected sinusoidal steady state is reached relatively quickly. We expect that these results and similar simulations can be used to help interpret efforts to experimentally demonstrate negative refraction with negative index (or left-handed) materials.[2]

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