Two-level Systems, Rabi Oscillations, and Composite Pulses

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Problem 1: Using the code you wrote for the time-evolution of a spin 1/2 particle in a magnetic field, we will now investigate what happens with pulse sequences. To do this, we will need to include the phase of the applied magnetic field and use $\vec{B}(t) = -B_1[\cos(\omega t + \phi)\hat{e}_x + \sin(\omega t + \phi)\hat{e}_y] - B_0\hat{e}_z$. This should be a simple modification to your existing code. The time-dependent Hamiltonian is now:

$$H = \frac{\hbar}{2}\begin{pmatrix} \omega_L & \omega_R e^{-i(\omega t+\phi)} \\ \omega_R e^{i(\omega t+\phi)} & -\omega_L \end{pmatrix} \tag{1}$$

We will use some terminology in the following work that is commonly used when discussing 2-level systems:

The pi-time t_π is commonly written as $t_\pi = \pi/\omega_R$, and a pulse of length t_π is known as a π-pulse. Similarly, a $\pi/2$-pulse is a pulse of half this duration. It's important to note that this is only true for a square pulse. More accurately, when the applied oscillating magnetic field is subject to a slowly varying (compared to ω) envelope function $g(t)$ such that a π- and $\pi/2$-pulse with corresponding pi- and pi/2-times, are defined such that:

$$\int_0^{t_\pi} w_R g(t)dt = \pi \qquad \int_0^{t_{\frac{\pi}{2}}} w_R g(t)dt = \frac{\pi}{2} \tag{2}$$

The general logic also holds true for other fractions of π. It's clear that, for the case of a constant envelope function, the π-time $t_\pi = \pi/\omega_R$, as expected.

(a) Apply two pulses to your system. First, apply a resonant ($\omega = \omega_L$) $\pi/2$-pulse. Then, wait a non-zero amount of time without applying any oscillating magnetic field B_1 (but maintain the constant magnetic field B_0). After this wait time, apply a second resonant $\pi/2$-pulse with the same phase as the first. Provide the excited state probability vs time, as well as the Bloch sphere animation. Be very aware of rounding errors that lead to incorrect pulse areas.

(b) Repeat (a), but now apply the seconds pulse $\pi/2$ out of phase with the initial pulse. Provide the excited state probability vs time, as well as the Bloch sphere animation.

(c) The coordinate representation $\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$ on the Bloch sphere for a given state $|\psi\rangle$ is given by $x = \langle\psi|\sigma_x|\psi\rangle$, $y = \langle\psi|\sigma_y|\psi\rangle$, and $z = \langle\psi|\sigma_z|\psi\rangle$, where the σ_i are the Pauli spin matrices. Show that:

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r} \tag{3}$$

for some $\vec{\omega}$. Explain what this result means, intuitively. How does this allow you to make sense of your results for parts (a) and (b)?

(d) Modify your result from (c) for the case where we removed the explicit time-dependence of the Hamiltonian. You don't have to re-do all the algebra. Again, explain intuitively what it means.

(e) Composite pulses are often used in NMR and atomic physics to compensate for experimental errors in pulse length or frequency mismatch ($\omega - \omega_L \neq 0$). There exists an enormous amount of literature on various pulse sequences and their effects on errors, but as an example here we will look at the relatively simple, but commonly used, BB1 family of compensated pulses. It is specifically designed to compensate for pulse length errors, which are most often the primary source of error in an atomic physics experiment.

BB1 works by performing three pulses in addition to the desired native pulse. These pulses can be applied before or after the native pulse. The correction pulse lengths have pulse lengths, in order, π, 2π, π, and their phases are ϕ, 3ϕ, ϕ. Here $\phi = \arccos(-\frac{\theta}{2\pi})$ and θ is the desired pulse angle.

Implement the BB1 sequence for a $\pi/2$-pulse using your code, and show how it outperforms the native $\pi/2$-pulse if all applied pulse angles are too large by 20%.